**Experimental Implementation of an Economic Model Predictive Control for Froth Flotation**

Paulina Quintanillaa,\*, Daniel Naviab, Stephen Neethlinga, Pablo Brito-Paradaa

a Department of Earth Science and Engineering, Royal School of Mines, Imperial College London, London SW7 2BP, United Kingdom

b Departamento de Ingeniería Química y Ambiental, Universidad Técnica Federico Santa María, Campus San Joaquín, Santiago 8940897, Chile

[p.quintanilla@imperial.ac.uk](mailto:p.quintanilla@imperial.ac.uk)

Abstract

We present the implementation of a novel economic model predictive control (E-MPC) strategy for froth flotation, the largest tonnage mineral separation process. A previously calibrated and validated dynamic model incorporating froth physics was used, which overcomes the limitations of previous simplified models reported in the literature. The E-MPC's optimal control problem was solved using full discretization with orthogonal collocation over finite elements, employing automatic differentiation via CasADi. This approach was applied in a 30-litre laboratory-scale flotation cell, significantly improving mineral recovery from 9% to 29% under feed flowrate disturbances while maintaining a minimum concentrate grade of 20%.

**Keywords**: Dynamic physics-based modelling, Economic model predictive control, Experimental validation, Froth flotation.

* 1. Introduction

The growing demand for minerals and metals in the shift towards cleaner energy sources poses a challenge due to decreasing ore quality. To meet demand effectively, improving

the efficiency of current mineral separation techniques while minimizing negative environmental impact is crucial. Froth flotation is the largest tonnage separation process. Given the large-scale nature of this process, even a small improvement in the separation efficiency can result in a substantial increment in mineral recovery (Ferreira & Loveday, 2000).

Effectively controlling the process is difficult due to its complex and dynamic nature (Quintanilla et al., 2021a) and process disturbances, such as feed flowrate, particle size, and feed grade. The traditional control method used in this process is Proportional-Integral (PI) control, primarily used for regulatory control. However, PI controllers alone are usually ineffective in optimizing key performance indicators, especially under process disturbances, leading to suboptimal outcomes. Advanced control and optimization strategies, particularly Model Predictive Control (MPC), have gained significant attention for improving process performance in froth flotation. MPC uses a dynamic model of the process to predict future behavior and optimize control actions, balancing performance while satisfying constraints. However, despite the potential benefits of MPC strategies in flotation, their full utilization has been hindered by the complexity of modeling process dynamics and instabilities. The kinetic models used in previous studies (e.g. Maldonado et al. (2007); Putz & Cipriano (2015); Riquelme et al. (2016)) are insufficient in modeling complex froth phase phenomena, which are critical drivers of the process performance. New advancements in flotation modeling for control that incorporate the froth phase phenomena can be found in Oosthuizen et al. (2021), and Quintanilla et al. (2021b, c).

Economic model predictive control (E-MPC) is a strategy to optimize control actions based on economic objectives. As such, it is a promising solution for enhancing flotation process efficiencies. E-MPC introduces the economic optimization layer into traditional model predictive control, allowing for direct integration of process economics and feedback control (Ellis et al., 2014). This approach considers both technical and economic variables as performance indices and uses nonlinear programming techniques to optimize the set points of control loops.

Our study validates an E-MPC strategy previously developed by Quintanilla et al. (2023a). The strategy was tested in a flotation rig described in Quintanilla et al. (2023b). We used a novel dynamic model incorporating froth physics, which was previously calibrated and experimentally validated. We selected the objective function based on sensitivity analyses, considering air recovery (a measurement of froth stability, directly linked to the flotation performance), separation efficiency (mineral recovery) and product quality (concentrate grade) as a proxy of economic performance.

* 1. Materials and Methods
     1. Model overview and definition of control variables

We use a nonlinear, dynamic model developed and experimentally validated by Quintanilla et al. (2021a, b). This model consists of a system of Differential and Algebraic Equations with a total of equations and variables, where K is the number of bubble size classes, and I is the number of mineralogical classes. Here, we assumed K=10 and two mineralogical classes (I=2): Chalcopyrite (valuable mineral) and gangue (waste rock with physical properties similar to quartz). As shown in Quintanilla et al. (2023a), the model is normalized to enhance the solver convergence and robustness. Important control variables of the model are given in Table 1.

Table 1: Variables used in control.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Variable** | **Definition** | **Variable** | **Definition** |  |
| Air recovery () | Fraction of air fed into the cell that overflows | Tails flowrate () | Flowrate of the tailings stream |  |
| Mass () | Mass of the mineralogical classes *i* | Superficial air velocity () | Air velocity injected into the flotation cell. |  |
| Gas holdup () | Volume fraction of gas of each bubble size class *k* | Grade () | Concentration of valuable minerals in the output stream (concentrate) |  |
| Pulp height () | Pulp level | Recovery () | The proportion of valuable minerals retrieved from the total available. |  |

* + 1. E-MPC strategy

The optimal control problem of the dynamic optimization is formulated as a nonlinear programming problem (NLP) using full discretization with orthogonal collocation over finite elements, and it was implemented in MATLAB R2021B with automatic differentiation via CasADi (Andersson et al., 2019). The optimal control problem was successfully implemented using the Interior Point Optimizer (IPOPT) solver. The sampling time was set at one second, and each iteration's solution time was, on average, 0.6 seconds. The general form of the NLP with normalized variables () is given by:

|  |  |
| --- | --- |
| s.t. | (1) |

where the normalized time () limits areand , the state vector is , the process variables vector is **y**, the algebraic variable vector is , the decision variable vector is (the superscript SP indicates that the variable is a set point for regulatory controllers), the weight vector is **,** is the dynamic model from Quintanilla et al. (2021b, c), and are the process constraints defined in Quintanilla et al. (2023a).

The objective function was selected via sensitivity analyses with respect to the decision variables, as discussed in Quintanilla et al. (2023b), and is defined as:

|  |  |
| --- | --- |
|  | (2) |

where is the dynamic concentrate grade, is the dynamic air recovery, is the mineral recovery at the end of the prediction horizon (), and are the decision variable vectors. It is important to note that the metallurgical recovery (*Rec*) is only meaningful at a steady state, which is why it is maximized at the end of the prediction horizon. To balance the trade-off between metallurgical recovery and concentrate grade, we propose to minimize the dynamic concentrate grade in the objective function (Eq. (2)) while imposing a process constraint where to ensure minimum quality.

* + 1. Experimental rig and E-MPC implementation

The experimental setup consists of a 30-litre tank, as described by Quintanilla et al. (2023b). The tank includes an airflow control system, peristaltic pumps for feed and pulp level regulation, and a sensor to measure the pulp level. Online measurement of air recovery is also available. The instrumentation links to Proficy Machine Edit ion software, which connects to MATLAB via an I/O Server. This configuration allows real-A diagram of a model

Description automatically generatedtime data acquisition to feedback the model and implement the controlled variables (and ), as shown in Figure 1.

The feed flowrate () is a measurable process disturbance. To test the robustness of the EMPC under disturbances, we performed step changes to the feed flowrate using four different values: 52.5, 56, 63, and 66.5 liters per minute [lpm]. Each feed flowrate was kept constant for 5 minutes, and each iteration was implemented every 1 minute in the laboratory-scale flotation rig.

Figure 1: Implementation framework of the E-MPC strategy at laboratory scale. The E-MPC determines the optimal control actions set points, which are then sent to the Programmable Logic Controller (PLC). The PLC signals the actuators (valves) to reach these set points.

* 1. Results and discussions

Figure 2 shows that mineral recovery was improved between +9% and +29% for the different conditions in feed flowrates. The concentrate grade was near 20% in most iterations, having some lower concentrate grades for the lowest feed flowrate (i.e., the first and third values). This issue is related to the five tuning parameters of the model remaining constant throughout all feed flow rate values.



Figure 2: Mineral recovery and concentrate grade for changes in feed flowrates .

As shown in Figure 3, air recovery generally followed the trends predicted by the E-MPC strategy, given the different values of superficial air velocity. Moreover, it can be observed that the highest air recoveries were obtained for the highest values of feed flowrates (63 and 66.5 [lpm]), which coincides with the highest pulp heights (see Figure 4). According to what was observed during the experiments, the increase in air recovery may be related to increments in overflowing froth velocity due to shallower froth depths.

Figure 3: Air recovery and superficial air velocity () for feed flow rate changes ().

Figure 4 shows the level control for the different pulp height setpoints sent from the E-MPC. While the trends of the pulp heights in the systems are the same as the setpoints, the values are usually different, which may be related to the differences between the parameters of the Proportional-Integral (PI) in the laboratory-scale system and the model used for the E-MPC. Those parameters are different because the sampling times are not the same in both cases, i.e. the PI controller had a sampling time of 1 second, while the model used in the E-MPC strategy corresponded to 10 seconds.

Figure 4: Control level using tail flowrates. Red lines are set points from E-MPC optimization, and blue lines are filtered pulp height (hp) in process.

* 1. Conclusions

This study validates an E-MPC strategy in a laboratory-scale flotation cell using a novel dynamic model that considers froth physics. The results of the experiments reveal that the E-MPC approach has led to higher metallurgical recoveries and has also shown its potential to handle feed flowrate disturbances efficiently. Further research will validate the E-MPC strategy in a laboratory-scale flotation bank (i.e. several tanks interconnected in series) to mimic the systems found in industrial-scale operations.

References

Andersson, J. A. E., Gillis, J., Horn, G., Rawlings, J. B., & Diehl, M. (2019). CasADi: a software framework for nonlinear optimization and optimal control. *Mathematical Programming Computation*, *11*, 1–36. <https://doi.org/10.1007/s12532-018-0139-4>

Ellis, M., Durand, H., & Christofides, P. D. (2014). A tutorial review of economic model predictive control methods. *Journal of Process Control*, *24*(8), 1156–1178. <https://doi.org/10.1016/j.jprocont.2014.03.010>

Ferreira, J. P., & Loveday, B. K. (2000). An improved model for simulation of flotation circuits. *Minerals Engineering*, *13*(14–15), 1441–1453. <https://doi.org/10.1016/S0892-6875(00)00129-1>

Maldonado, M., Sbarbaro, D., & Lizama, E. (2007). Optimal control of a rougher flotation process based on dynamic programming. *Minerals Engineering*, *20*(3), 221–232. <https://doi.org/10.1016/j.mineng.2006.08.015>

Oosthuizen, D. J., le Roux, J. D., & Craig, I. K. (2021). A dynamic flotation model to infer process characteristics from online measurements. *Minerals Engineering*, *167*, 106878. <https://doi.org/10.1016/j.mineng.2021.106878>

Perez-Correa, R., Gonzalez, G., Casali, A., Cipriano, A., Barrera, R., & Zavala, E. (1998). *Dynamic modelling and advanced multivariable control of conventional flotation circuits*. *11*(4), 333–346.

Putz, E., & Cipriano, A. (2015). Hybrid model predictive control for flotation plants. *Minerals Engineering*, *70*, 26–35. <https://doi.org/10.1016/j.mineng.2014.08.013>

Quintanilla, P., Navia, D., Neethling, S. J., & Brito-Parada, P. R. (2023a). Economic model predictive control for a rougher froth flotation cell using physics-based models. *Minerals Engineering*, *196*, 108050. <https://doi.org/10.1016/J.MINENG.2023.108050>

Quintanilla, P., Navia, D., Moreno, F., Neethling, S. J., & Brito-Parada, P. R. (2023b). A methodology to implement a closed-loop feedback-feedforward level control in a laboratory-scale flotation bank using peristaltic pumps. *MethodsX*, *10*. <https://doi.org/10.1016/j.mex.2023.102081>

Quintanilla, P., Neethling, S. J., & Brito-Parada, P. R. (2021a). Modelling for froth flotation control: A review. *Minerals Engineering*, *162*, 106718. <https://doi.org/10.1016/j.mineng.2020.106718>

Quintanilla, P., Neethling, S. J., Navia, D., & Brito-Parada, P. R. (2021b). A dynamic flotation model for predictive control incorporating froth physics. Part I: Model development. *Minerals Engineering*, *173*(November), 107192. <https://doi.org/10.1016/j.mineng.2021.107192>

Quintanilla, P., Neethling, S. J., Mesa, D., Navia, D., & Brito-Parada, P. R. (2021c). A dynamic flotation model for predictive control incorporating froth physics. Part II: Model calibration and validation. *Minerals Engineering*, *173*(November), 107190. <https://doi.org/10.1016/j.mineng.2021.107190>

Riquelme, A., Desbiens, A., Del Villar, R., & Maldonado, M. (2016). Predictive control of the bubble size distribution in a two-phase pilot flotation column. *Minerals Engineering*, *89*, 71–76. <https://doi.org/10.1016/j.mineng.2016.01.014>