Adaptive Offset Remover Based on Kalman Filter Integrated to a Model Predictive Controller

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Model predictive controller (MPC), even when well projected, is minded to unmeasured disturbances along operation. It is usual to observe offset between controlled variables and their set points in result of those disturbances. The undesirable offset may cause loss of quality and profits, unconformity of the final product or, at the furthest cases, the instability of the process. Some alternative methods to prevent offset were shown in literature. One of them is the application of state estimators to predict model mismatch in nonlinear processes. Huang et al. (2009) demonstrate the use of recursive observers to correct and stabilize MPC controllers. In the present work, an adaptive linear model is proposed to estimate the error between the predictive model and process response to help MPC to compute future control actions. Every sample time, this adaptive model is adjusted based on a Kalman Filter (a state observer), using past measured values of the process variables. The error found is then added to model predictions to find corrected actions for the control horizon of MPC.

The proposed MPC was applied to an experimental system of interacting tanks, controlling their levels by manipulating feed flow rates. The obtained results were compared to the original MPC controller (no corrections) in situations of servo and regulatory control, under occurrence of unmeasured disturbances and/or set point changes. As result, it was observed that controller response was improved in the presence of the adaptive model proposed, reducing significantly the offset in all tests carried out. It was found a reduction of more than 40 % of the Integral of Squared Errors (ISE) performance index, due to eliminating the lack of adjustment between plant and its mathematical modelling.

1. Introduction

Model predictive control (MPC) is a class of algorithms that optimize process operation based on a prediction model of the system behavior. This class of controllers became popular because it is able to handle with multiple variables and constraints in an intuitive way. The effectiveness of MPC relies on the accuracy of process modelling, but the mismatch between model and process inevitably exists (Tian et al., 2014). Other cause of reduction in MPC effectiveness is unmeasured disturbances, caused by non-modelling variables. So, that conventional MPC cannot guarantee steady-state offset-free control. The most widely used strategy to eliminate offset in industrial MPC is shifting the set point to compensate model-process mismatch, assuming that disturbance remains constant along prediction horizon, what can be applied in a restrict range of process (Tian et al., 2014). Robust MPC (RMPC) is also an alternative to eliminate offset from MPC. The RMPC usually makes a real-time parameter (or model) optimizing to compensate model mismatch.

Huang et al. (2009) proposed an offset free Nonlinear MPC (NMPC) formulation that integrated state and output disturbances from extended Kalman filter algorithm. The author demonstrated, mathematically and using simulation, the viability and stability of using state observer to map model mismatch to eliminate offset in NMPC.

From the above, the aim of this work is to develop an MPC with the same idea proposed by (Huang et al., 2009): using a state observer to predict model mismatch, improving the control action of the controller and reducing process offset. A system of two interacting tanks, that have their levels nonlinearly correlated, was chosen as a case-study to test the proposed MPC.

Although simple, level control problem of two interacting tanks is a good exercise to experiment multivariable control logic. Moreover, the tanks linked by a nonlinear valve added a nonlinear dependence between their

levels that hardly could be suitable handled with classical linear controllers (Johansson, 2000). Two MPC formulation, with and without prediction corrector, is compared and assessed by using Integral of Squared Error Criterion.

2. Methodology

2.1 Interacting Tanks System

Figure 1 shows the PI&D of the two tanks system in which the control was applied. The control of vessel V-101 and V-102 was proposed manipulating the power supply of the pumps P-101 and P-102. Hand valve HV-103, with opening previously set, communicated the two tanks. Valves HV-101 and HV-102 regulate the out flow from tanks V-101 and V102, respectively, and were previously set too.

Level data was obtained by piezoelectrics pressure transducers (LT-101 and LT-102) Siemens, series MPX5010 plugged at the bottom of the tanks. Delta PLC, model DVP20EX3 (YIC-101), read sensor data and transmits to a PC, where control action was calculated in a MPC algorithm programed in Matlab/Simulink dynamic simulation environment. The action law calculated by MPC algorithm is transmitted back to PLC that manipulates the power supplied to the pumps P-101 and P-102, closing the control loop. Level could be read too in visual graduated ruler indicator (LI-102 and LI-102). The level of Vessel V-103 is not read by control system since the tank is used just as a transferring vessel.
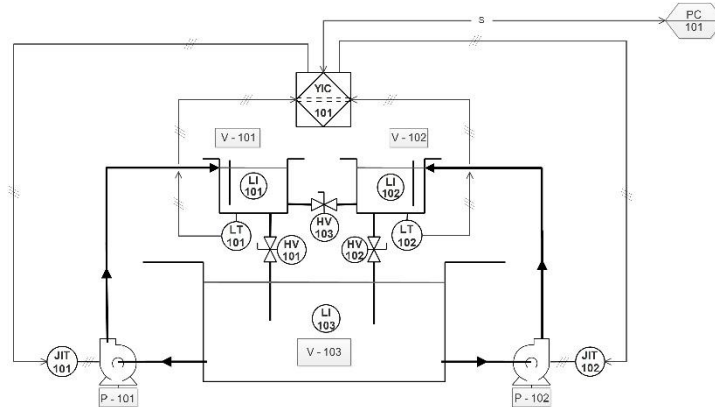

Figure 1: PI&D diagram of experimental system of interacting tanks.

2.2 Model Based Predictive Controller

Two MPC algorithms were proposed at this work: in the first one the control action was optimized based only in a prediction model. The second controller (MPC-KF) uses an adaptive model that corrects model prediction based on previously observation of the difference between model and system (Figure 2 – dashed line), this compensation is addicted to model prediction improving the optimization.
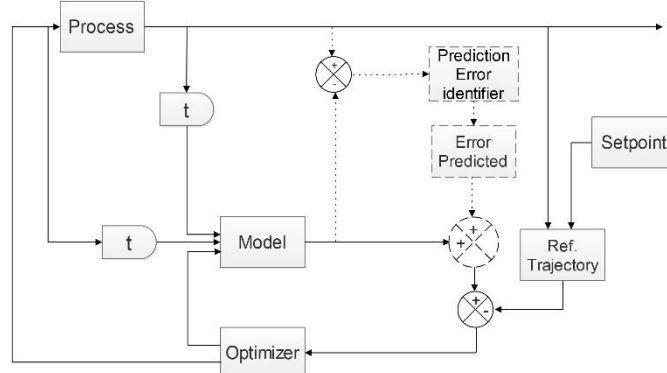

Figure 2: Block diagram of proposed MPC controllers.

A moving horizon MPC algorithm was proposed, based on logic present by (Camacho and Bordons, 2007), to control the level of the two tanks (V-101 and V-102), simultaneously, manipulating the energy supply to pumps P-101 (P_{b1}) and P-102 (P_{b2}). The MPC calculation is repeated every 5 seconds (approximately 10 % of time constant of process - τ). The prediction horizon (H_p) was set to ten sampling periods (equivalent to 50 s), and

the control horizon (H_c) to one sampling time (equivalent to 5 s). Every sampling time, the instantaneous tank levels are taken and a reference trajectory ($h_{i,ref}$) calculated by Eq(1) to make a gradual transition on desired set point ($h_{i,SP}$) (Camacho and Bordons, 2007).

$$h_{i,ref_t} = 0.5^t \cdot h_{i_0} + \left(1 - 0.5^t\right) \cdot h_{i,SP} \qquad \text{i refers to vessel V-101 (1) or V-102 (2)} \tag{1}$$

The optimization of control action is carried out by the optimization function "fmincon" from MATLAB optimization toolbox, using trust-region-reflective algorithm method (default option). The cost function, described in Eq(2), is minimized considering the error between predicted and reference level of the two tanks at prediction horizon and fluctuation in control action (Camacho and Bordons, 2007).

$$\text{Cost} = \sum_{t=1}^{H_p} \left(h_{1_t} - h_{1,ref_t}\right)^2 + \sum_{t=1}^{H_p} \left(h_{2_t} - h_{2,ref_t}\right)^2 + \sum_{t=1}^{H_c} \left(P_{b1_{t-1}} - P_{b1_t}\right)^2 + \sum_{t=1}^{H_c} \left(P_{b2_{t-1}} - P_{b2_t}\right)^2 \tag{2}$$

2.3 Model

The prediction model used is based on mass balance for each tank, as shown in Eq(3).

$$\frac{dh_i}{dt} = A_i \cdot \left(\dot{V}_{bj} + \dot{V}_{v,com} - \dot{V}_{vi}\right) \qquad \begin{array}{l}\text{i refers to vessel V-101 (1) or V-102 (2) and j refers to pumps}\\ \text{P-101 (1) or P-102 (2)}\end{array} \tag{3}$$

Pump flows ($\dot{V}_{bj}$) were described by Eq(4), where parameters a_j, b_j and c_j were calculated fitting experimental data to model as function of the supplied energy (P_{bj}).

$$\dot{V}_{b,j} = \left(\frac{a_j}{1 + \exp\left(b_j \cdot P_{bj}\right)}\right) - c_j \qquad \text{j refers to pump P-101 (1) or P-102 (2)} \tag{4}$$

Valves HV-101 ($\dot{V}_{v,1}$) and HV-102 ($\dot{V}_{v,2}$) flows were described by Eq(5) as function of tank level. Communication valve (HV-103) was modeled by Eq(6), as function of level difference. The flow direction was defined as positive from tank V-102 to V-101.

$$\dot{V}_{v,i} = k_i \cdot \sqrt{h_i} \qquad \begin{array}{l}\text{i refers to vessel V-101 (1) or V-102 (2) and it respective hand}\\ \text{valve HV-101 (1) or HV-102 (2)}\end{array} \tag{5}$$

$$\dot{V}_{v,com} = k_{com} \cdot \left(\frac{h_2 - h_1}{|h_2 - h_1|}\right) \cdot \sqrt{|h_2 - h_1|} \tag{6}$$

Valve constants (k_1, k_2 and k_{com}) were calculated by fitting open loop experimental data obtained by pre-setting valves opening.
The differential equation (Eq(3)) for each tank is solved using Euler numerical solution with dt = 1 s to obtain the future state predictions of the tank levels.

2.4 Kalman Filter Based Offset Corrector

The controller algorithm predicts system states every sampling time. These predictions are then compared to the system responses and the mismatch between them is used to real time identification of a model to predict this local divergence. Eq(7) describes the chosen model for the model/system mismatch used in the MPC-KF strategy.

$$\text{Err}_i = k_{d,i} \cdot \left(1 - \exp\left(^{-t}\!/_\tau\right)\right) \qquad \text{i refers to vessel V-101 (1) or V-102 (2)} \tag{7}$$

This model considers the mismatch as a first order dynamic with time constant equal to sampling period (τ = 5 s). The gains (k_d) of the model are obtained by a state observer algorithm based on a simplification of Kalman Filter algorithm, proposed by (Liguni et al., 1992). The observer algorithm reads the model mismatch over sampling periods of control and finds the best k_d that fits data to model.
The state observer is an extended Kalman Filter that estimates the state (k_d) from previous estimative and the observed states of the process (model/process mismatch). The observed and estimated states are correlated by linearization of Eq(7), and the future estimative consider that the model is locally constant. The obtained model parameter (k_d) is used to generate a vector of prediction errors that is summed to model prediction of MPC inside optimization algorithm.

2.5 Controller Performance

The comparison of controller performance was done for servo and regulatory control tests. Three different set points (40, 50 and 60 % of total level) were set for each tank in the servo problem, resulting nine combinations of references. Those combinations were sequentially applied every 500 s. The operating condition at the starting point of the experiment is a steady-state of levels at 50 % of nominal capacity.

For the regulatory test, the set points were 60 and 40 % for the tank V-101 and V-102, respectively. The valves were set at same configuration of servo control experiment. At time 200 s the first disturbance occurred: the valve HV-101 was partially closed. The controller had 1000 s to stabilize the plant, then a new disturbance was done: the valve HV-101 was completely closed and again, the controller had 1000 s to control the plant. At the end, valve HV-101 was set to initial configuration and the controller was observed for 500 s.

The performance of MPC and MPC-KF controllers was compared by using the Integral of Squared Error (ISE) Criterion, defined as Eq(8) (Seborg, 2011).

$$ISE_i = \sum_{t=0}^{t_{final}} \left(h_{i,SP_t} - h_{i_t} \right)^2 \quad \forall\, t \in N \quad \text{i refers to vessel V-101 (1) or V-102 (2)} \tag{8}$$

3. Results and Discussion

3.1 Servo Control Experiment

Figure 3 shows the results of servo problem run for both controllers, MPC and MPC-KF. Observing level set points (Figure 3 – a and Figure 3 – b) both controllers kept levels controlled with small oscillation and offset along experiments (between 40 and 60 % of level). The offset observed from MPC controller were caused by prediction model mismatch, while the MPC-KF results show the reduction of this offset.

The controller action in pumps P-101 and P-102 are show in Figure 3 – c and Figure 3 – d, respectively. Smooth control actions are observed, what is a good result because oscillation on action response reduces actuator life.
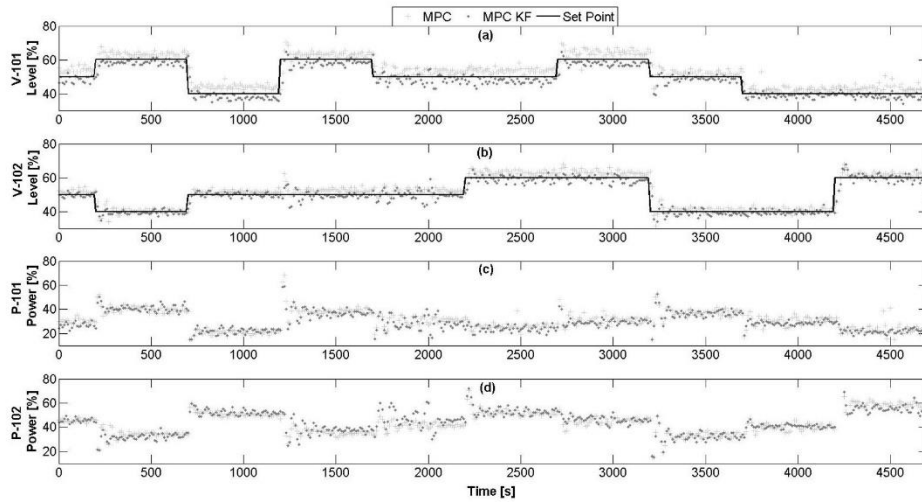

Figure 3: Controller Responses for Servo Control Experiments: (a) Tank V-101 level; (b) Tank V-102 level; (c) Pump P-101 supplied power; (d) Pump P-102 supplied power.

At moment of altering set point it is observed a peak on pump response. Those peaks result from fast response to the offset change, but causes no significant overshoot to level variables. Visually comparing the two controllers is hard to see difference between their actuation.

The improvement in control can be observed quantitatively by the integral of squared error, shown in Table 1. The ISE was reduced in 40.2 % for tank V-101 and 27.0 % for tank V-102.

Although graphics show very similar response, the quantitative result highlights the improvement of control by using MPC-KF. The mismatch at this run is just from model imprecisions.

Table 1: Integral of squared error from servo control experiments.

Vessel	ISE_{MPC}	ISE_{MPC-KF}	Reduction
V-101	13.96 dm²	8.32 dm²	40.4 %
V-102	5.30 dm²	3.87 dm²	27.0 %

This problem could be solved by improving the model, what could be costly. Other limitation could be the computational time required for the optimization of control action at MPC when complex models are employed.

3.2 Regulatory Control Experiment

For the regulatory control experiment, disturbance was done at valve HV-101. Figure 4 shows the results of the experiments for MPC and MPC-KF. It could be seen that MPC controller could not realize the change at tank V-101, so that the optimizing algorithm calculated an action that could not keep the set point (Figure 4 – a and Figure 4 – b). The model of prediction correction of the MPC-KF could adjust a model that improves controller prediction, and it was observed the reduction of offset compared to the first controller. As expected, the second disturbance results in larger offset then the first one since the HV-101 completely closed changed the tank dynamics (tank V-101 outflow is more dependent of tank V-102 level). It is important to see that after opening HV-101 again (initial condition) both controllers could find the set point again. Even after large disturbances, the prediction correction algorithm can go back to initial condition if the disturbance is removed. As disturbance was just applied to the outflow valve of tank V-101, the dynamic of tank V-102 did not change (Figure 4 – b). Analyzing the action response of controllers (Figure 4 – c and Figure 4 – d), is hard to visualize difference between then at figures, but those little differences were sufficient to reduce significantly the offset.
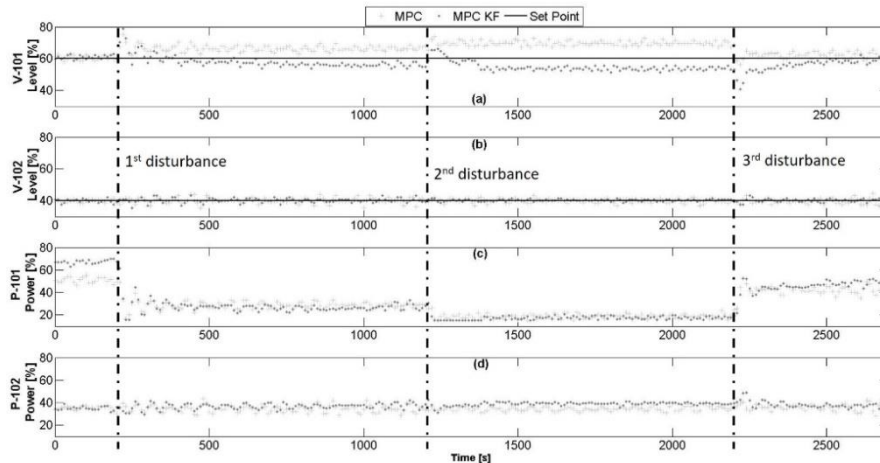

Figure 4: Controller Responses for Regulatory Control Experiments: (a) Tank V-101 level; (b) Tank V-102 level; (c) Pump P-101 supplied power; (d) Pump P-102 supplied power.

Quantitative results could be seen at Table 2. The integral of squared error of the levels was reduced 48.4 % for tank V-101 and 49.2 % for tank V-102. Although an expressive percentage improvement is reached in tank V-102 ISE, the numerical value is low, indicating that both controllers had almost no offset for the level of this tank.
It is important to observe that the state observer does not interfere on the controller logic, so it possibly does not interfere on controller stability as well. The improvement is clear at level control of tank V-101, but the offset could not be eliminated for regulatory test. Anyway, the MPC-KF controller could keep the level below the set point, different from original MPC that let the level of tanks stay higher.

Table 2: Integral of squared error from regulatory control experiments.

Vessel	ISE_{MPC}	ISE_{MPC-KF}	Reduction
V-101	40.53 dm²	20.92 dm²	48.4 %
V-102	1.93 dm²	0.98 dm²	49.2 %

3.3 Adaptive Algorithm Learning

Figure 5 shows learning progress of prediction parameters of corrector model (k_d) on servo (Figure 5 – a) and regulatory (Figure 5 – b) runs. Data plotted shows the effort of Kalman Filter algorithm to find the best solution for this parameter. As algorithm searches for the best local solution, the gain changed at each set point for servo problem. For regulatory problem, after disturbance occurs it is seen that the algorithm gradually looks for the optimal gain and then finds it, just changing again after a new disturbance. As the disturbance was applied just in the valve HV-101, the calculated gain to correct the dynamic of tank V-102 did not change along the run.
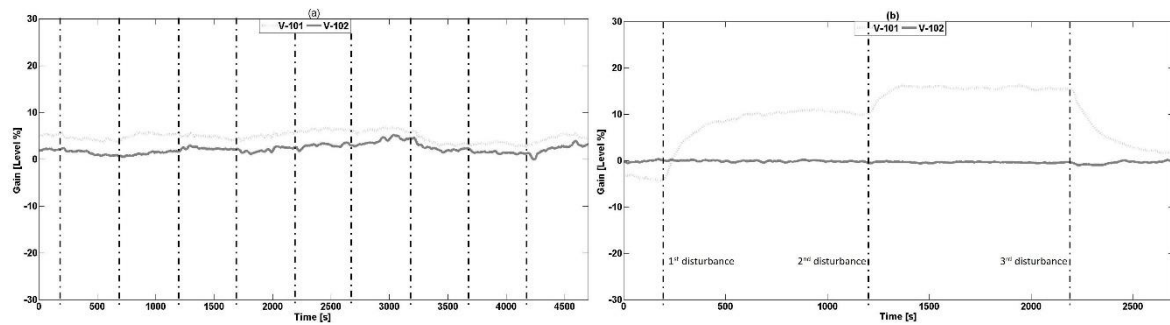

Figure 5: Kalman Filter gain learning of mismatch model for (a) Servo control; (b) Regulatory control problem.

Is important to observe that, at the end of regulatory problem run, the original configuration of the system was restored and the corrector algorithm tends to find the original gain, near to zero.

4. Conclusions

Results show that the model prediction correction strategy to reduce offset from MPC controller was promising. It could reduce the sum of squared error in more than 40 % for servo and regulatory problems. Is important to note that no oscillation occurred in pump action or at tank levels by the correction algorithm. The Kalman Filter algorithm clearly could find an optimal local solution for the problem as seen at gain evolution graphics, keeping the solution until a new disturbance (or change in set point) was done.

As seen at block diagram, this strategy is not invasive at MPC logic, and do not change the prediction model. Those points highlight the potential of practical use.

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