



# Optimizing a Tri-objective Fuzzy Agricultural Food Load Planning Problem for Point-to-point Short-haul Road Transportation with a Pareto-based Discrete Firefly Algorithm

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In this paper, we define the mathematical model of a tri-objective fuzzy agricultural food load planning problem, which is a blend of the multi-objective optimization, the multidimensional knapsack problem, and the multiple knapsack problem under fuzziness. For the imprecise model, profit and shelf life for each agricultural food product are fuzzified and represented by triangular fuzzy numbers, and then defuzzified by the graded mean integration representation method. To solve this problem, we develop a pareto-based discrete firefly algorithm. In this algorithm, both the initial configuration and movements of fireflies are redefined. Meanwhile, a two-phase repair operator is proposed to guarantee the feasibility and quality of solutions. The  $(\mu+\lambda)$ -PAES (Pareto Archived Evolution Strategy) with a 3-dimensional adaptive grid algorithm is adopted to optimize the population of fireflies. Finally, the effectiveness of the proposed algorithm is evaluated based on 14 problem instances.

## 1. Introduction

Given that refrigeration stops or reduces the rate at which biochemical changes such as browning reactions and pigment degradation occur in food, many agricultural food products should be transported by refrigerated vehicles (James and James, 2010). Thus, an optimal load plan is critical for maintaining the food quality and improving the profitability of the shipment. However, in the field of food engineering, which has a close relationship with chemical engineering (Chen and Yoo, 2006), the focus has been kept on vehicle routing problem while studies about load planning are quite few. Therefore, it is worthwhile to model the agricultural food load planning problem for point-to-point short-haul transportation and develop its solution approach.

It is obvious that this problem can be regarded as a complex variant of the knapsack problem. So far, there has been a large amount of literature concerning the algorithms for solving knapsack problems. These algorithms can be broadly divided into two major fields as exact algorithms and heuristic algorithms. Given that exact algorithms are usually infeasible when the problem is too complex, we will use a non-exact algorithm to get near-optimal solutions.

As a famous meta-heuristic algorithm, the firefly algorithm is applicable to almost all engineering areas (Fister et al. 2013). Yang (2012) proposed the multi-objective firefly algorithm by extending the basic firefly algorithm which was proposed by himself to solve multi-objective optimization problems. Given that the original firefly algorithm was developed to solve continuous optimization problems, some modifications of it have been proposed to solve discrete optimization problems such as the two-dimensional min-cost covering problem (Lu and Wang, 2015) and the fuzzy cold storage problem (Lu and Wang, 2016).

In terms of multi-objective optimization, the Pareto Archived Evolution Strategy (PAES) can be used as a simple baseline algorithm (Knowles and Corne, 2000). It uses a non-dominated solutions archive to determine whether a candidate solution should be accepted or not. As an important variant of PAES, the  $(\mu+\lambda)$ -PAES aims to mutate one of the  $\mu$  current solutions to generate  $\lambda$  mutants per iteration. Meanwhile, an adaptive grid algorithm which is an integral part of PAES is usually used as the crowding procedure to optimize the population of fireflies. Therefore, we attempt to propose a pareto-based discrete firefly algorithm (PDFA) which integrates the firefly algorithm with the  $(\mu+\lambda)$ -PAES in this paper.

The rest of the paper is organized as follows. In Section 2, the mathematical model of the tri-objective fuzzy agricultural food load planning problem is given. In Section 3, the pareto-based discrete firefly algorithm is designed and presented in detail. In Section 4, computational performance of the proposed algorithm is presented through a series of simulation experiments. Finally, conclusions are presented in Section 5.

## 2. Problem formulation

### 2.1 Mathematical description of the proposed model

It is assumed that there are many agricultural food products to be transported by some single-temperature refrigerated trucks for short-haul transportation. Let  $N = \{1, 2, \dots, n\}$  be the set of products and  $M = \{1, 2, \dots, m\}$  be the set of vehicles. The profit, weight, volume, shelf life, and optimal temperature range for storage of the product  $j$  are denoted by  $\tilde{p}_j$ ,  $w_j$ ,  $v_j$ ,  $\tilde{s}_j$ , and  $[\underline{t}_j, \bar{t}_j]$  respectively. Shelf life is the time during which an agricultural food product remains safe and nutritious for consumption under appropriate storage conditions. Due to the impreciseness of the profit and the shelf life for each of the agricultural food product,  $\tilde{p}_j$  and  $\tilde{s}_j$  are given as fuzzy numbers.  $q_i^w$ ,  $q_i^v$  and  $T_i$  are weight limit, volume limit, and compartment temperature of the refrigerated truck  $i$ . The binary decision variable  $x_{ij}$  denotes whether the product  $j$  is assigned to the refrigerated truck  $i$  or not. Thus, the agricultural food load planning problem is formulated as follows:

$$\max I_p = \sum_{i=1}^m \sum_{j=1}^n \tilde{p}_j x_{ij} \quad (1)$$

$$\min I_s = \sum_{i=1}^m \sum_{j=1}^n \tilde{s}_j x_{ij} \quad (2)$$

$$\max I_m = \sum_{i=1}^m \sum_{j=1}^n x_{ij} \text{sim}(T_i, [\underline{t}_j, \bar{t}_j]) \quad (3)$$

subject to:

$$\sum_{j=1}^n w_j x_{ij} \leq q_i^w, \sum_{j=1}^n v_j x_{ij} \leq q_i^v, \quad \forall i \in M \quad (4)$$

$$\sum_{j=1}^n w_j > \sum_{i=1}^m q_i^w, \sum_{j=1}^n v_j > \sum_{i=1}^m q_i^v, \quad (5)$$

$$w_j < q_i^w, v_j < q_i^v, \quad \forall i \in M, \forall j \in N \quad (6)$$

$$x_{ij} \in \{0,1\}, \quad \forall i \in M, \forall j \in N \quad (7)$$

$I_p$  and  $I_s$  stand for the total sum of profits and shelf lives of all products to be loaded respectively. They state that the total profit is to be maximized and products with short shelf lives are preferred.  $I_m$  aims to minimize the difference between the optimal temperature range of those products and the compartment temperature of their refrigerated trucks. Let  $\underline{\varphi} = \min \{ \underline{t}_j \mid j \in N \}$  and  $\bar{\varphi} = \max \{ \bar{t}_j \mid j \in N \}$ . To measure the similarity between a range of a crisp value, we use the following formula which was proposed by Slonim and Schneider (2001).

$$\text{sim}(T_i, [\underline{t}_j, \bar{t}_j]) = \frac{\int_{\underline{t}_j}^{\bar{t}_j} \left(1 - \frac{|x - T_i|}{\bar{\varphi} - \underline{\varphi}}\right) dx}{\bar{t}_j - \underline{t}_j} = \begin{cases} 1 - \frac{\bar{t}_j + \underline{t}_j - 2T_i}{2(\bar{\varphi} - \underline{\varphi})}, & T_i \leq \underline{t}_j \\ 1 - \frac{(\bar{t}_j - T_i)^2 + (\underline{t}_j - T_i)^2}{2(\bar{\varphi} - \underline{\varphi})(\bar{t}_j - \underline{t}_j)}, & \underline{t}_j < T_i < \bar{t}_j \\ 1 - \frac{2T_i - \bar{t}_j - \underline{t}_j}{2(\bar{\varphi} - \underline{\varphi})}, & \bar{t}_j \leq T_i \end{cases} \quad (8)$$

Constraint (4) enforces the capacity of weight constraint and the capacity of volume constraint. Generally, this model can be regarded as a 0-1 integer programming problem with constraints (5)-(6). The decision variable is defined in constraint (7).

### 2.2 Integrating graded mean integration representation method into the proposed problem

In the tri-objective fuzzy agricultural food load planning problem, we fuzzify  $\tilde{p}_j$  and  $\tilde{s}_j$  as triangular fuzzy numbers. Hence, they can be written as  $\tilde{p}_j = (p_j - \Delta_{jl}, p_j, p_j + \Delta_{jr})$  and  $\tilde{s}_j = (s_j - \Delta'_{jl}, s_j, s_j + \Delta'_{jr})$ , where  $\Delta_{jl}$ ,  $\Delta_{jr}$ ,  $\Delta'_{jl}$ ,  $\Delta'_{jr}$  are determined by decision makers.

Graded mean integration representation (GMIR) method is a popular defuzzification technique proposed by

Chen and Hsieh (1998). The GMIR of  $\tilde{p}_j$  is denoted by  $\Phi_G(\tilde{p}_j)$  and defined as:

$$\Phi_G(\tilde{p}_j) = \frac{\int_0^1 h[L^{-1}(h)+R^{-1}(h)] dh}{2 \int_0^1 h dh} = \frac{\int_0^1 h \{ [p_j - (p_j - \Delta_{jl})]h + p_j - \Delta_{jl} + [p_j - (p_j + \Delta_{jr})]h + p_j + \Delta_{jr} \} dh}{2 \int_0^1 h dh} = \frac{6p_j - \Delta_{jl} + \Delta_{jr}}{6} \quad (9)$$

The GMIR of  $\tilde{s}_j$  can be calculated in a similar way.

### 3. Description of the pareto-based discrete firefly algorithm

#### 3.1 Encoding scheme

In this algorithm, we use the vector  $X = (x_1, x_2, \dots, x_n)$  to denote a solution. For example, let  $x_j = i$  if the agricultural food product  $j$  is assigned to the refrigerated truck  $i$ , and  $x_j = 0$  if the product  $j$  is assigned to none of those vehicles. The light intensity of a firefly is denoted by  $I = (I_p, I_s, I_m)$ .

#### 3.2 Initialization of firefly population

In the MOFA, initialized fireflies are distributed as uniformly as possible (Yang, 2012). In this procedure, we generate initial population by Equation 10 based on the greedy heuristic algorithm proposed by Hiley and Julstrom (2006). In order to do this, we firstly define the  $PRT(j)$  of an agricultural food product  $j$ :

$$PRT(j) = \frac{\frac{\Phi(\tilde{p}_j)}{\Phi(\max\{\tilde{p}_j | j \in N\})} + 1 - \frac{\Phi(\tilde{s}_j)}{\Phi(\max\{\tilde{s}_j | j \in N\})}}{\theta_1 w_j + \theta_2 v_j} \quad (10)$$

Next, let  $S = (s_1, s_2, \dots, s_n)$  be a sequence that sorts by Equation 10 in descending order. To avoid duplicate solutions, select each food product with a high probability to load. At the end of this procedure, add non-dominated solutions to an archive  $F$ . We use  $pop\_size$  to denote the population size.

#### 3.3 Movement of fireflies

We use the Hamming distance to evaluate the distance between firefly  $i$  and firefly  $j$ , which is stated as:

$$r_{ij} = \sum_{k=1}^n |\text{sign}(F_i.x_k - F_j.x_k)| \quad (11)$$

Let  $\varepsilon$  be a constant and  $m\_iter$  be the number of iterations. The movement of firefly  $j$  which is attracted to another more attractive firefly  $i$  is determined by  $\delta_1$  and  $\delta_2$ , where

$$\delta_1 = [\beta_0 \times \exp(-\gamma \times r_{ij}) \times r_{ij}] \quad (12)$$

$$\delta_2 = [\alpha^{m\_iter} \times \text{rand}(0, \varepsilon)] \quad (13)$$

Let  $F'$  be a firefly. The pseudocode of this procedure is shown in Algorithm 1.

Algorithm 1: Firefly movement of the PDFFA

1: $pop\_size \rightarrow border; \emptyset \rightarrow D; 0 \rightarrow dn, up;$	17: <b>else if</b> ( $F'.x_{D_i} \neq 0 \ \&\& \ F_i.x_{D_i} \neq 0$ )
2: <b>for</b> $i = 0 : border - 1$	18: $F_i.x_{D_i} \rightarrow F'.x_{D_i};$
3: <b>for</b> $j = 0 : border - 1$	19: <b>end for</b> $t$
4: <b>if</b> $i \neq j \ \&\& \ i, j < border \ \&\& \ F_i.I \neq F_j.I$	20: <b>for</b> $d = 0 : \delta_2$
5: $F_j \rightarrow F';$	21: $\text{rand}(1, m) \rightarrow F'.x_{\text{rand}(0, n-1)};$
6: <b>for</b> $k = 0 : n - 1$	22: <b>end for</b> $d$
7: <b>if</b> ( $F_i.x_k \neq F'.x_k$ )	23: call the repair operator
8: $D.add(k); r_{ij}++;$	24: <b>if</b> (no member of $F$ dominates $F'$ )
9: <b>end if</b>	25: <b>if</b> ( $F'$ dominates one or more members of $F$ )
10: <b>end for</b> $k$	26: discard members dominated by $F'$ ;
11: reorder the elements of $D$ randomly;	27: <b>end if</b>
12: <b>for</b> $t = 0 : \min(\delta_1, D.size())$	28: $F.add(F');$ update $border$ ;
13: <b>if</b> ( $F_i.x_{D_i}$ equals 0 && $dn < \delta_1/2$ )	29: <b>end if</b>
14: $0 \rightarrow F'.x_{D_i}; dn++;$	30: <b>end if</b>
15: <b>else if</b> ( $F'.x_{D_i}$ equals 0 && $up < \delta_1/2$ )	31: <b>end for</b> $j$
16: $F_i.x_{D_i} \rightarrow F'.x_{D_i}; up++;$	32: <b>end for</b> $i$

### 3.4 Repair operation

An abnormal encode individual may be obtained after moving. Therefore, we develop the repair operator which is shown in Algorithm 2 to guarantee the feasibility of solutions.

*Algorithm 2: Repair operation of the PDFA*

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1: for  $t = 0 : m-1$ 
2:   while ( $firefly.FW_t > q_t^w \parallel firefly.FV_t > q_t^v$ )
3:     randomly remove a product from the vehicle  $t$ ;
4:   end while
5:   if ( $q_t^w > firefly.FW_t \parallel q_t^v > firefly.FV_t$ )
6:     for  $k = 0 : n-1$ 
7:       if ( $firefly.x_{s_k} = 0 \ \&\& \ firefly.FW_t + w_{s_k} \leq q_t^w \ \&\& \ firefly.FV_t + v_{s_k} \leq q_t^v$ )
8:         Assign the product  $N_{s_k}$  to the vehicle  $t$ ;
9:       end if
10:      if ( $q_t^w - firefly.FW_t \leq \tau_1 \parallel q_t^v - firefly.FV_t \leq \tau_2$ )
11:        break ;
12:      end if
13:    end for  $k$ 
14:  end if
15: end for  $t$ 
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### 3.5 Density Optimization

When the degree of crowding increases, performance of the algorithm will significantly degrade and thereafter final solutions cannot be obtained within a reasonable time period. The density-estimation metric and the crowded-comparison operation in NSGA-II (Non-dominated Sorting of Genetic Algorithm-II) is a commonly used approach to cope with this problem. However, it consumes too much computational time in our tests while large-sized instances are conducted. Therefore, given that many grid-based evolutionary multi-objective optimization algorithms have demonstrated good performance to solve many-objective optimization problems (Yang et al. 2013), we use an adaptive grid algorithm to remove excessive fireflies from the crowded regions. When the population size exceeds  $Max\_pop$ , a map of grid with high adaptability is generated to envelope the population. The grid is divided into  $g^3$  cubes and only one solution can be retained in each cube except the three solutions with the largest  $I_p$ ,  $I_m$ , and lowest  $I_s$  respectively.

### 3.6 Termination condition

Repeat 3.3 - 3.5 until an iteration variable reaches the specified iteration limit  $Max\_it$ .

## 4. Algorithm Simulation

### 4.1 Computational Instances and Parameter Settings

To validate the proposed mathematical formulation and evaluate the performance of the PDFA, we conduct several experiments on different instances. We code the PDFA in C# and compile with Microsoft Visual Studio 2012 compiler in a PC having 3.20 GHz Intel G3420 processor and 8 GB RAM.

In the experiments, data are randomly generated for four refrigerated trucks and 3000 agricultural food products, where  $p_j$ ,  $w_j$ ,  $v_j$ ,  $s_j$ ,  $\underline{t}_j$ ,  $\bar{t}_j$ ,  $\Delta_{jl}$ ,  $\Delta_{jr}$ ,  $\Delta_{jl}'$ ,  $\Delta_{jr}'$  are generated with uniform distribution  $U(10,100)$ ,  $U(5,15)$ ,  $U(10,30)$ ,  $U(2,7)$ ,  $U(0,10)$ ,  $U(\underline{t}_j, 10)$ ,  $U(0, p_j/10)$ ,  $U(0, p_j/10)$ ,  $U(0.1, s_j/10)$ ,  $U(0.1, s_j/10)$ . Let  $q^w = \{3500, 4000, 4500, 5500\}$ ,  $q^v = \{8000, 8500, 9000, 10000\}$ ,  $T = \{2, 4, 6, 8\}$ . For the system parameters, we set  $\alpha = 0.9998$ ,  $\beta_0 = 1$ ,  $\gamma = 0.003$ ,  $\theta_1 = 2$ ,  $\theta_2 = 1$ ,  $\tau_1 = 9$ ,  $\tau_2 = 9$ ,  $\varepsilon = 10$ ,  $Max\_it = 10000$ . Given that  $pop\_size = 10-25$  is fairly suitable for most applications and  $pop\_size = 50$  can handle almost any problem (Gandomi et al., 2016), we set  $Max\_pop = 50$ . Given that balancing convergence and diversity is critical for the evolutionary multi-objective optimization, we set different values of  $g$  to find the relations between them.

### 4.2 Experimental Results

Computational results are shown in Table 1. In this table, the columns " $F_p^{best}$ ", " $F_s^{best}$ ", " $F_m^{best}$ " denote the best solution in terms of  $I_p$ , the best solution in terms of  $I_s$ , and the best solution in terms of  $I_m$  respectively. To visualize the diversity of solutions, Figure 1. illustrates the Pareto front of six problem instances. Furthermore,

Figure 2 depicts the convergence curves of the PDFA with  $Max\_it = 20000$  over the instance of No. 14 to further find the efficiency of the proposed algorithm.

Table 1: Computational results of PDFA for 14 instances

No.	n	m	g	pop_size	$F_p^{best}$	$F_s^{best}$	$F_m^{best}$	CPU(s)
1	2000	1	3	13	34280, 1633, 251	17603, 749, 162	28672, 1981, 325	371
2	2000	1	4	20	34948, 1624, 247	17631, 723, 166	28724, 1985, 327	751
3	3000	1	3	12	36299, 1580, 270	16710, 612, 161	32745, 1833, 313	352
4	3000	1	4	21	36643, 1679, 277	19998, 784, 177	30273, 1976, 350	898
5	2000	2	3	14	65094, 3748, 548	38683, 2015, 414	57442, 3532, 585	349
6	2000	2	4	19	65135, 3731, 550	41953, 2102, 429	59424, 3482, 591	872
7	3000	2	3	13	70623, 3787, 581	45301, 2103, 442	63620, 3424, 617	357
8	3000	2	4	21	68428, 3402, 566	46542, 2143, 451	62301, 3808, 658	817
9	2000	3	3	11	86899, 5390, 868	63859, 3880, 718	82905, 5715, 957	265
10	2000	3	4	13	87303, 5417, 877	60019, 3578, 699	83331, 5684, 963	518
11	3000	3	3	11	100071, 5750, 940	62956, 3346, 717	92044, 5624, 1004	443
12	3000	3	4	18	99830, 5517, 905	62372, 3236, 708	90599, 5519, 1004	928
13	3000	4	3	11	129924, 8007, 1299	87899, 5147, 1025	121931, 8059, 1484	382
14	3000	4	4	18	130041, 7978, 1302	86264, 5031, 1040	121112, 8061, 1508	974

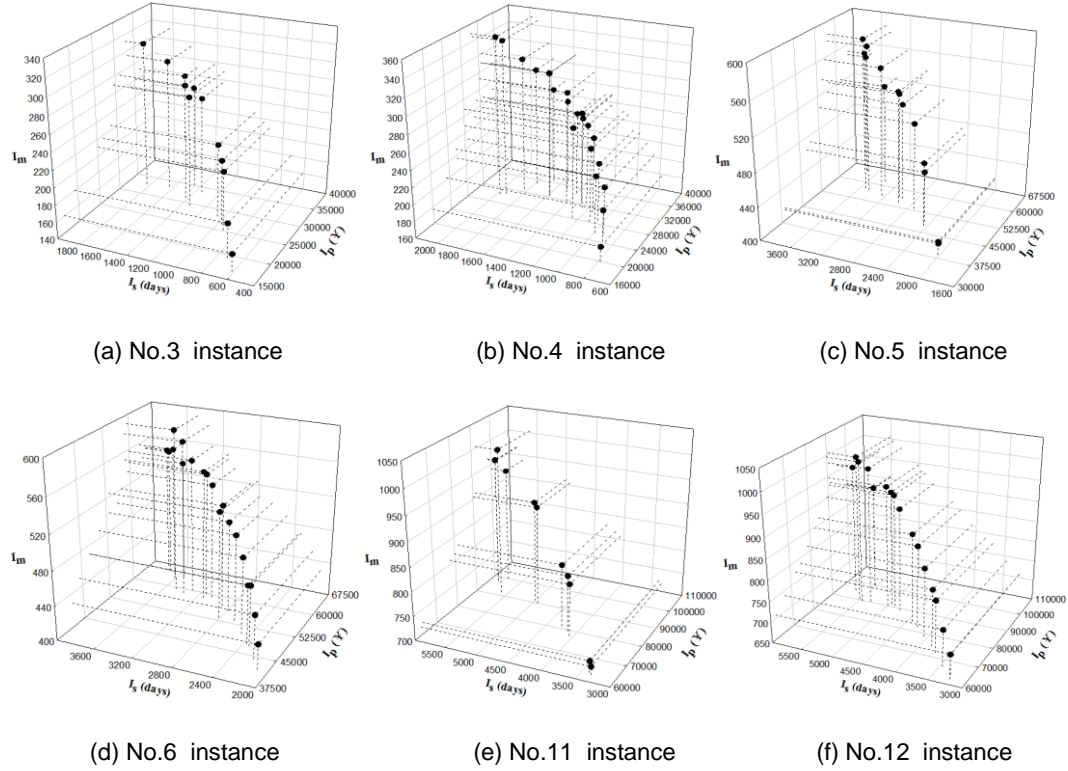


Figure 1: The Pareto front of 6 test problems for the PDFA.

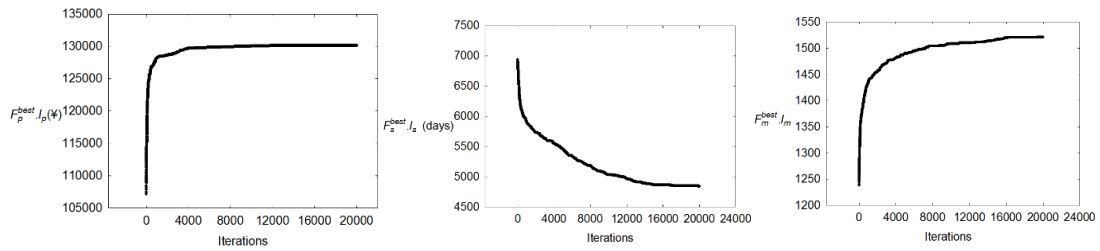


Figure 2: Convergence curves of the instance with  $n=3000, m=4, g=4, Max\_it=20000$ .

From Table 1, it can be obviously seen that the PDFA can solve the problem we proposed effectively in a reasonable time period. Figure 1 indicates that larger value of  $g$  leads to better diversity of solutions at the cost of computational time. Figure 2 clearly shows that the algorithm has a good convergence ability.

## 5. Conclusions

In the short-haul road transportation with one source and one destination, an optimized load plan is essential for food quality and profitability when agricultural food products are too many to be loaded all together by given refrigerated trucks. Therefore, this paper presents the modelling of tri-objective fuzzy agricultural food load planning problem that takes into account two factors of the agricultural food products: (1) different temperature demands. (2) fuzziness of profits and shelf lives. Specifically, profit and shelf life of each product are represented by triangular fuzzy numbers and defuzzified by the GMIR method.

To solve the proposed model, we have proposed the pareto-based discrete firefly algorithm, which has four major improvements compared with the basic firefly algorithm: (1) a greedy heuristic algorithm is used to generate initial population of fireflies to speed up convergence. (2) movement function of the firefly is redefined. (3) a two-phase repair operator is used to improve the quality of solutions. (4) the  $(\mu+\lambda)$ -PAES algorithm and the 3-dimensional adaptive grid algorithm are used to accept, reject, and discard fireflies. Furthermore, experimental studies on a set of 14 instances show that the PDFA can solve the tri-objective fuzzy agricultural food load planning problem effectively and efficiently. Therefore, this algorithm is a useful tool to manage food cold chain intelligently. For example, it can be implemented and then integrated as a module of the decision support system for agricultural product supply chain which was proposed by Qiu et al. (2015). Furthermore, the model we proposed can be blended with the vehicle assign problem which was proposed by Barany et al. (2010) for more complex problems about agricultural food products delivery.

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