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Towards the Development of Generalized Correlations for the Design of Compact Heat Exchangers

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This work looks at the design of finned surfaces, explores existing generalised correlations and further shows the development of new ones. The aim is to develop correlations with the following features: a) the range of validity of the Reynolds number is extended from laminar to turbulent regimes, and b) the correlations are a function of the fin geometry. In this work new generalised correlations for finned surfaces of the plain triangular type are developed and validated against experimental data and other existing generalised correlations. The new correlations are implemented in a design methodology. The results indicate that compared with experimental data, the average absolute error is 5 % for the friction and 7 % for Colburn factor. In design, the exchanger size results in smaller volumes down to 41 % compared with the use of published correlations.

1. Introduction

Compact plate and fin heat exchangers (PFHE) are suitable for many applications that include high pressure and temperatures (Mortean et al., 2016). They can be constructed in almost any kind of material which makes them capable of withstanding reactive and corrosive environments (Hathaway et al., 2018). The design methodologies of this type of exchangers are mature but the information on the thermohydraulic performance of the finned surfaces still faces some challenges. Some of them are the need to develop new correlations as a function of the basic fin geometry to cover a larger range of Reynolds number, and that need to improve on the accuracy with respect to experimental data (García and Picón, 2021). Currently, the designer depends on experimentally determined friction and Colburn factor data for a range of fins and this information is only available for existing commercial surfaces. Some research has been done to develop new surfaces and correlations. Aasi and Mishra (2021) studied the thermohydraulic performance of a PFHE using rectangular plain fins, introduced the concept of thermal efficacy and applied ANN (Artificial Neural Network) to predict the thermo-hydraulic performance of the unit. In the field of cryogenics, Hu and Li (2021) developed an expression to predict the local heat transfer coefficient; their comparison versus the experimental data was within a ±20 %. Other authors have studied the relation between the shape and geometry of the fin with the thermohydraulic performance. For instance, Tariq et al. (2021) found that perforations and slots enhance the convection coefficient and reduce the pressure drop compared to conventional fins. Do Nascimento et al. (2020) using CFD simulations and multi-objective optimization techniques analysed offset strip-fins and developed correlations between thermal performance and Reynolds number. Yang et al. (2019) studied the surface efficiency in a double stack arrangement for air and water. They found a deviation of the Colburn factor of 3.7 % for air and 3.2 % for water compared to experimental data.

There is a strong relation between fin geometry, flow regime and the thermohydraulic performance of a PFHE. In terms of design, it is of paramount importance to count with the means to predict friction and Colburn factors as a function of the fin geometry and flow conditions. Most of the work done on the prediction of the friction and Colburn factor is based on expressions obtained from experimental and numerical analysis. Usually, those expressions are valid for laminar and turbulent flow without considering the transition zone. The approach in the present work proposes a model to predict the friction and Colburn factor for the laminar, transition and turbulent

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flow in PFHE and due to space limitations, only the case of triangular plain surfaces is covered. In this paper the experimental data of Kays and London (1984) are used to validate the generalised correlations. The methodology is demonstrated on a case of study.

2. Generalised correlations.

Churchill and Usagi (1972) demonstrated that in the case of heat transfer is possible to obtain generalised correlations by taking the *pth* root of the sum of the *pth* power of the limiting solutions for large and small values of the independent variable. Using this principle, the development of a general friction factor f correlation for plain fin surfaces can be expressed as:

$$f^n = {f_{lam}}^n + {f_{turb}}^n \tag{1}$$

Where, f_{lam} and f_{turb} are the friction factor in the laminar and turbulent zone. In the laminar region, the pressure drop depends on the fluid velocity, so, it is possible to obtain an analytic solution for the friction factor as:

$$f_{lam} = C_{f,lam}/Re \tag{2}$$

Where $C_{f,lam}$ is a constant that depends on the duct geometry. Carreón (2010) studied the triangular surfaces in the collection proposed by Kays and London (1984) and related the value of the constant $C_{f,lam}$ with the aspect ratio; this is, the fin height divided by the fin basis. The analysis showed that there exists a linear relation as follows:

$$f_{lam} = [24 - 9.77\,(A_R)^{0.5}]/Re \tag{3}$$

The friction factor for the turbulent zone is obtained from the Blasius correlation:

$$f_{turb} = C_{fT1}/Re^{C_{fT2}} \tag{4}$$

Performing a similar analysis, in turbulent flow for circular tubes the constant C_{fT1} equals to 0.078 and $C_{fT2} = 0.25$. The expression to predict the friction factor in turbulent zone is:

$$f_{turb} = 0.078/Re^{0.25} \tag{5}$$

A similar analysis for heat transfer can be developed for the laminar regime where the developing zone is considered. The Leveque equation is used (Bennett, 2020).

$$Nu_{Dev} = KGz^{1/3} \tag{6}$$

Where *K* is the Leveque constant and *Gz* is the Graetz number. The Graetz number is calculated by:

$$Gz = RePr\left(\frac{d_h}{L_f}\right) \tag{7}$$

Where Re and Pr are the Reynolds and Prandtl numbers, d_h is the hydraulic diameter and L_f is the fully developed flow length. The Nusselt predicted (Nu_{Pred}) considering the transition from developing flow to fully developed flow is given by:

$$Nu_{Pred} = \left(Nu_{\lim}^3 + Nu_{Dev}^3\right)^{1/3} \tag{8}$$

Introducing Eq(6) into Eq(8)

$$Nu_{Pred} = \left(Nu_{\lim}^3 + KGz^{1/3}\,\right)^{1/3} \tag{9}$$

For triangular surfaces, Carreón (2010) proposed an expression to calculate the limiting Nusselt number (Nu_{lim}) as a function of the characteristic angle 2θ.

$$Nu_{lim} = -1.152 \cdot 10^{-8}(2\theta)^4 + +5.323 \cdot 10^{-6}(2\theta)^3 - 9.48 \cdot 10^{-4}(2\theta)^2 + 6.48 \cdot 10^{-2}(2\theta) + 1.02 \tag{10}$$

Carreón (2010) used Eq(10) to obtain the limiting Nusselt for several commercial surfaces contained in Kays and London (1984). He also found a relation between the characteristic angle 2θ and the K constant as:

$$K = 2.78 - 0.033(2\theta) \tag{11}$$

Using the expression for K and introducing Eq(11) into Eq(9) and rearranging gives:

$$Nu_{Pred} = \left[{Nu_{lim}}^3 + \left(2.78 - 0.033(2\theta)\right)Gz^{1/3}\,\right]^{1/3} \tag{12}$$

2.1 Turbulent region.

The entrance effects in the turbulent region are considered by introducing a correction factor (F_{ie}):

$$F_{ie} = 1 + \left(0.68 + \frac{3000}{Re^{0.81}}\right) / \left(\frac{L_f}{d_h}\right)^{0.9} Pr^{1/6} \tag{13}$$

Thus, the Nusselt number considering the inlet effects Nu_∞ in the turbulent zone is expressed as (Al-Arabi, 1982):

$$Nu_\infty = F_{ie} Nu_{Turb} \tag{14}$$

To determine the heat transfer in the turbulent zone (Churchill, 1977) an expression based on the Colburn factor is used:

$$j_{turb} = a\sqrt{f} Re^b \tag{15}$$

where a is the coefficient and b the exponent obtained from the fitting of experimental data for commercial surfaces, f is the friction factor from Eq(1) and Re is the Reynolds number. To obtain *a* and *b*, experimental data for the turbulent regime are analyzed. Triangular surfaces 10.27T, 11.4T and 12.00T (Kays and London, 1984) were studied. The values of the constants are *a*=0.01625 and *b*=0.0675. Substituting *a* and *b* into Eq(10) the resultant expression is:

$$j_{turb} = 0.01625\sqrt{f} Re^{0.0675} \tag{16}$$

The Colburn factor (j) is represented by:

$$j_{turb} = St Pr^{2/3} \tag{17}$$

where:

$$St = Nu_\infty / Re\ Pr \tag{18}$$

So, the expression for the Nusselt number in the turbulent region considering the inlet effects is:

$$Nu_\infty = 0.01625 F_{ie}\sqrt{f} Re^{1.0675} Pr^{1/3} \tag{19}$$

Taking into consideration the effects of the transition zone, Churchill (1977) stablished that the Nusselt for the laminar, transition and turbulent may be obtained using the following expression:

$$Nu_{pred} = \left[{Nu_{lam}}^m + \left(\frac{1}{{Nu_i}^2} + \frac{1}{{Nu_\infty}^2}\right)^{-m/2} \right]^{1/m} \tag{20}$$

where ${Nu_i}^2$is the Nusselt at the transition zone and is calculated as follows:

$$Nu_i = Nu_{lam,Re_{Crit}} \cdot e^{\left(\frac{Re - Re_{crit}}{730}\right)} \tag{21}$$

where $Nu_{lam,Re_{Crit}}$ is the Nusselt evaluated at the critical Reynolds (Re_{crit}). Carreón (2010) found that the value of *m*=3 in Eq(20), gives the best prediction for the laminar, transition and turbulent regimes. For the surfaces classified as triangular by Kays and London (1984), Eq(20) is used to calculate the Nusselt number in all flow regimes as a function of Reynolds number and fin geometry. Once the Nusselt number (*Nu*) is determined, the heat transfer coefficient *(h*) is obtained as a function of thermal conductivity (*k*) and hydraulic diameter (d_h) from:

$$Nu = h d_h / k \tag{22}$$

In the next sections, these generalized models are validated against experimental data, then they are in used in design. The methodology proposed by García and Picón (2020) is used and the results are compared with other published correlations.

3. Model validation

To evaluate the accuracy for the new model, the prediction of the friction factor and Nusselt number are compared with the experimental data from Kays and London (1984) and with correlations for triangular surfaces that depend on the plate spacing (δ) and fin pitch (F_{pitch}). The expressions are presented in Table 1.

Table 1: Correlations for triangular heat transfer surfaces

Expression		Range of validity	Std dev.	Notes
		Triangular surfaces		
$j = 0.718Re^{-0.625}\left[\delta/F_{pitch}\right]^{0.765}\left[F_{th}/F_{pitch}\right]^{0.765}$	(23)	$100 < Re < 1{,}000$	±12 %	(Chennu, 2018)
$j = 0.789Re^{-1.1218}\left[\delta/F_{pitch}\right]^{1.235}\left[F_{th}/F_{pitch}\right]^{-0.764}$	(24)	$1{,}000 < Re < 10{,}000$	±12 %	(Chennu, 2018)
$f = 3.12Re^{-0.852}\left[\delta/F_{pitch}\right]^{0.156}\left[F_{th}/F_{pitch}\right]^{-0.184}$	(25)	$100 < Re < 1{,}000$	±11 %	(Chennu, 2018)
$f = 2.69Re^{-0.918}\left[\delta/F_{pitch}\right]^{0.355}\left[F_{th}/F_{pitch}\right]^{-0.175}$	(26)	$1{,}000 < Re < 10{,}000$	±11 %	(Chennu, 2018)

Figure 1 and Table 2 show the comparison of the friction and Colburn factor between experimental data, generalised correlations, and the new model. The absolute error deviation with respect to experimental data is determined. The deviation for the friction and Colburn factor for the new model is calculated from:

$$f_{Dev} = \left|\left(f_{Exp} - f_{New}\right)/f_{Exp}\right| \cdot 100\ [\%] \quad (27)$$

$$j_{Dev} = \left|\left(j_{Exp} - j_{New}\right)/j_{Exp}\right| \cdot 100\ [\%] \quad (28)$$

The maximum absolute error using the new generalised correlation is 11 % for the friction factor and 20 % for Colburn factor while the average absolute errors are 5 % and 7 %. In the case of other correlations, the maximum absolute error for the friction factor is 88 % and for the Colburn factor is 65 %; the average absolute errors are 72 % and 45 %. The new model predicts the thermo-hydraulic performance in a more accurate way compared with the experimental results.

Table 2: Average deviation for Colburn and friction factor for triangular surface 46.95T

Re	f_{Exp}	j_{Exp}	f_{Che}	j_{Che}	$f_{Dev,Che}$	$j_{Dev,Che}$	f_{New}	j_{New}	$f_{Dev,New}$	$j_{Dev,New}$
300	0.0662		0.0276	0.0093	58 %		0.0733	0.0170	11 %	
500	0.0419	0.0110	0.0179	0.0068	57 %	38 %	0.0441	0.0121	5 %	10 %
600	0.0357	0.0089	0.0153	0.0061	57 %	32 %	0.0368	0.0107	3 %	20 %
1000	0.0228	0.0068	0.0099	0.0044	57 %	35 %	0.0225	0.0076	1 %	12 %
1500	0.0165	0.0052	0.0050	0.0034	70 %	35 %	0.0159	0.0058	3 %	11 %
2000	0.0137	0.0045	0.0038	0.0029	72 %	36 %	0.0130	0.0048	5 %	8 %
3000	0.0109	0.0037	0.0026	0.0022	76 %	40 %	0.0107	0.0037	2 %	2 %
4000	0.0095	0.0034	0.0020	0.0019	79 %	46 %	0.0097	0.0031	3 %	10 %
5000	0.0087	0.0033	0.0017	0.0016	81 %	51 %	0.0092	0.0031	5 %	5 %
6000	0.0083	0.0032	0.0014	0.0014	83 %	55 %	0.0088	0.0031	7 %	2 %
7000	0.0079	0.0031	0.0012	0.0013	85 %	58 %	0.0085	0.0031	8 %	1 %
8000	0.0076	0.0030	0.0011	0.0012	86 %	60 %	0.0083	0.0030	8 %	1 %
10000	0.0072	0.0030	0.0009	0.0010	88 %	65 %	0.0079	0.0029	9 %	1 %
				Average	72 %	45 %		Average	5 %	7 %

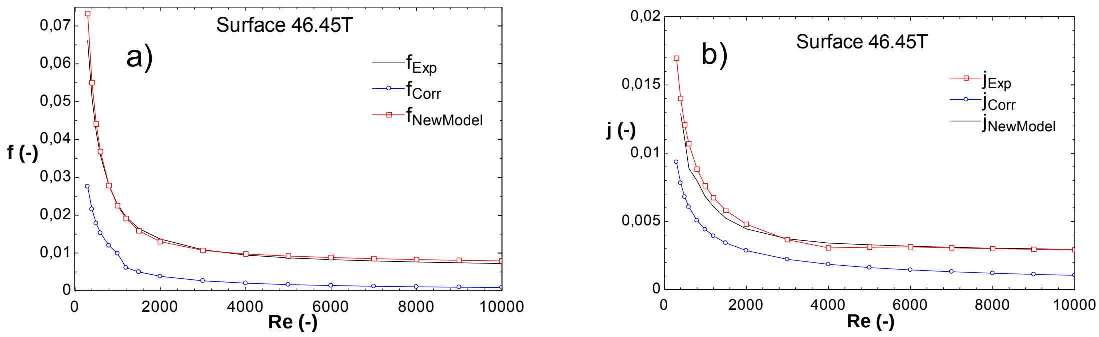

Figure 1: Comparison between experimental data, correlations, and proposed model for surface 46.45T a) f vs Re, b) j vs Re

4. Case study

The case study is taken from García and Picón (2021), where a heat exchanger to cool methanol using plate and fin technology is to be designed. Table 3 shows the operating data, physical properties, and fin geometry for the design. In the study by García and Picón (2021), generalised correlations are used to predict the heat transfer coefficients and the design is carried out using the minimum and maximum fin density. Table 4 contains the results obtained using the proposed approach. For the case in which the hot and cold streams have the minimum fin densities (4 fins/in) the volume of the unit is 1.14 m^3 which is 41.8 % smaller than the one predicted using correlations. In the maximum fin density scenario (28.2 fins/in), the volume for the unit is $V_T = 0.042\ m^3$.

Table 3: Operating data and physical properties for the case study

Process data	Cold stream	Hot stream
Mass flow rate (kg/s)	101.40	30
Pressure drop (Pa)	10,000	25,000
Inlet temperature (K)	303.15	363.15
Outlet temperature (K)	313.15	313.15
Density (kg/m^3)	995.0	750
Heat capacity (kJ/kgK)	4.20	2.84
Thermal conductivity (W/mK)	0.59	0.19
Viscosity (kg/m s)	0.00034	0.0008
Thermal conductivity material of construction (W/mK)	16.30	
Heat load (kW)	4,260	
Plate spacing (m)	0.0065	
Fin thickness (m)	0.0003	

Table 4. Volume dimensions for the case study using triangular fins.

Dimension	Correlation		New model		Percentual deviations	
	F_{in}(fins/in)		F_{in}(fins/in)			
	Hot side	Cold side	Hot side	Cold side	Hot side	Cold side
	$F_{in} = 4$	$F_{in} = 28.2$	$F_{in} = 4$	$F_{in} = 28.2$	$F_{in} = 4$	$F_{in} = 28.22$
Volume (m^3)	1.96	0.035	1.14	0.042	-41.84 %	+20 %
Width (m)	0.405	0.68	0.58	0.87		
Height (m)	0.405	0.68	0.58	0.87		
Length (m)	14.73	0.09	4	0.06		
Free flow area (m^2)	0.058	0.061	0.1238	0.1046		
Reynolds number (-)	2,520	198	1,172	114.7		
	20,040	1,580	9,322	912.6		
Surface area (m^2)	778.3	62.63	454.5	76.1	-41.6 %	+17.7 %
	778.3	62.63	454.5	76.1	-41.6 %	+17.7 %
Pressure drop (-)	7,791	8,637	2,315	9,090		
	10,000	10,000	10,000	10,000		

Table 5. Volume dimensions for case study using triangular fins.

Dimension	Correlation		New model			
	F_{in}(fins/in)		F_{in}(fins/in)			
	Hot side	Cold side	Hot side	Cold side		
	$F_{in} = 10.7$	$F_{in} = 20$	$F_{in} = 10.7$	$F_{in} = 20$		
Volume (m^3)	0.39		0.23		-41 %	
Width (m)	0.75		0.80			
Height (m)	0.75		0.80			
Length (m)	0.85		0.39			
Free flow area (m^2)	0.16	0.12	0.20	0.14		
Reynolds number (-)	381.7	1,788	309.3	1,449		
Surface area (m^2)	301	512.60	173.20	295	73.78 %	73.76 %
Pressure drop (-)	1,257	10,000	972.50	10,000		

García and Picón (2021) obtained a design for a set of heat exchanger dimensions using a fin density for the hot and cold side of 10.7 and 20 fins/in. For these fin densities, the size of the unit obtained using the new generalised correlation is 41 % smaller, this is due to the under prediction of the Colburn factor with respect to the experimental data that most published correlations give. The results obtained in this paper open the way to explore the use of modern techniques for the prediction of the thermal performance of secondary surfaces such as machine learning or artificial neural networks. This will be the focus of future work.

5. Conclusions

A generalised expression to predict the friction and Colburn factor of triangular surfaces for laminar, transition and turbulent flow as a function of fin geometry was developed. The model is accurate compared with experimental data as the mean absolute error for the friction and Colburn factor are 5 % and 7 %. Compared to existing generalised correlations, the new model gives designs that are 41 % smaller. An advantage of the approach is the use of only one expression for the Colburn and friction factor to make predictions across the laminar and turbulent regimes. The methodology applied in the development of the new correlation can easily be extended to cover other types of secondary surfaces commonly employed in the design of heat exchangers of the plate and fin type.

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