**Slippage at the solid-liquid interfaces: implications for colloidal transport**

 **in confined geometries**

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**1.Introduction**

Transport of colloidal systems in viscous flows plays an important role in chemical engineering problems involving microfluidic devices, biochemical reactors, sol-gel processes, filtration, sedimentation and other separation operations.

The characteristic sizes of colloids (typically order of 1-1000 nm), much greater than that of the fluid molecules is however small enough to consider valid low Reynolds-number hydrodynamics for colloidal systems, making the Stokes equations, with no-slip boundary conditions at the solid (particle, wall) surfaces, the natural candidate for describing colloidal hydrodynamics and transport, and predicting the relevant emerging parameters: the resistance coefficient η (friction factor) characterizing the dissipation properties as regards momentum balance, and the diffusion coefficient D accounting for the intensity of fluctuations in particle motion. These two parameters are, for isothermal systems, related to each other by the simplest form of fluctuation-dissipation relation, namely ηD=kBT, where kB  is the Boltzmann constant and T the temperature of the system. The simplest approximation as regards diffusive transport properties consists in considering the system isotropic and diluted so that both η and D are scalar constant quantities (specifically, η=6πμRp for spherical particles with radius Rp translating in a fluid with viscosity μ). However, in dealing with colloidal systems in a confined fluids, whenever the characteristic size of the particle Lp is comparable with the characteristic size of the system L, both η and D become position-dependent tensorial quantities. This is the case of particle transport in microfluidic devices, micro/nano-particle motion in porous media, chromatographic analytical methods, colloidal dynamics near a solid collector, dynamics of concentrated suspensions etc. . However, since the Stokes equations in the presence of no-slip boundary conditions at solid boundaries predict a singular behavior of the resistance of two surfaces getting in touch, i.e., η∼1/h, h being the distance between the surfaces, thus implying that D=0 at the solid surface, many paradoxes and difficulties arise in modeling surface phenomena. Specifically, since η(h) is characterized by a non-integrable singularity at the touching point, Stokes hydrodynamics with no-slip boundary conditions predicts an infinite ``touching time’’, for a particle to reach the surface, and similar paradoxes arise in dealing with surface chemical reactions or coalescence phenomena. Moreover, since D=0 for h=0, mixed boundary conditions accounting for surface chemical reactions cease to be valid. A way for overcoming these difficulties is to eliminate or mollify the singularity at the touching point by introducing slippage at the surfaces by means of the Navier slip boundary conditions. In fact, as shown in [1], if the slip length is the same at both the surfaces the singularity of the resistance coefficient becomes integrable as it is characterized by a logarithmic scaling η∼-ln(h), providing a finite touching time. Starting from numerical simulations and using also the semianalytical approach proposed by Goren [2] we investigate in this presentation the quantitative behavior of the transport parameters η(h) and D(h) of a spherical particle close to a planar surface, proposing also solutions for solving the above mentioned paradoxes.

**2. Methods**

Hydrodynamic Finite Element Simulations have been used to characterize the scaling of the resistance matrix near a planar surface. In order take computational advantage from the axial symmetry of the problem of a sphere translating perpendicularly to a planar wall, cylindrical coordinates ( r, φ, z ) have been used reducing the computation to a two dimensional ( r, z ) domain representing the flow region, with an empty disk, representing the spherical particle placed at distance h from planar wall. A finer mesh has been adopted along the perimeter of the disk representing the sphere, and the maximal length of the elements has been imposed to be less than 0.1 Rp. Both P2P1 and P3P2 finite elements have been used depending on the position of the particle. Furthermore, we solved the infinite system of Goren’s equations furnishing the drag force on the particle [2] over the entire range of positions and for any values of the slip length at the wall λw and at the particle λp, truncating it to 500 equations (that was sufficient for the h-values considered to achieve convergence).

**3. Results and discussion**

If the no-slip boundary condition is imposed on just one of the two surfaces (particle or wall) the singular scaling η∼ 1/ h is preserved independently of the value of the slip length imposed on the other surface. In the latter case, we can distinguish among three different regimes: (i) the scaling η∼ 1/ h for λ<h<10-1 λ, (ii) an apparent logarithmic behavior for 10− 2 λ< h <λ and (iii) the asymptotic regime where η ∼ 1/h for h <10− 2 λ.



**Figure 1.** Dimensionless hydrodynamics resistance coefficient η(h)/ ( 6 π µ R p ) vs h/Rp obtained by keeping fixed the slip length λi /R p = 10− 3 on the i-th surface (i = w, p) and varying the slip length on the other obtained by solving the Goren’s equations.

Keeping fixed the slip length at one of the surfaces ( λi= 10− 3 , i = w, p) and increasing the slip length on the other λj , j = p, w, the logarithmic scaling occurs for d < λj , see Figure 1. This means that an arbitrarily small slip on both the surfaces is sufficient to determine an asymptotic logarithmic scaling of the transversal resistance and thus the occurrence of a finite value of the touching time [3].

**4. Conclusions**

We have thoroughly analyzed how the lack of integrability of mass-transport models (diffusion equation) in the presence of a surface chemical reaction depend on the simplifying assumption of no-slip boundary conditions. The inclusion of slippage effects at all the solid surfaces (walls and particle) transforms the non-integrable 1/h-singularity in the transversal resistance coefficient η(h) near a planar wall into a logarithmic singularity ( η∼− ln(h)) resolving the above mentioned integrability problem. However, there are physical reasons to conjecture that a more refined modeling of fluid-particle interactions (including compressibility and acoustic effects) could completely eliminate the singularity occurring for η(h) at the wall [3].

**References**

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2. S. L. Goren, J. Colloid Interface Sci. 69 (1979) 78-85.
3. G. Procopio and M. Giona, Fluids 7 (2022) 105.