Optimal Control Scheme on Anaerobic Processes in Biodigesters

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This paper deals with the development and test optimal control strategies that adapt their operations to confront some unstable circumstances in biological biomass processes, such as variations in the operation conditions, broadly affecting the industrial scale implementation. The complexity on biological reactions inside the process requires appropriate models that represent the phenomena in order to design and build control systems to optimize the process, and keep them away from disturbances. A two-reactor chain mass-balance model has been considered as an approximation to represent biological reactions between the different groups of bacteria immerse on anaerobic process. The model incorporates biokinetic rates, which plays a central role in the dynamical growth speed of bacteria population. An optimal control strategy is proposed to test the improvement on reaction times, compared with standard Proportional-Integral-Derivative (PID) controllers, widely used on the industry. The results show the optimal control strategies react in advance the disturbances; keeping the process on desirable operational conditions. A performance index is used to measure the advantages to use optimal control strategies. Simulations were performed on Matlab.

1. Introduction

Anaerobic digestion treatment plant consists in biological process where organic material, biomass (composed by carbohydrates, proteins, lipids and more complex compounds), is transformed by bacteria to biogas (methane and carbon dioxide) in absence of oxygen (Elizalde-Blancas et al., 2013). Anaerobic digestion demonstrates several advantages compared with aerobic treatment process; has a high capacity to degrade concentrated and difficult substrates, produce very few sludges, and requires little energy to operate (Xu et al., 2016). However, despite the advantages, this type of plants are not extensively spread out around the globe. Some problems affect the performance and stability of biogas produced due the uncontrolled feedstock purity, inadequate elimination of contaminants, economic construction-feasibility, constantly clogged pipes, and massive sediments in the digester (Ferreira et al., 2018). However, the challenge is to maintain the process on desirable operating conditions; where some variables affect considerably the produced biogas. The main problem are the different components of biomass, which may inhibit the anaerobic degradation process and thus may cause problems in downstream, decreasing the biogas production efficiency. Specifically, metabolic products inhibit the process; ammonia ($\text{NH}_3$), ammonium ($\text{NH}_4^+$), dinitrous oxide ($\text{N}_2\text{O}$), nitrite ($\text{NO}_2^-$) and nitrate ($\text{NO}_3^-$) (Akindele et al., 2018). The four groups of microorganisms falls into specific degradation steps, they interact based on different physical and chemical requirements inside the reactors, where the coexistence produce synergetic interactions. Due to the process complexity, this paper is focused on design strategies to develop technical solutions, where control strategies and reliable mathematical models which represent the complexity of the organic matter, guarantee appropriate operational conditions. Efforts to propose methods for parameter estimation and model validation has been a notable activity of research from the half of the past century (Kythreotou et al., 2014). However, the non-linearities and the difficulties to analyse data collected from different phenomena results in diverse modelling approaches, giving a high uncertainty variability on the parametric estimation (André et al., 2018). This paper presents a standard model
as an approximation of the biological reactions in biomass. Biokinetic growth of bacteria is considered as a resource to represent the phenomena. The aim is to use this plant as an starting point to test the optimal controllers and to evaluate the improve in performance compared with the standard PID controllers used on the industry (Pachauri et al., 2017).

In recent years, some strategies based on modelling optimization have been proposed to address anaerobic digestion treatment plants (Ghanimeh et al., 2018). Linear Quadratic Regulator (LQR) plays a crucial role in optimal control systems with a lot of applications including chemical process control, airplane flight control, motor control, etc. (Li et al., 2008). The LQR controllers focuses on minimizing the quadratic function (or cost function) through a state feedback (controlled variables), allowing to keep track references in a suitable way. This method looks for a control signal ($u(t)$) to be applied on states in order to minimize the system bad performance. The LQR control strategies typically works out on dynamic systems basically in centralized and decentralized schemes. In decentralized control, subsystems are controlled by themselves in a useful way that makes easier to find and amend mistakes. However, the lack of information between systems results in performance decrease, which by the way, induce to provide not as finest operational conditions as it would be desirable. In contrast, centralized control is not frequently used in the industry due to different issues: Inflexibility, obstacles on both physical and organizational requirements, and challenges detecting and solving problems in real time. However, performance of this type of control is optimal because it considers the dynamics of the system as a whole: the flow of information between subsystems. Previous works (Shuai et al., 2011, Christian and Jason, 1988) make comparison of centralized and decentralized control performances, resulting in the fact that centralized control has better performance than decentralized one. The plant study consists of two continuous stirred tank reactors (CSTRs). This process was studied by Venkat (2006) using a Model Predictive Control (MPC). In this article, the goal is to prove centralized control implementation is better than decentralized control schemes and both of them better compared with the PID control strategies. In the literature so far, despite emphasizing the differences on performance for these controller strategies, there is not information on studies evaluating comparatively their performances. This fact makes this article of great interest to quantify the controller performances. The paper is organized as follows. The section 2 the chemical model with bionkinetics considerations is described. Then, in section 3, the centralized and decentralized Linear Quadratic Regulator (LQR), and PID control strategies are described. In section 4, the simulation results are shown. Finally, in section 5, concluding and further research remarks are presented.

2. Two reactors

Consider a chemical process represented by a mathematical model from mass-energy balance equations.

![Figure 1. Two reactor chain with biological biomass process](image)

A linear model in state space will be developed around an operating point; this linear model will enable the implementation the LQR control schemes. The problem shall be addressed to control the final product of the process (output mass fraction of B) based on centralized and decentralized control strategies.

2.1 Model description

A plant consisting of two continuous stirred tank reactors (CSTRs) is considered (see Figure 1). $F_{A_1}$ and $F_{A_2}$ flows go into CSTR-1 and CSTR-2 tanks respectively, with associated concentration and temperature, as well as the recirculation flow coming from the flash distiller CSTR-1 which is related with valve $D$ opening. Heat exchangers $Q_1$, $Q_2$ along with the above variables will provide to the plant a direct effect on heights, concentrations and temperatures in each subsystem. This result is clear on $F_1$ and $F_2$ are the output flows. In each of the CSTRs, the desired product $B$ is produced through the irreversible first order reaction $A \xrightarrow{k_1} B$. An undesirable side reaction $B \xrightarrow{k_2} C$ results in the consumption of $B$ and in the production of the unwanted side
product C. Such reactions are directly linked to temperature changes in the process. The product stream from CSTR-2 is the product resulted of the process. It is assumed that the mathematical modelling of the system allows the following nonlinear ordinary differential equations (ODEs) as showed by Venkat (2006):

CSTR-1

\[
\frac{dH_1}{dt} = \frac{1}{\rho A_1} [F_A + D + F_1] \\
\frac{dx_{A_1}}{dt} = \frac{1}{\rho A_1 H_1} \left[ F_A \left( x_{A_1} - x_{A_1} \right) + D \left( x_{A_2} - x_{A_1} \right) \right] - k_1 x_{A_1} \\
\frac{dx_{B_1}}{dt} = \frac{1}{\rho A_1 H_1} \left[ F_A \left( x_{B_1} - x_{B_1} \right) + D \left( x_{B_2} - x_{B_1} \right) \right] - \theta_a \\
\frac{dT_1}{dt} = \frac{1}{\rho A_1 H_1} \left[ F_A (T_0 - T_1) + D (T_d - T_1) \right] - \omega_a
\]

where:

\[
\begin{align*}
\theta_a &= k_1 x_{A_1} + k_2 x_{B_1} \\
\omega_a &= \frac{1}{c_p} \left[ k_1 x_{A_1} \Delta H_1 - k_2 x_{B_1} \Delta H_2 \right] - \frac{Q_1}{\rho A_1 c_p H_1} \\
k_1 &= k_{11} e^{\left( \frac{E}{RT_1} \right)} \\
k_2 &= k_{22} e^{\left( \frac{E}{RT_1} \right)} \\
x_{A_1} &= a_A x_{A_1} + a_B x_{B_1} + a_C (1 - x_{A_1} x_{B_1}) \\
x_{B_1} &= a_A x_{A_1} + a_B x_{B_1} + a_C (1 - x_{A_1} x_{B_1})
\end{align*}
\]

CSTR-2

\[
\frac{dH_2}{dt} = \frac{1}{\rho A_2} [F_A + F_{A_2} + F_2] \\
\frac{dx_{A_2}}{dt} = \frac{1}{\rho A_2 H_2} \left[ F_A \left( x_{A_2} - x_{A_2} \right) + F_{A_2} \left( x_{A_2} - x_{A_2} \right) \right] - \eta \\
\frac{dx_{B_2}}{dt} = \frac{1}{\rho A_2 H_2} \left[ F_A \left( x_{B_2} - x_{B_2} \right) + F_{A_2} \left( x_{B_2} - x_{B_2} \right) \right] - \theta_b \\
\frac{dT_2}{dt} = \frac{1}{\rho A_2 H_2} \left[ F_A (T_1 - T_2) + F_{A_2} (T_0 - T_2) \right] - \omega_b
\]

where:

\[
\begin{align*}
\eta &= k_{1m} x_{A_2} \\
\theta_b &= k_{1m} x_{A_2} + k_{2m} x_{B_2} \\
\omega_b &= \frac{1}{c_p} \left[ k_{1m} x_{A_2} \Delta H_1 - k_{2m} x_{B_2} \Delta H_2 \right] - \frac{Q_2}{\rho A_2 c_p H_2} \\
k_{1m} &= k_{11} e^{\left( \frac{E}{RT_1} \right)} \\
k_{2m} &= k_{22} e^{\left( \frac{E}{RT_1} \right)}
\end{align*}
\]

The inputs considered on the model are \( D[Kg/s] \) the flowrate from a feedstock, \( F_{A_1}[Kg/s] \) the flowrate from CSTR-1, \( F_{A_2}[Kg/s] \) the flowrate from CSTR-2, \( Q_1[KJ/s] \) the cooling duty from CSTR-1 and \( Q_2[KJ/s] \) the cooling duty from CSTR-2. The states are \( H_1[\text{m}] \) the liquid level in the CSTR-1, \( H_2[\text{m}] \) the liquid level in the CSTR-2, \( T_1[K] \) the temperature in CSTR-1, \( T_2[K] \) the temperature in CSTR-2, \( x_{A_1} \) the product stream mass fraction of \( A \) in CSTR-1, \( x_{A_2} \) the product stream mass fraction of \( A \) in CSTR-2, \( x_{B_1} \) the product stream mass fraction of \( B \) in CSTR-1, \( x_{B_2} \) the product stream mass fraction of \( B \) in CSTR-2. The model parameters are given in Table 1. The desired operating variables (height, product mass fractions and temperatures) are
obtained from the system in the steady-state after setting the above ODEs equal to zero. The next step is to achieve a linear model in state space as follows (Venkat, 2006):

\[ \dot{x} = Ax + Bu \]
\[ y = Cx + Du. \]

### Table 1. Model parameters on the steady-state

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>0.15 ( Kg m^{-3} )</td>
<td>( A_2 )</td>
<td>3 ( m^2 )</td>
<td>( \alpha_A )</td>
<td>3.5</td>
</tr>
<tr>
<td>( x_{B_1} )</td>
<td>0</td>
<td>( \Delta H_2 )</td>
<td>-40 ( KJ/K_g )</td>
<td>( \alpha_B )</td>
<td>0.1</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>0.3 ( m^2 )</td>
<td>( x_{A_1} )</td>
<td>1</td>
<td>( \alpha_C )</td>
<td>0.5</td>
</tr>
<tr>
<td>( C_p )</td>
<td>0.15 ( KJ/K_g K )</td>
<td>( T_d )</td>
<td>313 ( K )</td>
<td>( D )</td>
<td>4.6374 ( K_g/s )</td>
</tr>
<tr>
<td>( x_{B_{21}} )</td>
<td>0</td>
<td>( k_{11} )</td>
<td>0.2 ( 1/s )</td>
<td>( F_{A_1} )</td>
<td>0.0354 ( K_g/s )</td>
</tr>
<tr>
<td>( \Delta H_2 )</td>
<td>-40 ( KJ/K_g )</td>
<td>( k_{22} )</td>
<td>0.018 ( 1/s )</td>
<td>( F_{A_2} )</td>
<td>1.5289 ( K_g/s )</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>313 ( K )</td>
<td>( E_1 )</td>
<td>50</td>
<td>( Q_1 )</td>
<td>-200 ( K_g/s )</td>
</tr>
<tr>
<td>( x_{A_2} )</td>
<td>1</td>
<td>( E_2 )</td>
<td>150</td>
<td>( Q_2 )</td>
<td>-200 ( K_g/s )</td>
</tr>
</tbody>
</table>

3. Linear quadratic regulator (LQR)

The design of state feedback controller requires the establishment of a new state (error signal) for each reference variable to be introduced into the system, in order to fix up tracking references. An LQR is implemented for a chemical plant with integrators that can reduce the steady-state error to zero when there are set point changes (Espinosa, 2003):

\[ \dot{e} = y_{ref} - y \]

Based on the previous expression it is possible to obtain the representation of the open loop system as follows:

\[
\begin{bmatrix}
\dot{x}_e \\
y
\end{bmatrix} =
\begin{bmatrix}
A & 0 \\
-C & 0
\end{bmatrix}
\begin{bmatrix}
x_e \\
y
\end{bmatrix} +
\begin{bmatrix}
B \\
-D
\end{bmatrix} u +
\begin{bmatrix}
0 \\
1
\end{bmatrix} y_{ref}
\]

\[ y = Cx + Du \]

\[ k = [k_c \ k_i] \]

The aim of the optimal controller without constraints shall be to minimize the objective function shown below (Espinosa, 2006).

\[ J = \int_{t_0}^{t_f} \frac{1}{2} |x(t)'Qx(t) + u(t)'Ru(t)| dt \]

There are other objective functions, but perhaps the most popular is the quadratic one by two reasons: to be an easy reference and to be directly related to the system energy content. While for controlling the plant with a decentralized LQR it is required to design a single controller for each subsystem, the centralized LQR control the system as a whole. A set of variables are defined in order to apply the LQR control scheme to the system: For CSTR-1, the input variables are \( F_{A_1} \) and \( Q_1 \), the state variables are \( H_1, x_{A_1}, x_{B_1}, T_1 \). In addition, it has been chosen \( H_1 \) and \( T_1 \) as the controlled variables. For CSTR-2, the input variables are \( F_{A_2} \) and \( Q_2 \), the state variables are \( H_2, x_{A_2}, x_{B_2}, T_2 \) and \( H_2 \) and \( T_2 \) as controlled variables. Heights and temperatures were chosen as variables to control since they improve the system efficiency. When decentralized control is used, there are two LQR controllers working separately without any flow information between them; changes in non-controlled inputs on each subsystem will be seen as disturbances. The LQR controller schemes will be established by the feedback state. The energy required to conduct the system to a specific set point will be established by the tuning parameters \( Q \) and \( R \) (the weighting matrices that balance the relative importance of input and state in the cost function problem optimization); those are correlation matrices which denote the worth of controlling some states over other ones. The matrices \( Q \) and \( R \) were chosen to prioritize reference tracking; this is achieved assigning the values of \( Q \), especially those related with error, to be greater than the other values in \( R \). It is important to highlight the fact that the design of the controller is based on reference tracking and disturbances rejection, seeking by these means variables which quickly stabilize the operating point without
the capability for controlling the output variables. The PID control scheme have been designed based on the Ziegler-Nichols tuning criteria.

4. Results

Mass fraction of $B$ is the output of the process, in consequence, the most important operating point to evaluate is $x_{B_{2}}$, the mass fraction of $B$ inside on CSTR-2. The simulation results consider a decrease on 40% in the mass fraction of $B$ ($x_{B_{2}}$) at 5 seconds of simulation. The level in CSTR-1 ($h_{1}$) and temperature in CSTR-2 ($T_{2}$) were analyzed to measure the system operation. The results show the reaction of the variables between the controllers. The centralized LQR control scheme has better disturbance rejection than the decentralized one and PID control strategy, however, the results do not show substantial evidence to determine which is more efficient.

![Figure 2. Variables H1 and T2 after change set point mass fraction B on CSTR-2](image)

Consider now the inputs of the system. When $x_{B_{2}}$ decreases, it is necessary to increase $F_{A}$ to regulate $H_{1}$. The change in both: flows and concentrations, generates changes on the temperature level on the systems, regulated by the action of the heat sources $Q_{1}$ and $Q_{2}$. The effort to control (inputs) is higher on the decentralized control scheme as well as on the PID control scheme than the centralized one; resulting in an increase of energy. So there is a need to quantify mathematically the performance of the control schemes and compare them using the following index (Venkat, 2006):

$$J_{cost} = \frac{1}{k} \sum_{j=0}^{k} \sum_{i=1}^{M} \frac{1}{2} [x_{i}(j)'Q_{i}x_{i}(j) + u_{i}(j)'R_{i}u_{i}(j)]$$  \hspace{1cm} (12)

where $k$ is the number of steps obtained in the simulation, $M$ the inputs, the states, and $Q$ and $R$ matrices on each single step. It is necessary to compare them with an additional test which is shown below (Venkat, 2006):

![Figure 3. Variables Q2 and XB2 after change set point mass fraction B on CSTR-2](image)
The following table shows the difference between the performance index between the controllers.

<table>
<thead>
<tr>
<th>Controller</th>
<th>$J_{cost}$</th>
<th>$J_{cen}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centralized</td>
<td>923.7585</td>
<td>-</td>
</tr>
<tr>
<td>Decentralized</td>
<td>973.8955</td>
<td>5.42</td>
</tr>
<tr>
<td>PID</td>
<td>1123.9873</td>
<td>19.67</td>
</tr>
</tbody>
</table>

The decentralized LQR control scheme results in an increase of 5.42% in the performance index compared with the centralized LQR control scheme. The PID controller decrease 19.67% in the performance index compared with the centralized LQR control scheme.

5. Conclusions

The simulation results were designed in order to make both regulation and reference changes. The results shown the control schemes based on optimization have high performance compared with PID controllers. The centralized LQR controller increase the performance index on 5.42%, however, the difference with the PID control was 19.67%. Therefore, here there is a starting point for future research in the study and improvement of control schemes on biological systems using biomass. This type of control on large-scale systems have two main advantages: designing simplicity and low computational cost. To evaluate the performance of each one of them, it is possible to conclude that it is not enough to examine the output as well as the energy consumption of the process.

References


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