

A Goal Programming Approach for Optimizing Natural Gas Transportation Network

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Natural gas (NG) is widely transported through the high-pressure pipeline network. For maintaining the high pressure in the gas pipeline network, compressor stations are installed at fixed intervals along with the pipeline. The compression procedures involve huge amounts of energy and investment. So designing any pipeline network in NG transportation also needs to include conflicting goals of various criteria with the priority like maximization of demand satisfaction and minimization of compression energy and investment cost. In this paper, a goal programming approach dependent upon lexicographic methodology is proposed for solving the multi-criteria problem in NG transportation where each criterion is incorporated on a priority basis to achieve targeted goals. The relevance of the proposed model is explained via an example. Maximization in demand satisfaction is a prime objective and remaining targeted goals are attained on a priority basis. The deviation variable in the proposed model is solved in GAMS by using the CPLEX solver. These deviation variables provide flexibility to the decision-maker to change their priorities according to the social, environmental, and economical constraints.

1. Introduction

Natural gas (NG) is a low-carbon, environmentally friendly, and high-quality energy source. Because of its convenience and economic feasibility, the pipeline is considered one of the most common ways to supply a huge volume of NG from sources station to demand end (Wang et al., 2018). Normally, in pipeline transportation the primary aims of decision-makers is to maximize demand satisfaction or maximum delivery, the economic benefit, reduce the environmental effect satisfy multiple objectives concurrently (Wu et al., 2018). The regulatory body must first define the pipeline project design criteria for designing a transport network for an attractive return on investment as well as the needs of current and potential customers.

The goal of multi-objective optimization is to identify the best operational strategies that achieve two or multiple goals simultaneously. Martin et al. (2006) used a mixed-integer linear programming (MILP) framework to investigate the optimum supply chain's sensitivity to increases in fuel and installation cost (pipes, loading tanks, etc.) as well as changes in energy consumption. Tabkhi et al. (2010) suggested an algorithm for reducing the high-pressure gas network's expenditure and operational costs. Further, Cheboub (2015) addresses gas pipeline line pack control, which involves making trade-offs between reducing energy consumption in compression stations and enhancing gas line pack. Da Silva et al. (2016) suggested a MILP framework in a multi-objective optimization system to identify the least transportation cost along with optimizing compressor station (CS) gas flow. Recently, Su et al. (2019) suggested a multi-objective optimization model using grid concept for the trade-off between consistency and energy demand in pipeline network for the policy-making process capacities at altered situations. Sukharev et al. (2019) proposed multiple-criteria decision analysis based on the Monte Carlo method for cross-country gas. Schlossera et al (2019) proposed an energy-efficient model utilizing pinch analysis for vapor-compression heat pumps for various industries.

Since multiple objectives need to be achieved, the researcher developed a mathematical model which represents those purpose. The goal programming (GP) techniques have become a widely used approach in multi-criteria decision-making problems. Each criterion in the GP technique is given a goal or target value to reach, and undesirable deviations from this set of target levels are then reduced by the decision variables.

Carvalho et al. (2012) proposed the MILP model which provides a Pareto frontier set of solutions representing optimal trade-offs between economic and environmental goals. Recently, Osiadacz and Isoli (2020) proposed a bi-objective methodology for a trade-off between reducing compressor operating costs and increasing gas network capability. From the literature review, it can be concluded that the optimization in gas transportation is well studied while the multi-objective problems which consider demand satisfaction, investment cost, and compression energy simultaneously in pipeline transportation are not focused simultaneously. In this paper, an optimization model based on lexicographical GP is proposed to minimize investment cost, compression energy requirement in transporting NG, and maximize demand satisfaction. In this view, the problem statement and model formulation are discussed in the next sections.

2. Problem Statement and Mathematical Formulation

A GP approach is introduced in NG transportation which considers three sustainability measures i.e. demand satisfaction, energy consumption, investment cost. Consider a gas allocation network consist several existing CS which are supplying natural gas flow to demand compressor station. For an allocation network, the following parameter is given.

- Maximum available flow (F_i) of each existing CS i th ($i=1, 2, 3, \dots, n$) and demand CS j th ($1, 2, 3, \dots, m$).
- Pressure (P_i) of each existing CS and demand station (P_j).
- Projected goal G_k to be achieved for k th ($k=1, 2, 3, \dots, x$) criteria along with its priority.
- Cost Factor (C_{ij}) corresponding to each route.

A symbolic representation of the problem statement is shown in Figure 1. Table 1 shows the nomenclature section.

Table 1: Nomenclature

Set/Parameter/ Variable	Context
I	i/i Source CS
J	j/j Demand CS
K	k / k index of criteria
$\mu^{(ij)}$	Energy index of supply CS/ demand CS
λ	Compression Energy Index
F	Volumetric flow rate (m^3/s)
F_o^*	Volumetric flow rate at atmospheric condition (Sm^3/s)
P_0	Pressure at atmospheric condition (kPa)
G_k	Projected goal to be achieved for k^{th} criteria
P_j, P_i	Demand and supply pressure, (kPa)
F_i	Maximum limit of supply flow from CS, (Sm^3/s)
C_{ij}	The investment cost for supplying flow from i^{th} station to j^{th} demand
f_{ij}	Flow supply from compression station (i) to demand station (j), (Sm^3/s)
d_k^- / d_k^+	Positive / Negative deviation variable for over-achievement for k^{th} criteria

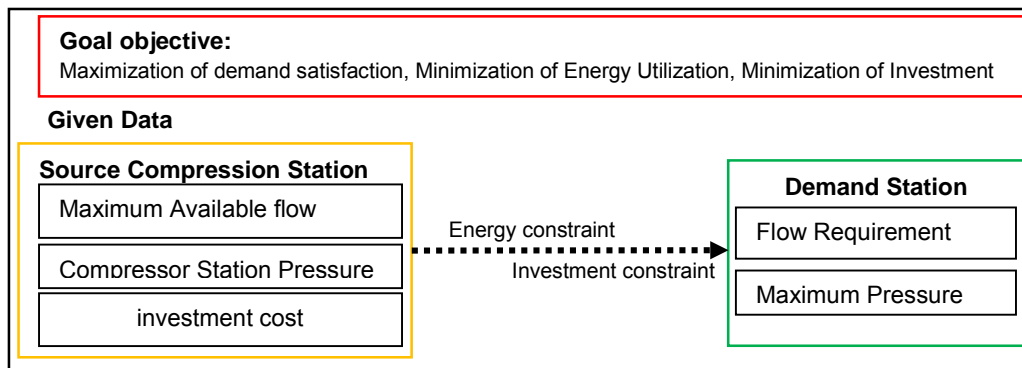


Figure 1: Representation of the problem

Following constraints comprises the model. It consists of material balance and, thermodynamic related constraints.

2.1 Material balances:

The flow from CS (i) to demand (j) is symbolized by f_{ij} . The flow balance for each source CS is addressed by Eq(1).

$$\sum_{j=1}^m f_{ij} = F_i \quad \forall i \in I \quad (1)$$

2.2 Thermodynamic related Constraints:

The compression work or energy requirement is directed by the initial and final states along with the volumetric flow. For isothermal compression, the energy requirement (E) can be expressed as Eq(2): (Shukla and Chaturvedi, 2020)

$$\text{Net Energy, E} = \begin{cases} F_0^* \left[\left(P_0 \ln \left(\frac{P_i}{P_0} \right) \right) - \left(P_0 \ln \left(\frac{P_j}{P_0} \right) \right) \right] & \text{For isothermal process} \\ \left(\frac{n}{n-1} \right) P_0^{\frac{1}{n}} P_i^{\frac{n-1}{n}} F_0^* \left[\left(\frac{P_i}{P_0} \right)^{\frac{n-1}{n}} - 1 \right] & \text{For polytropic process} \end{cases} \quad (2)$$

Where F_0^* represents the volumetric flow rate at atmospheric condition, P_0 is the atmospheric pressure, P_j and P_i are the demand and supply pressures while n is the polytropic index.

The Energy Index (EI) can be estimated utilizing Eq(3).

$$\mu_{(i/j)} = \begin{cases} P_0 \ln \left(\frac{P_{(i/j)}}{P_0} \right) & \text{For isothermal process} \\ \left(\frac{n}{n-1} \right) P_0 \ln \left(\frac{P_{(i/j)}}{P_0} \right)^{\frac{(n-1)}{n}} & \text{For polytropic process} \end{cases} \quad \forall i \in I, j \in J \quad (3)$$

Where $\mu_{(i/j)}$ shows the energy index for the compressor station.

The quantity $(\mu_j - \mu_i)$ can be stated as the compression energy index, λ_{ij} (CEI) and expressed as Eq(4);

$$\lambda_{ij} = (\mu_j - \mu_i) \quad \forall i \in I, j \in J \quad (4)$$

GP introduces an auxiliary deviation variable that acts as a 'facilitator' to frame the model. Goal constraint is defined in Eq(5) and Eq(6).

$$\sum_{i=1}^n \sum_{j=1}^m f_{ij} + d_k^- - d_k^+ = G_k^1 \quad \forall k \in K \quad (5)$$

$$\sum_{i=1}^n \sum_{j=1}^m (f_{ij} * \lambda_{ij}) + d_k^- - d_k^+ = G_k^2 \quad \forall k \in K \quad (6)$$

There are two types of deviation variables; negative deviation (d_j^-) for under performance of the objective and positive deviation (d_j^+) for over performance of the objective or vice versa depending on nature of objective. These negative and positive deviations depict the gap between the goal and the obtained result. Unlike the linear programming model that directly determines the solution via optimizing objectives; the GP tries to reduce undesirable deviations between the goal's expectation level and the solution.

2.3 Objective functions:

The objective function for achieving multiple goals based on priority order can be expressed by Eq(7).

$$Z = \text{minimize} \{ d_{k=2}^-, d_{k=1}^-, d_{k=3}^+, d_{k=4}^+ \} \quad (7)$$

The GP model minimizes unwanted deviations in a lexicographical way. This approach determines the optimal solution of the overall problem by sequentially solving several sub-problems which occur on a priority basis for each goal. According to priority, a sub-problem is solved for minimizing the unwanted deviation variable of the current goal. Then, this deviation variable value becomes a constraint for the next sub-problem which is solved for minimizing the unwanted deviation of subsequent goals on a priority basis.

3. Proposed Algorithm

The algorithm for the proposed methodology in NG transportation concludes the following steps.

Step 1: Calculate the energy index of supply CS and demand CS using Eq(3).

Step 2: Calculate the compression energy index using Eq(4).

Step 3: Formulate the model with multiple goals which minimize unwanted deviations.

Step 4: To achieve the current goal solve the model for minimizing unwanted deviation.

Step 5: Repeat Step 4 for satisfying each goal while using the previous unwanted deviation as a constraint.

Step 6: Tabulate the result of the deviation variable.

Flow chart showing a proposed algorithm for obtaining optimal network for NG transportation.

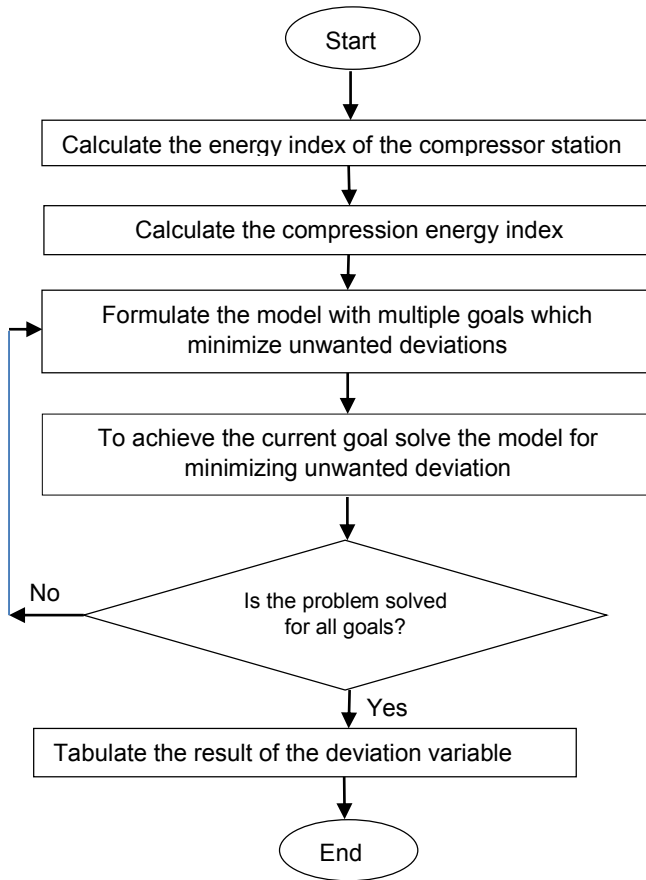


Figure 2: Flowchart for the goal programming approach

4. Illustrative Example

The suggested procedure is explained via an illustrative example in this section. In this example, two demands CS at 500 kPa and 620 kPa are to be satisfied by four existing CS. Table 2 shows the available flow at a specific pressure is given:

Table 2: Data for Source Compressor Station

Compressor Station	Pressure (kPa)	Maximum available Flow (Sm ³ /s)
X_1	200	300
X_2	300	240
X_3	130	360
X_4	140	490

Table 3: Cost factor and Compression Energy Index for demand 1 and demand 2

Compressor Station	Cost Factor for demand 1 (\$·s/Sm ³)	Cost Factor for demand 2 (\$·s/Sm ³)	CEI for demand 1 (kJ/Sm ³)	CEI for demand 2 (kJ/Sm ³)
X_1	10	15	92.58	114.65
X_2	8	13	51.76	73.56
X_3	15	20	62.44	84.24
X_4	7	12	22.61	44.41

The compression process is assumed to be isothermal. The compression energy index (CEI) for supplying the flow is calculated Using Eq(3) and Eq(4). Table 3 shows the CEI's and cost factors corresponding to the various transporting route is given.

Table 4 shows the value of criteria i.e. each demand, total investment cost, and total compression energy for demands.

Table 4: Values (Goal) of criteria

Criteria	Value
Demand D ₁ (Sm ³ /s)	600
Demand D ₂ (Sm ³ /s)	680
Investment Cost (\$)	90,000
Total Compression energy (kJ/s)	18,000

The goal constraint equation for satisfying the demand 1 is given in Eq(8).

$$f_{11} + f_{21} + f_{31} + f_{41} + d_1^- - d_1^+ = 600 \quad (8)$$

The equation limits demand satisfaction to the desired goal. Hence (d_1^-) should be minimized. Similarly, the demand balance equation on the priority-based satisfying the demand 2.

$$f_{12} + f_{22} + f_{32} + f_{42} + d_2^- - d_2^+ = 680 \quad (9)$$

This equation, limits demand satisfaction to the desired goal. Hence (d_2^-) should be minimized.

The equation for achieving the goal to restrict compression energy consumption is given in Eq(10).

$$92.85f_{11} + 1.76f_{21} + 62.44f_{31} + 22.61f_{41} + 114.65f_{12} + 73.56f_{22} + 84.24f_{32} + 44.21f_{42} + d_3^- - d_3^+ = 90,000 \quad (10)$$

Their energy consumption is the desired goal, so the unwanted deviation i.e. (d_3^+) should be minimized.

The equation for achieving the goal to get a maximum investment of \$18000 is given in Eq(11).

$$10f_{11} + 8f_{21} + 15f_{31} + 7f_{41} + 15f_{12} + 13f_{22} + 20f_{32} + 12f_{42} + d_4^- - d_4^+ = 18,000 \quad (11)$$

This equation is for attaining profit of desired goal, so the unwanted deviation i.e. (d_4^+) should be minimized.

For the multi-criteria problem, minimizing the deviation of projected goals on a priority basis using the Lexicographic method. The GP model is solved in GAMS by using a CPLEX solver. The interpretation of the result shown in Table (5) is as follows:

Table 5: Result for deviation variables of each criterion

Variable	d_1^-	d_1^+	d_2^-	d_2^+	d_3^-	d_3^+	d_4^-	d_4^+
Value	50	0	0	0	0	2,199.4	1,490	0

To design of allocation network in NG transportation needs to combine several conflicting goals simultaneously. This multi-criteria decision variable is solved using the GP model and the solution provides the deviation in criteria utilized by the decision-maker for designing an optimal allocation network. In this example, the priority is satisfying the demand 2. The result shows the zero deviation i.e. demand 2 is satisfied. For demand 1, a negative deviation of 50 Sm³/s shows of flow is required i.e. supplied flow from supply stations to demand 1 is lesser than the targeted demand. In the next priority for the goal of compression energy requirement, a positive deviation of 2,199.4 kJ/s of energy is obtained. It means [energy consumed by the CS is greater than the required goal. So for satisfying the energy demand, external source CS needs to be installed. For targeting the investment cost, a positive deviation of zero shows that the goal is achieved, and a negative deviation of \$ 1,490.5 total investment is lesser than the required goal. The proposed model provides flexibility to the decision-maker to change their priorities according to environmental and economical constraints.

5. Conclusion

Transportation of NG consists of several conflicting goals that need to be addressed simultaneously. In this paper, the multi-criteria problem in NG transportation is solved using GP in a lexicographical manner. GP is used as it seems to be most suitable and appropriate tool in multi-objective problems where predefined goal or priority is given. In gas pipeline transportation, three objectives are considered which are maximization of

demand satisfaction, minimization of investment cost associated with flow transportation, and compression energy requirement. The prioritized criteria will govern the result of optimization and also we can draw the following conclusion.

- For demand 2, the goal objective for satisfying the demand is completely attainable with the available source CS.
- The model suggests that the achievement of the goal set for demand 1 is not attainable without any additional external CS.
- The positive deviation shows that goal associated to meet the energy consumption in the natural gas transportation network is not fulfilled which means external CS is needed to fulfill the energy demand.
- For the goal of investment cost, a positive deviation of zero shows that the goal is achieved and a negative deviation of \$ 1,490 shows that the investment is lesser than the required goal. So the decision-maker can utilize the investment budget for installing the external CS.

The result shows the model can efficiently find an optimum transportation network of NG within a given criterion. It can also help monitor the amount of compression energy used, the cost of expenditure, and the fulfillment of demand. The developed model allows policymakers to discuss various scenarios with various probable norms under various priority rankings.

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