

# Transient Heat Transfer Analytical Model for Low-melting-point-metal Phase Change Material Heat Sink

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This paper presents a low-melting-point-metal phase change material/air natural convective cooling combined heat sink for the thermal control requirement with transient thermal shock. A theoretical transient heat transfer analytical model is established, and the general process for the design of the heat sink is proposed. The thermal control performance and advantages of this heat sink were demonstrated through numerical simulation studies of specific cases, and the reliability of the analytical model was verified by comparison with the numerical results. The transient heat transfer analytical model established in this paper can provide rapid performance prediction and evaluation for the low-melting-point-metal phase change material/air natural convective cooling heat sink, and provide guidance for practical thermal design.

## 1. Introduction

Phase change materials (PCM) are a class of materials that undergo a solid-liquid phase transition at a specific temperature with a large amount of latent heat absorption or release. PCM has broad application in the field of thermal energy storage and thermal control of electronic devices (Fleischer, 2015). In thermal control of electronic devices, PCM absorbs the heat generated by the device during work, and release the heat to the environment after the device stops working, thereby prevent the device from overheating (Yang et al., 2018). Traditional PCM used for thermal control of electronic devices are mainly organic PCM, the main shortcoming of organic PCM is their low thermal conductivity, poor heat transfer capability, large volume expansion in phase change. Many efforts have been done to improve this situation, including the addition of high thermal conductive nanoparticles (Liu et al., 2016); and the provision of high heat transfer paths, such as internal fins, metal foam, heat pipes (Li et al., 2016).

In recent years, low-melting-point metal PCM has received widespread attention (Ge et al., 2013). Low-melting-point-metals are a class of metallic materials with melting points near room temperature, which are generally gallium-based alloys (Salyan and Suresh, 2017). They have advantages of high thermal conductivity (generally 10-40 W/m·K), excellent heat transfer performance, large heat storage density, and small volume change during phase change, generally less than 3 % (Yang et al., 2017).

A theoretical model is of great importance in practical thermal design, it can provide rapid evaluation of the design, without the need to spend much time and money as that in experiment and simulation. Until now, there is no analytical model for the design of low-melting-point-metal PCM heat sink in the literature. In this paper, a low-melting-point metal/air natural cooling combined heat sink is proposed, and a theoretical, analytical model for the transient heat transfer process of the heat sink is established.

## 2. Heat transfer analytical model

### 2.1 Problem description

Figure 1 shows the schematic diagram of the problem to be analysed in this paper. It is assumed that the electronic device (heat source) has two working states: normal working state and pulse working state. In the

normal working state, the heating power of the heat source is maintained at a low-value  $q_n$ , and conventional natural cooling heat sink can meet the cooling demand. During the pulse working state, the heating power of the heat source suddenly increases to a high-value  $q_p$ , the original natural cooling heat sink may not be able to cope with the large heat pulse, which causes the device overheating. This problem can be well solved by the use of phase change material, as shown in Figure 1. This paper is aimed at providing a theoretical model for the two heat sinks and evaluating their thermal control performances.

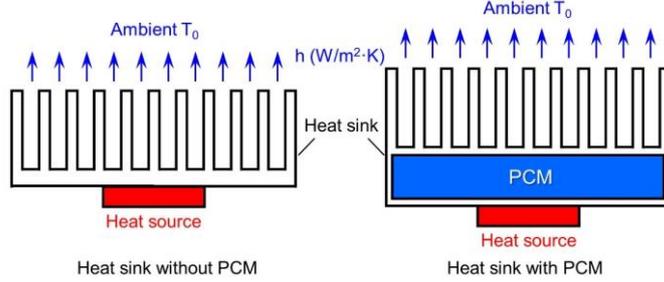


Figure 1: Schematic diagram of the heat sink

## 2.2 Transient heat transfer analytical model

### 2.2.1 Natural cooling heat sink

For the air-cooled heat sink, its convective heat transfer capacity is relatively weak, and the entire heat sink can be analysed using the lumped heat capacity method. In theory, the lumped heat capacity method is generally applicable to the case of small Biot numbers ( $Bi < 0.1$ ). For the rectangular plate-fin heat sink structure analysed here, the  $Bi$  number can be defined as:

$$Bi = \frac{h_{eq} D_{eq}}{k}, \quad h_{eq} = h \frac{(n-1)w_c L + 2\eta_f n H_f L}{WL}, \quad D_{eq} = d + \frac{nw_f H_f}{W} \quad (1)$$

where,  $k$  is the thermal conductivity of the heat sink,  $h_{eq}$  is the equivalent convective heat transfer coefficient between the heat sink and environment, and  $D_{eq}$  is the equal thickness of the heat sink.  $d$  is the thickness of the heat sink substrate;  $W$ ,  $L$  are the width and length of the overall heat sink;  $n$ ,  $w_t$ ,  $w_c$ ,  $H_f$  are the number of fins, thickness, spacing and height;  $h$  is the natural convection heat transfer coefficient between the heat sink and the environment;  $\eta_f$  is the fin efficiency and is defined as:

$$\eta_f = \frac{\tanh(mH_f)}{mH_f}, \quad m = \sqrt{\frac{2h}{k_f w_f}} \quad (2)$$

where,  $k_f$  is the thermal conductivity of the fin material. Under the assumption of lumped heat capacity method, the entire heat source and heat sink can be regarded as a uniform body with a constant temperature of  $T$ , the energy conservation equation for the whole heat sink is:

$$\frac{d(T \cdot \sum_i m_i c_{p,i})}{dt} = q - hA(T - T_0), \quad \sum_i m_i c_{p,i} = m_{source} c_{p,source} + m_{sink} c_{p,sink} \quad (3)$$

where,  $q$  is the heating power of the heat source,  $A$  is the external heat dissipation area of the heat sink,  $T_0$  is the ambient temperature,  $m_i c_{p,i}$  ( $i = source$  or  $sink$ ) represents the heat capacity of the heat source or heat sink. Assuming that  $h$  and  $c_{p,i}$  are constant, and defining the following parameters:

$$\theta = T - T_0, \quad a = \frac{hA}{\sum_i m_i c_{p,i}}, \quad b = \frac{q}{\sum_i m_i c_{p,i}} \quad (4)$$

Eq(3) and Eq(4) can be expressed as follows:

$$\frac{d\theta}{dt} + a\theta = b, \quad \theta = \frac{b}{a} + C \cdot e^{-at}, \quad C = (T_1 - T_0 - \frac{b}{a}) \cdot e^{at_1}, \quad \theta = \frac{b}{a} + (T_1 - T_0 - \frac{b}{a}) \cdot e^{-a(t-t_1)} \quad (5)$$

Figure 2 schematically shows the temperature response curve of a heat sink in three phases of a conventional heat load, a pulsed heat load, and a depulsed heat load. During the regular heat load phase (time period 0- $t_n$ , heat source power  $q_n$ ), the heat source temperature rises exponentially and reaches a stable value  $T_n$ . At time  $t_n$ , the power of the heat source suddenly rises to  $q_p$ , the temperature of the heat source increases rapidly in a new exponential form, and reaches the highest temperature  $T_{p,NC}$  at the end of the heat pulse ( $t_n + t_p$ ). After that, the heat source power returns to the average value  $q_n$ , and the heat source temperature then decreases exponentially, and eventually approaches the stable value  $T_n$ .

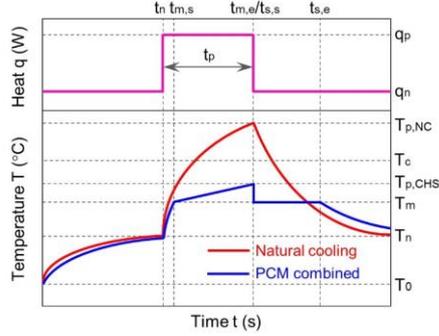


Figure 2: Temperature response of the heat sink under thermal shock

### 2.2.2 Metallic phase change material/air natural cooling combined heat sink

The overall heat capacity of the combined heat sink is:

$$\sum_i m_i c_{p,i} = m_{source} c_{p,source} + m_{sink} c_{p,sink} + m_{PCM} c_{p,PCM} \quad (6)$$

The temperature variation of the heat sink during the phase change process of the PCM can be approximated by the following equation:

$$T = T_m + \frac{q_p \cdot t^2 (t - t_{m,s})}{\rho_{l,PCM} k_{l,PCM} \Delta H} \quad (7)$$

Where,  $q_p$  is the heat flux flowing into the PCM during the thermal shock;  $\rho_{l,PCM}$ ,  $k_{l,PCM}$ , and  $\Delta H$  are the density, thermal conductivity, and fusion latent heat of the PCM. The temperature of the heat source during the melting process is a linear function of time, Eq(7). At the end of the thermal shock ( $t = t_n + t_p$ ), the heat source temperature reaches the maximum value  $T_{p,CHS}$ . If  $T_{p,CHS}$  is lower than the critical heat source temperature  $T_c$ , this combined heat sink is effective.

After the heat pulse, the temperature of the external fins of the heat sink and the stable region of the PCM is still maintained at  $T_m$ . The temperature gradient in the liquid phase region of the PCM will soon disappear, and the temperature of the entire heat sink will be restored. It is assumed that this process is completed in a short time; after the heat pulse, the whole heat sink temperature is instantly reduced to the PCM melting point  $T_m$ . The heat sink transfers the heat of the heat source to the environment and dissipates the latent heat stored in the PCM at the same time. The latent heat absorbed by the PCM can be calculated using the following equation:

$$q_m = q_p - hA \left( \frac{T_{p,CHS} + T_m}{2} - T_0 \right), \quad Q_m = q_m (t_{m,e} - t_{m,s}), \quad Q_m + q_n (t_{s,e} - t_{s,s}) = hA (T_m - T_0) (t_{s,e} - t_{s,s}) \quad (8)$$

where,  $q_m$  and  $Q_m$  are the power and total heat absorbed by the PCM in the form of latent heat;  $t_{m,s}$ ,  $t_{m,e}$  are the time when the melting process starts and ends.  $t_{s,s}$ ,  $t_{s,e}$  are the time when the solidification process begins and ends. During the solidification process, the temperature of the entire heat sink is maintained at the melting point temperature  $T_m$  of the PCM. After the solidification process, the temperature variation of the heat sink can be predicted by the lumped heat capacity method, it decreases to  $T_n$  exponentially, as expressed in Eq(5).

### 2.2.3 Design process of the metallic PCM heat sink

The theoretical analysis model introduced above can be used for design and optimization of practical problems. The specific steps are shown in Figure 3. The selection of PCM is the first step in the design. The first principle is to choose a material with a suitable melting point. The melting point  $T_m$  of the PCM must be higher than the

limit temperature  $T_n$  and lower than the critical temperature  $T_c$ . The amount of PCM can be estimated according to the following equation:

$$m_{PCM} = \frac{(q_p - q_n)t_p}{\Delta H} \quad (9)$$

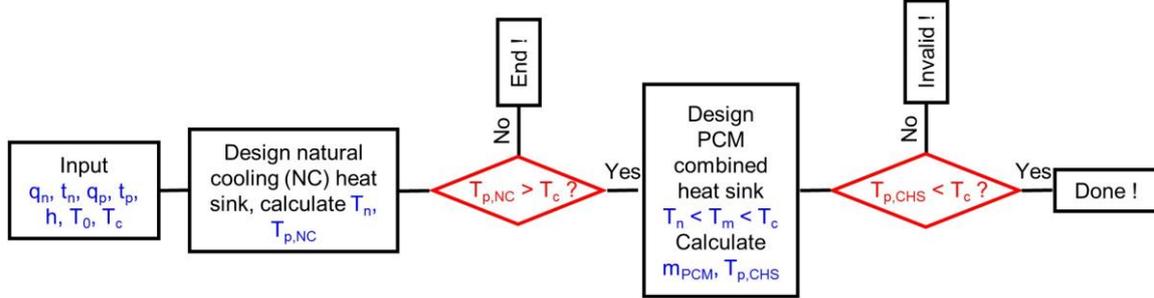


Figure 3: Design process of the phase change material combined heat sink

### 3. Numerical simulation of typical cases

#### 3.1 Numerical calculation model

The entire calculation domain includes a heat source, heat sink and PCM; only heat transfer equation needs to be solved. Enthalpy method is used to consider the phase transition process:

$$\frac{\partial(\rho H)}{\partial t} = \frac{\partial}{\partial x_j} \left( k \frac{\partial T}{\partial x_j} \right) + \dot{q} \quad (10)$$

where,  $H$  represents the total enthalpy, which is the sum of the sensible enthalpy  $H_s$  and latent enthalpy  $H_l$ , Commercial software ANSYS Fluent is used for numerical calculation, the main thermal conditions of the case studied in this paper are listed in Table 1, and corresponding natural air cooling heat sink is designed as listed in Table 2. The  $Bi$  number of the heatsink is 0.045, which meets the prerequisites of the lumped heat capacity method.  $Bi_{31.6}In_{48.8}Sn_{19.6}$  (E-BiInSn) is chosen as the PCM, the melting point of which is 60.2 °C. It can be obtained from Eq(9) that the amount of PCM is 129 g, the PCM thickness in heat sink is 10.8 mm. The heat source is a square silicon wafer ( $20 \times 20 \times 2 \text{ mm}^3$ ), and the heat sink structural material is aluminium. The main thermophysical properties of the materials are listed in Table 3. 1/4 model is used for simulation due to its symmetry, as shown in Figure 4.

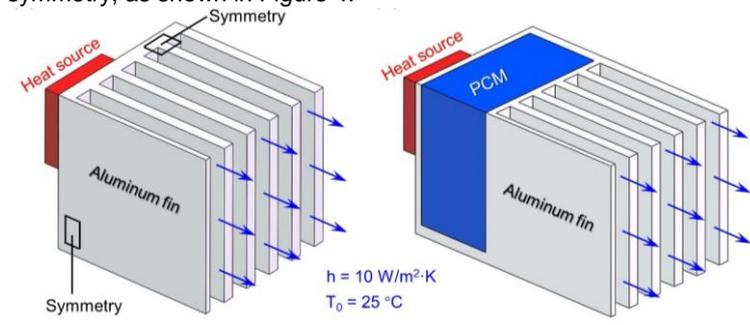


Figure 4: Numerical simulation model

Table 1: Thermal conditions of typical case

Parameter	Value	Parameter	Value
Normal heating power $q_n$ (W)	4	Thermal shock power $q_p$ (W)	40
Normal working time $t_n$ (s)	1,200	Thermal shock time $t_p$ (s)	100
Heat transfer coefficient $h$ (W/m²·K)	10	Ambient temperature $T_0$ (°C)	25
Heat source size (mm×mm×mm)	20 × 20 × 2	Critical temperature of heat source $T_c$ (°C)	85

Table 2: Dimensions of the natural air cooling heat sink

Parameter	Value	Parameter	Value
Base area $W \times L$ (mm $\times$ mm)	41.5 $\times$ 41.5	Fin thickness $w_f$ (mm)	1.5
Base thickness $d$ (mm)	2	Fin pitch $w_c$ (mm)	2.5
Number of fins $n$	11	Fin height $H_f$ (mm)	20

Table 3: Main thermophysical properties of the heat source and heat sink materials

Structure material	$\rho$ (kg/m $^3$ )	$c_p$ (J/kg $\cdot$ K)	$k$ (W/m $\cdot$ K)	$T_m$ ( $^{\circ}$ C)
Heat source (silicon)	2,328	700	148	-
Heat sink (aluminum)	2,719	871	202.4	-
PCM (E-BiInSn)	8,043	270 (solid)/297 (liquid)	19.2 (solid)/14.5 (liquid)	60.2

### 3.2 Numerical simulation results

Figure 5 shows the temperature distribution of the two heat sinks. For the natural air cooling heat sink, the maximum temperature is 44.8  $^{\circ}$ C during the normal working state, and the temperature difference in the entire structure is below 1.2  $^{\circ}$ C, which is consistent with the premise of the lumped heat capacity assumption. The temperature rise rate of the low-melting-point metal PCM heat sink is slightly slower than the natural air cooling heat sink in the normal working state, at time  $t_n$ , its temperature is slightly lower, and the maximum temperature is 43.8  $^{\circ}$ C, the temperature difference within the heat sink is 2  $^{\circ}$ C.

After the thermal shock, the temperature distribution of the two heat sinks shows a significant difference. The natural air cooling heat sink cannot deal with the thermal shock of 40 W (100 s), and the temperature of the heat source rapidly rises to 119  $^{\circ}$ C, which is much higher than the critical temperature of 85  $^{\circ}$ C. The low-melting-point metal PCM heat sink can cope well with this thermal shock, during its phase change process, only the temperature of the liquid PCM rises, the temperature of the solid phase region and the aluminium fin keeps unchanged (about 60.2  $^{\circ}$ C). The PCM heat sink can control the temperature of the heat source within 77  $^{\circ}$ C during the thermal shock, which meets the thermal design requirements.

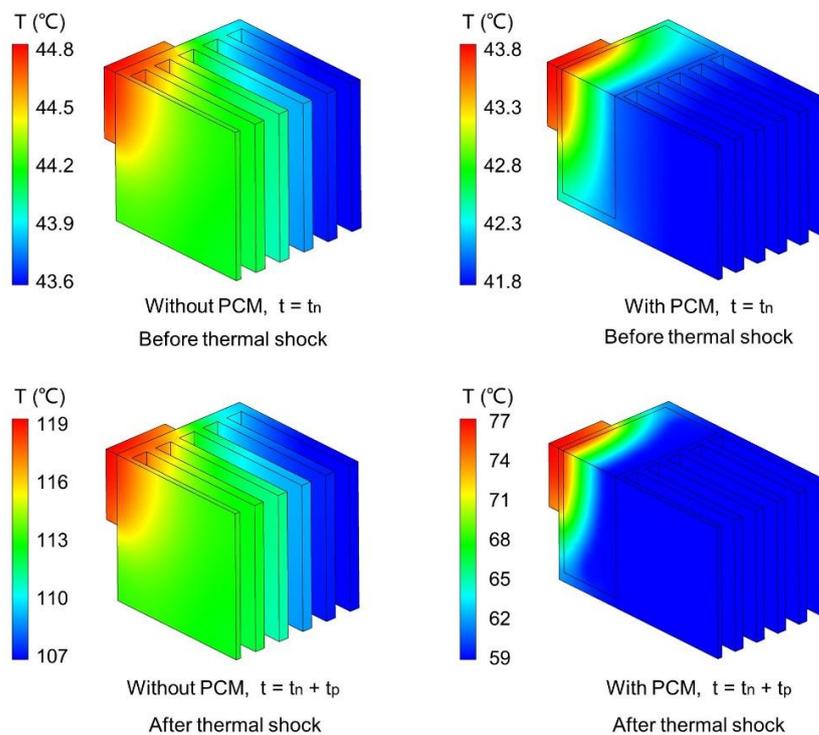


Figure 5: Temperature contours of the heat source and heat sink before and after thermal shock

### 3.3 Comparison of theoretical model and numerical results

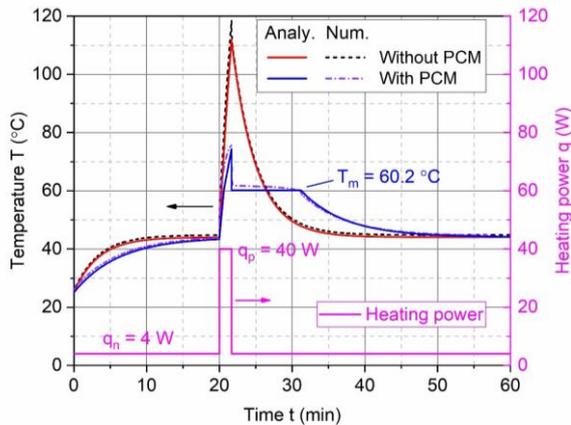


Figure 6: Comparison of the theoretical model and the numerical simulation results

To verify the reliability of the theoretical analysis model established here, it is compared with the numerical calculation results, as shown in Figure 6. The normal working time lasts 1,200 s, with heat power of 4 W; the power of thermal shock is 40 W, which lasts 100 s. The temperature responses of the heat sink with and without PCM are monitored and compared. It can be seen that the theoretical transient temperature response curve and the results obtained by numerical simulation are incident, which can well match the engineering design requirement. Therefore, in practical applications, The transient thermal analysis model established here can be used for rapid heat sink design and performance evaluation.

### 4. Conclusions

A low-melting-point-metal phase change material/air natural cooling combined heat sink is proposed in this paper, which can be used for thermal control of electronics with thermal shock. Theoretical analysis model for the transient heat transfer performance of the heat sink is established, and the general design process for the heat sink is provided. Numerical simulation of specific cases shows that low-melting-point-metal PCM gallium can effectively suppress the temperature rise of the heat source and prevent the device from overheating through its phase change heat absorption under the thermal shock condition. Comparison between the theoretical analysis model and the numerical results shows that the transient heat transfer analysis model established in this paper can provide a reliable prediction of the thermal performance of the PCM heat sink, which can be used for rapid performance prediction and evaluation in the practical thermal design.

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### References

- Fleischer A.S., 2015, Thermal energy storage using phase change materials: fundamentals and applications, Springer International Publishing, Cham, Switzerland.
- Ge H., Li H., Mei S., Liu J., 2013, Low melting point liquid metal as a new class of phase change material: An emerging frontier in energy area, *Renewable and Sustainable Energy Reviews*, 21, 331-346.
- Liu L., Su D., Tang Y., Fang G., 2016, Thermal conductivity enhancement of phase change materials for thermal energy storage: A review, *Renewable and Sustainable Energy Reviews*, 62, 305-317.
- Li Z.W., Lv L.C., Li J., 2016, Combination of heat storage and thermal spreading for high power portable electronics cooling, *International Journal of Heat and Mass Transfer*, 98, 550-557.
- Salyan S., Suresh S., 2017, Liquid metal gallium laden organic phase change material for energy storage: An experimental study, *International Journal of Hydrogen Energy*, 43(4), 2469-2483.
- Yang X.H., Tan S.C., Ding Y.J., Wang L., Liu J., Zhou Y.X., 2017, Experimental and numerical investigation of low melting point metal based PCM heat sink with internal fins, *International Communications in Heat and Mass Transfer*, 87, 118-124.
- Yang X.H., Tan S.C., He Z.Z., Liu J., 2018, Finned heat pipe assisted low melting point metal PCM heat sink against extremely high power thermal shock, *Energy Conversion and Management*, 160, 467-476.