An MILP based-approach to logistics of air separation supply chains with a heterogeneous fleet of trucks

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Abstract

This article addresses the production and distribution scheduling of industrial-sized gas problems when a heterogeneous fleet of trucks is considered. In the work, an MILP-based approach is proposed to deal with the characteristics of the problem. In the distribution part, for each size of truck considered, a set of feasible routes is previously generated and selected. Then, the model is solved by considering the routes selected, choosing the ones that satisfy all the conditions imposed while optimizing production and distribution activities. It is worth remarking that the approach takes into account the lead time on production and distribution decisions. Finally, an illustrative example is presented and solved to show the impact of multi-size trucks. The results obtained demonstrate the effectiveness of the proposed approach in reasonable CPU times.

**Keywords**: Supply chain management, Production routing problem, Mathematical modelling.

* 1. Introduction

When examining the operation and management of industries specializing in industrial gas production, a critical aspect is identified in the design and management of the supply chain (SC). In the SC, decisions regarding production and distribution of products to customers are critical. Ramaswamy et al., (2020) provide a comprehensive analysis about the design and management of industrial gas supply chains. Additionally, Barbosa-Povoa and Pinto, (2020) conduct an exhaustive study of the most significant problems and challenges within SC, offering proposals for their resolution.

* 1. Problem statement

This work deals with the production and distribution problem of an industrial gas supply chain that operates under a vendor-managed inventory (VMI) policy. The aim is to optimize the operation of the SC, seeking to minimize the resulting total cost. One particularity of this problem, is the consideration of a heterogeneous fleet of trucks. Due to the characteristics of customers (i.e., location, demands, tank sizes), the adoption of multi-size trucks is necessary. In some cases, customers who are located inside a city or that have low-capacity tanks, cannot receive a delivery from big-sized trucks and smaller trucks are needed. By taking this issue into account a more efficient utilization of the truck’s capacities is expected. As mentioned above, the company must generate optimal production and distribution strategies that allow it to visit customers on time to meet their demands. For this reason, knowing the monthly demands of customers and their maximum and minimum inventory capacities, the company has to decide the plant’s production rate and the sizes of trucks to use, in terms of supplying the requested product. An important aspect of the problem under consideration is that trucks can either make multiple short trips per day or a single trip of several days. The duration of the trip is determined by factors such as the distance to the clients involved, the sequence in which customers are visited, and the loading and unloading times of the products. Given that trips can extend over several days, this problem explicitly takes into account the impact of lead time on: a) the date when the product is released from the plant and reaches the customer, and b) the time during which the trucks are unavailable for other trips.

* 1. Solution strategy and main assumptions

To address the problem, the MILP model and the route generator algorithm introduced in Bonino et al., (2023) are the basis for the current paper. Thus, the route generator algorithm is still employed for the creation of all potential trips between customers and plants. Regarding the mathematical formulation, the previous equation blocks and decision variables introduced in Bonino et al., (2023) are used to face the new problem. Also, to include the sizes of trucks in the previous developed model, a new set indicating the different available sizes is added. The truck sizes taken into account are: “Small”, “Medium” and “Big”. This new index impacts in the variables that consider: a) amount of product *i* delivered by plant *p* using a specific route*s* at day *t*:“$\overbar{DL}\_{i,p,s,size,t}$”, b) amount of product *i* delivered to customer *c* from plant *p* using a specific route *s* at day *t*: “$\hat{DL}\_{i,p,s,c,size,t}$”, c) trucks in the plant *p* for the product *i* available on a given day *t*: “$TU\_{i,p,size,t}$”, d) the activation or deactivation of the route *s* associated with plant *p* and product *i* at day *t*: “$A\_{i,p,s,size,t}$”. It is worth noting that the resulting increase in the model size is related to the new set added and not to the number of trucks available.

The following assumptions underlie the proposed solution strategy: a) Trucks can only be used a given number of hours per day. More time implies a multiday trip, b) Routes of more than one day are considered, c) Routes of several customers are considered, d) Lead time is taken into account when scheduling a customer visit, e) Trucks making trips longer than the limit of hours per day are not available for use until their return to the plants, f) The total ending inventory must be greater than or equal to the total beginning inventory, g) A truck can have multiple trips in one day only if the total time of all trips is less than or equal to the limit of hours per day, h) A customer can be visited at most twice on the same day, i) All the parameters and variables representing quantity of product are standardized to medium-sized truck units,

* + 1. Model formulation

The extended MILP model is designed to minimize the costs associated with production and distribution operations when considering a heterogeneous fleet of trucks. The introduction of a new index indicating the size of the trucks has an impact on the variables mentioned earlier and, consequently, on all equations related to them. Unfortunately, due to space limitations, the complete model cannot be included. Nevertheless, to illustrate the impact of the changes in constraints, Equations (1) and (2) are given as example. Both constraints are performed for trucks that are engaged in trips with durations longer than the maximum number of working hours per day (*hd*). Constraint (1) calculates the number of trucks for each size that are available to perform multi-day trips. $LC\_{i,p,size}$ is a parameter that indicates the total number of trucks of a given size available in plant *p* to transport product *i*. *SPp, SIi, SASt* are the sets of routes departing from plant *p*, transporting product *i*, and available routes on day *t*, respectively. *IPi* are the set of plants producing product *i*. Constraint (2) calculates the number of unavailable trucks for transporting the product *i* since the plant *p* in the period *t* ($TU\_{i,p,size,t}$). To calculate the trucks not available in period t, equation (2) takes into account the parameter *Durations* in order to consider the trucks that are in use in period *t* because they are yet performing a given route *s*.

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| --- | --- |
| $$\sum\_{s \in \left(SP\_{p}∧SI\_{i}∧SAS\_{t}),(τ\_{s}>hd)\right)}^{}A\_{i,p,s,size,t} \leq (LC\_{i,p,size}-TU\_{i,p,size,t}) ,$$$$  ∀i\in I, ∀p\in IP\_{i}, ∀t, ∀size\in Size$$ | (1) |
|  |  |
| $$TU\_{i,p,size,t}=\sum\_{\begin{array}{c}s \in \left(SP\_{p}∧SI\_{i}∧SAS\_{t} ∧\left(Duration\_{s}>1\right)\right),\\tt :(\left(tt\geq t-Duration\_{s}+1) ∧ (tt\leq t-1)\right) \end{array}}^{}A\_{i,p,s, size, tt}$$$$∀i\in I, ∀p\in IP\_{i},∀t:t>1, ∀size\in Size$$ | (2) |
|  |  |

* + 1. Strategy for matching routes with trucks considering their sizes

In this section, the proposed strategy is presented in order to establish an efficient relationship between the generated routes and the available truck sizes. In this work the truck capacities ($Capacity\_{size}$) considered are 0.5, 1, and 1.4 for small, medium, and large trucks, respectively. To match routes and truck sizes, the characteristics of the clients involved in the trips are considered. Customer location is a key aspect because not all trucks can deliver product to customers due to accessibility issues. On the other hand, tank capacity and daily customer demand substantially affect the minimum and maximum delivery connected to the routes. For the generated routes that do not have limitations (for example, due to the location of customers), Algorithm 1 establishes the set of routes that can be used by the trucks of the different sizes. It is important to note that for each route generated the minimum and maximum amount of product ($DeliveryMin\_{s}$,$DeliveryMax\_{s}$) that the route can deliver is determined. These parameters are computed considering the daily customer demands and the difference between the tank capacity and the reserve levels of the customers included on the routes, respectively.



The procedure is initiated by generating empty lists for the different truck sizes, where the assigned routes will be stored. For each route, a comparison is made between the minimum delivery and the capacities of different trucks to determine if, in any case, the lowest quantity to be shipped exceeds the available capacity for a given truck size. If the minimum shipment for a route is less than the capacity of the truck size under analysis, the maximum shipment is then compared with the available capacity of the next size of truck. If the highest delivery of the route is greater than the 80% of the capacity of the next truck, the route is assigned to that truck size; otherwise, the route is assigned to the initial size.

The algorithm presented outlines the assignment of a single truck size per route. Nevertheless, with slight adjustments, it can also be extended to consider two truck sizes per route. In this case, the sizes are determined by those ensuring the minimum delivery of the route, along with the inclusion of the next larger truck size.

* 1. Illustrative example and computational results

To demonstrate the advantage of considering a heterogeneous fleet of trucks, an illustrative example is presented. Figure 1, illustrates the logistic decisions expected with a heterogeneous truck fleet. The case addressed involves one production plant, one product, 14 customers with different demands and storage tank sizes, three vehicles of different sizes, and a time horizon of 28 days. The geographical locations of the plant and customers are depicted in Figure 2. As observed in the image, customers have different daily demand and tank sizes. All the customers are located to less than 24 hours of the plant. The maximum number of working hours per day (*hd*) considered is 12.

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| --- | --- |
|  |  |
| **Figure 1**. Trips with heterogeneous fleet  | **Figure 2**. Plant and customer location |

To assess the performance of the mathematical formulation, nine different scenarios were tested. In each scenario, the number of trucks considered varies, as well as the number of possible truck sizes per route. Table 1 provides a detailed description of the conditions in each scenario. The model was implemented in GAMS 32.0 and all scenarios were solved with GUROBI 10.0.0 on a desktop computer with 2.80GHz Intel(R) Core(TM) i9-10900F processor and 16 GB RAM on Windows 10. The maximum run time for each scenario was set to 900 seconds.

**Table 1**. Scenarios tested

|  |  |  |
| --- | --- | --- |
| **Scenario** | **Number of trucks per capacity** | **Truck sizes per route** |
| **Small** | **Medium** | **Big** | **All posible (\*)** | **1 size** | **Up to 2 sizes** |
| S1 | 3 | - | - | ● | - | - |
| S2 | - | 3 | - | ● | - | - |
| S3 | - | - | 3 | ● | - | - |
| S4 | 1 | 1 | 1 | ● | - | - |
| S5 | - | 2 | 1 | ● | - | - |
| S6 | - | 1 | 2 | ● | - | - |
| S7 | 1 | 1 | 1 | - | ● | - |
| S8 | 1 | 1 | 1 | - | - | ● |
| S9 | - | 1 | 2 | - | - | ● |
| (\*)In S1, S2, and S3, the allowed truck sizes per route are small, medium, and large, respectively. |

The results obtained for the scenarios are presented below in Tables 2 and 3. For the addressed example, the goal is to satisfy all customer demands through the company's production. If it is not possible, and external purchases are needed to meet the total demand, the result is considered to be infeasible. The scenarios S1 and S7 were found to be infeasible, and therefore, no results will be displayed in the tables. The infeasibility is attributed to the fact that both scenarios only consider a single truck size per route, and the assigned trucks cannot fulfill the entire demand.

The performance of the model for the scenarios tested is shown in Table 2. It includes the size of the resulting MILP model, the processing time, and the value of the objective function with the associated optimality GAP. Table 3 provides a detailed analysis of the solution in terms of distribution.

**Table 2**. Model performance

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **S2** | **S3** | **S4** | **S5** | **S6** | **S8** | **S9** |
| **Equations** | 27,372 | 28,632 | 74,606 | 51,619 | 51,619 | 43,974 | 43,974 |
| **Continuous var.** | 12,293 | 12,853 | 35,813 | 24,333 | 24,333 | 20,497 | 20,497 |
| **Binary var.** | 3,332 | 3,472 | 10,416 | 6,944 | 6,944 | 5,768 | 5,768 |
| **Cost [$]** | 89,551.7 | 83,600.4 | 85,183.5 | 84,356.7 | 82,817.7 | 85,977.8 | 82,812.7 |
| **CPU Time [s]** | 900 | 900 | 900 | 900 | 900 | 900 | 900 |
| **GAP [%]** | 1.79 | 1.06 | 1.79 | 1.44 | 0.75 | 2.31 | 0.98 |

Table 2 shows that the variation in model sizes between scenarios is predominantly due to the number of enabled routes to solve the problem. In terms of solution quality, it can be observed that within 900 seconds, results close to the optimum were obtained, where the optimality GAPs range from 0.75% to 2.31%. Regarding costs, the best solution was obtained for scenario S9, which considered a medium-sized truck and two big-sized trucks. In this scenario, each route is allowed to have up to two truck sizes assigned, when it is possible. The results obtained for S9, compared with the ones achieved for S2 and S3, show the advantages of considering heterogeneous fleets. S2 and S3 assume all medium and big trucks, respectively. While S3 outperforms S2 by considering larger trucks for the trips, both scenarios are surpassed by S9 given that the flexibility to choose the appropriate truck size for each trip.

For each scenario, Table 3 displays the number of trips, customer visits, and average truck utilization for each considered size. When contrasting S2 with S9, it is noticeable that in S2, there were 62 trips with 82 customer visits. In S9, with the inclusion of larger truck sizes, the number of trips decreased to 48, and the visits reduced to 67. This reduction is attributed to the capability of transporting a larger quantity of product in each trip, if needed.

**Table 3**. Solutions obtained for transportation decisions

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **S2** | **S3** | **S4** | **S5** | **S6** | **S8** | **S9** |
| **Trips** | 62 | 45 | 58 | 52 | 48 | 59 | 48 |
| **Visits** | 82 | 70 | 75 | 73 | 66 | 74 | 67 |
| **Truck utilization [%]** | **Small** | - | - | 88.46 | - | - | 89.29 | - |
| **Medium** | 96.19 | - | 98.19 | 92.80 | 80.75 | 98.59 | 85.92 |
| **Big** | - | 94.66 | 99.32 | 99.80 | 99.10 | 98.45 | 98.91 |
| **Possible number of routes** | 119 | 124 | 292 | 243 | 243 | 206 | 206 |

Figures 3 and 4 illustrate the inventory profile for customer 12. This customer demands 0.5 units daily of product and can receive a maximum product delivery of 2.5 units. Figure 3 shows the inventory evolution obtained for S2, where only medium trucks with a capacity of 1 are considered. The graph indicates that to meet the customer's demand, 14 visits with full trucks are scheduled. In Figure 4, the evolution of C12 in scenario S9 is presented. In this case, with the inclusion of big trucks with a capacity of 1.4, only 10 visits (using big-sized trucks) are performed, resulting in a number of visits less than in S2. This highlights that the use of heterogeneous trucks enables a more efficient allocation of trucks to trips, optimizing the available capacities.

The perceived reduction in trips for C12 in the S9 scenario also results in reduced distribution costs. In the case presented in the figures, the scheduling of visits for C12 in S2 incurs a total cost of $13,013, while for S9, the cost is $11,154. This trend in distribution costs explains the difference in total cost between S2 and S9. While the production cost for both scenarios is $33,043.5, the distribution costs are $56,508.2 and $49,769.2 for S2 and S9, respectively. A lower transportation cost is observed in S9 due to the consideration of heterogeneous trucks.



**Figure 3**. C12 inventory in S2



**Figure 4**. C12 inventory in S9

* 1. Conclusions

In this work, an MILP-based model was presented to optimize daily production and delivery decisions when considering multiple truck sizes and multiday trips. Considering the distribution decisions, it is important to remark that the way in which the types of trucks are represented in the formulation and the use of strategies for linking routes with the different truck sizes allow obtaining very good quality solutions in short CPU times. From a practical point of view, the analysis of various scenarios addressed shows the advantage of incorporating trucks of different sizes in order to improve transport efficiency and reduce distribution costs.

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