

Simultaneous Minimization of Cost and Energy in Gas Allocation Network

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Pipelines permit to transport large volumes of a wide range of products from supply nodes to demand nodes. They have been considered as the most effective and safest way of transporting gases. Transportation of gas through the pipeline is an energy-intensive process. Energy management in gas allocation networks (GANs) is needed. In this paper, a fuzzy mathematical model is developed for targeting multi-objective transportation problem i.e. simultaneous minimization of energy budget and investment budget in GANs where the model parameters exhibit fuzzy relaxed boundaries. The proposed procedure is explained via an illustrative example where the process data is represented through interval-typed triangular fuzzy data. The result shows that there is almost 38 % reduction in energy budget while 2.26 % increase in the investment budget. The proposed model also utilizes fuzziness in supply pressures while optimization and calculates the optimum values of pressures within its limit. For example, if pressures are maintained at their mean values in the illustrated example, the energy requirement increases by 11 %. The presented model provides the optimal balance between the conflicting goals. The novelty of this work is its ability to combine investment budget, energy budget and environmental aspect using a fuzzy non-linear programming approach to provide a suitable functioning point or exact value of pressure at which CS is supplying in GANs.

1. Introduction

Pipeline networks are the most common way of fluid transportation in many industrial and engineering systems such as natural gas transportation (Hossam et al., 2011). Between supply and demand points pressure drop occurs due to gas expansion, friction loss, elevations and temperature changes. Compression is required to overcome those pressure losses that occur over the length of the pipeline. Adding compressor stations (CS) at intervals along the pipeline network is one of the solutions used to achieve and maintain the required pressure. The compression procedures involve huge amounts of energy that are typically supplied by the compressor with shaft works (Yilmazoglu et al., 2014). Energy is one of the major resources need to be conserved (Chaturvedi, 2017) and due to the growing concern over global warming and sustainable energy usage, every region needs to be energy efficient (Hua et al., 2011). The reduction of the energy used in pipeline operations will not only have a tremendous economic impact but an environmental one. The more efficient use of compressors stations is the least greenhouse emissions are dissipated in the atmosphere (Sorrell, 2015). Wang et al. (2018) proposed a MILP model for optimizing gas transmission networks using equations of mass balance of network nodes, the pressure drop of pipelines and pressure increase of compressors.

Transportation network problems have been well studied in a deterministic approach, but they are inadequate partly to exactly demonstrate the real-world problem (Azadeh et al., 2015). In real-world decision-making problems, dealing with fuzziness is one of the most important features for making the optimum Gas Allocation Network (GAN). For explaining the fuzziness in multi objective programming, there are several well-known theories such as fuzzy set theory, probability theory and other mathematical tools (Tang et al., 2004). A fuzzy multi-objective analysis has been developed by Agrawal and Singh (2001) for energy allocation in India. In their work nine objectives functions are considered for allocation of ten energy sources using fuzzy goal

programming. Alikhani and Azar (2013) used a combined stochastic goal programming model under a fuzzy environment for gas resource quota allocation. Their method draws upon the existing chance constrained programming and triangular fuzzy numbers by allowing analysis on trade-offs among objective functions and the risk of violating constraints that comprise uncertain parameters. Fan and Klemes, (2020) utilized the Pinch analysis for optimizing Emission and transporting cost, as well as the energy content of the biomass resources in biomass network allocation. Fuzzy model parameters defined mathematically by using membership functions. The relationship between each membership function is defined by using fuzzy operators. A comprehensive analysis and ordering of method explanation for fuzzy multi-objective optimization are proposed by Arikan (2013). Following the Zimmerman max-min approach (1978) several researchers present fuzzy multi objective approaches that are based on mathematical modeling. Further, the computational work of Liu and Liu (2003) proposed the expected value model for developing a fuzzy random multi-objective programming problem. The benefit of this fuzzy model is that it provides a compromise solution combining the different objectives. Chiang (2005) pointed out that it is better to represent the availability and demand as interval-valued fuzzy numbers instead of normal fuzzy numbers and proposed a method to find the optimal solution of single objective transportation problems.

From the above literature review, it can be analyzed that there is still a need for a systematic approach which that is capable to address the conflicting goal (energy budget and investment budget) in a fuzzy environment for GAN. Addressing the two objectives simultaneously gives the opportunity to the decision-maker to calculate the compromised solution. This paper describes the methodology for determining the optimized solution for GAN with their relaxed boundaries. The suggested procedure is explained via an illustrative example where the process data is represented through interval-typed triangular fuzzy data. This work presents a multi-objective operational algorithm for the simultaneous minimization of investment budget and energy budget in GAN where the constraints are defined with fuzzy interval.

2. Problem Definition and Mathematical Formulation

A schematic representation of the problem definition is presented in Figure 1.

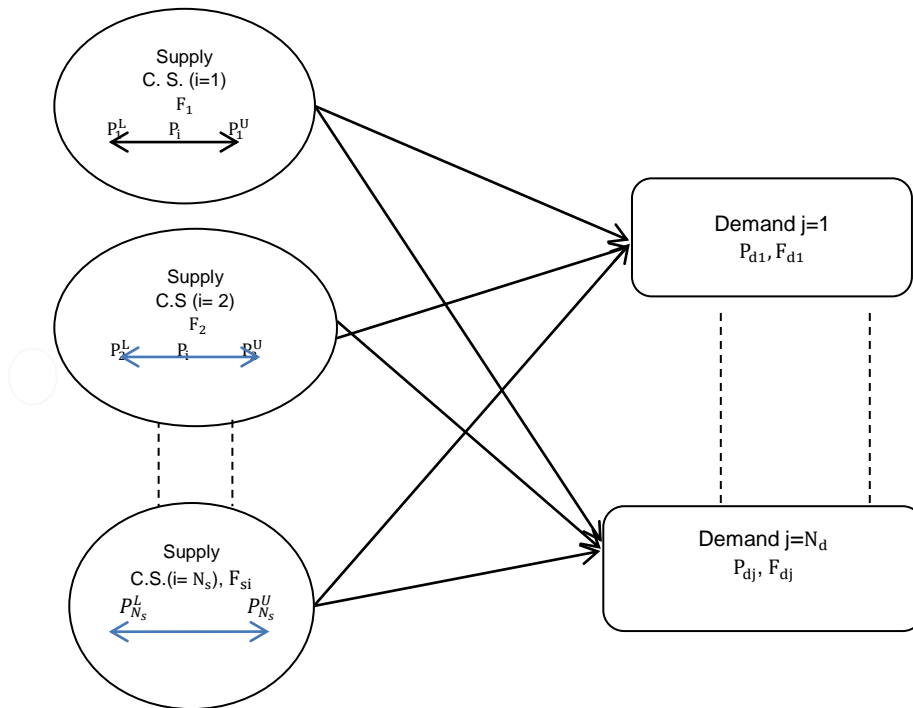


Figure 1: A schematic representing a generalized for GAN

The GAN usually comprises several CS's for supplying the gas at several demand nodes at their required pressure. Now the problem statement of this paper can be stated as follows:

- The set of N_s compressor station (CS) is characterized where each CS $i \{1, 2, \dots, N_s\}$ with available at pressure (P_{si}) which are fuzzy i.e. CS have relaxed boundaries in the parameter with a most

acceptable value (MAV) within this limit. The specific investment (C_i) per unit flow for installing CS is also given.

- The set of N_d demands is defined where each demand $j \{1, 2, \dots, N_d\}$ has a specific flow requirement (F_{dj}) at a specified pressure (P_{dj}).

The objective of this study is to determine a compromised optimized solution for GAN which involved in installing new compressor stations. The fuzzy optimization approach can be used in deciding an optimal balance between the conflicting goals which is investment budget and energy budget for decision-makers. Note that, supplying flow from a station having a higher pressure than the demand pressure does not require any compression work. The mathematical model comprises the following sets, variables, parameters, and constraints.

2.1 Constraints

Material Balance Constraints: The flow from CS (i) to demand (j) is symbolized by f_{ij} . The flow balance for CS and demand station may be addressed as follows Eq(1) and Eq(2):

$$\sum_{i=1}^{N_s} f_{ij} = F_{dj} \quad \forall j \in J \quad (1)$$

$$\sum_{i=1}^{N_d} f_{ij} = F_{si} \quad \forall i \in I \quad (2)$$

where F_{dj} is flow requirement of demand and F_{si} is maximum limit of supply flow from CS.

Thermodynamic Constraints: The compression work or energy requirement is directed by the initial and final states along with the volumetric flow. For isothermal compression, the energy requirement (E) can be expressed as Eq(3):

$$\text{Net Energy, } E = \begin{cases} F_0^* \left[\left(P_0 \ln \left(\frac{P_j}{P_0} \right) \right) - \left(P_0 \ln \left(\frac{P_i}{P_0} \right) \right) \right] & \text{For isothermal Process} \\ \left(\frac{n}{n-1} \right) P_0^{\frac{1}{n}} P_i^{\frac{(n-1)}{n}} F_0^* \left[\left(\frac{P_j}{P_i} \right)^{n-1/n} - 1 \right] & \text{For polytropic Process} \end{cases} \quad (3)$$

Where F_0^* represents the standard volumetric flow rate being compressed, n is polytropic index and P_0 is the pressure under standard conditions. P_j and P_i are the demand and supply pressures. The Pressure index of an isothermal process and polytropic process (Bandyopadhyay et al., 2014) can be calculated by using Eq(4):

$$\text{Pressure Index, } \mu_{(i/j)} = \begin{cases} P_0 \left(\ln \left(\frac{P_{(i/j)}}{P_0} \right) \right) & \text{For Isothermal Process} \\ \left(\frac{n}{n-1} \right) P_0 \ln \left(\frac{P_{(i/j)}}{P_0} \right)^{(n-1)/n} & \text{For Polytropic Process} \end{cases} \quad (4)$$

Where $\mu_{(i/j)}$ signify the pressure index for stations. Energy requirement for supplying flows balance (f_{ij}) from various supply pressure levels (P_i) pressure level to demand at a pressure (P_j) pressure level ($P_j > P_i$) may be expressed as in Eq(5):

$$\sum_{i=1}^{N_s} f_{ij} (\mu_j - \mu_i) = E_j \quad \forall j \in J \quad (5)$$

Fuzzy Pressure Constraints: To address fuzziness in GAN, a triangular fuzzy method is utilized in this work. Zimmermann (1978) explained the utilization of membership function in a fuzzy set for the multiple objective functions in decision making. In this work, a fuzzy non-linear program (NLP) has been developed for GAN with relaxed boundaries using "max-min" aggregation. The primary goal of the work is to maximize the degree of satisfaction (ϵ) by each fuzzy constraint with a compromised solution. The interval of fuzzy satisfaction lies between the interval $[0, 1]$ where the 0 significances to unsatisfactory while the value of 1 shows the complete satisfaction. The value between 0 and 1, shows the fractional satisfaction in fuzzy linear programming.

$$0 \leq \epsilon \leq 1 \quad (6)$$

Optimization in GAN with fuzziness, each constraint should be satisfying partially the degree of satisfaction using Zimmerman's max-min approach. (Zimmerman 1978)

For the pressure index triangular linear fuzzy method is applied:

$$\frac{\mu_i - \mu_i^U}{\mu_i^M - \mu_i^U} \geq \epsilon \quad \forall i \in I \quad (7)$$

$$\frac{\mu_i^L - \mu_i}{\mu_i^L - \mu_i^M} \geq \epsilon \quad \forall i \in I \quad (8)$$

The formulation can be solved using the fuzzy NLP model to determine the optimization in GAN i.e. the optimum pressure for flowing gas by the compressor while satisfying the demand within the fuzzy limit.

2.3 Objective functions:

The formulation can be solved using the fuzzy NLP model to obtain the simultaneous minimization of cost and energy while satisfying the demands.

For the energy budget and investment budget linear fuzzy is applied as follows:

$$\frac{E_m - E^U}{E^L - E^U} \geq \epsilon \quad (9)$$

$$\frac{C_m - C^U}{C^L - C^U} \geq \epsilon \quad (10)$$

The actual energy budget (E_m) and investment budget (C_m) can be calculated using Eq(11) and Eq(12):

$$\sum_{i \in I} \sum_{j \in J} f_{ij} (\mu_j - \mu_i)^+ = E_m \quad (11)$$

$$\sum_{i \in I} F_i C_i = C_m \quad (12)$$

The primary goal of formulation is to maximize the degree of satisfaction (ϵ) that ultimately gives a compromised solution.

3. Targeting Algorithm

The targeting algorithm for determining fuzzy non-linear programming for GAN concludes the following steps.

Step1: Calculate the pressure index of supply CS and demands along with their minimum and maximum limit using for isothermal process and) for a polytropic process Eq(4).

Step 2: Energy budget limit and investment budget limit is chosen as the decision-maker choice.

Step 3: Formulate the GAN problem using fuzzy non-linear programming.

Step 4: Specify the linear membership function of each objective function.

Step 5: Introduce the auxiliary variable ϵ to transform the problem. The variable ϵ can be interpreted as representing the overall degree.

Step 6: Solve the NLP problem and obtain a compromised solution.

4. Illustrative Example

In this section, the applicability of the proposed model is validated through an illustrative example. The example shows a fuzzy optimization model for GAN while fuzziness in pressure. The model is solved using GAMS/ CONOPT solver. For this example, demand pressure (P_d) and the flow rate requirements are tabulated in Table 1. Available pressure limit and maximum flow rate of new compressor stations (Y_1, Y_2, Y_3 and Y_4) is given in Table 2.

Table 1: Demand Data for illustrative Example

Demand	Pressure (kPa)	Required Flow (Sm ³ /s)
Z_1	200	2,300
Z_2	220	1,600

For any given GAN, the parameters can be explained in terms of a fuzzy interval, which ranges from minimum to maximum as per their working capacity. For example, the pressure limits for the new compressor station (Y_1) ranges from 120 to 140 (130 ± 10) kPa. As the problem is presented in triangular linear fuzzy where decision-maker opinion based on one point with accompanying relaxed boundaries at both sides. The minimum limit (120 kPa) of the pressure corresponds to the least pressure at which station is working while the maximum limit (140 kPa) involves the highest pressure while 130 kPa is a conservative pressure. Initially, the pressure index of new stations and demands is calculated.

Table 2: Source Data for illustrative Example

Source Compressor Station	Pressure (kPa)	Flow limit (Sm ³ /s)	Cost (\$s/Sm ³)
Y_1	130 ± 10	1,000	10
Y_2	150 ± 20	1,200	15
Y_3	160 ± 25	800	20
Y_4	170 ± 15	1,100	40

The feasible energy limit for satisfying the demand is 79,551.17 kJ/s to 174,679.92 kJ/s and the corresponding investment budget is \$ 90,000 to \$ 84,000. It can be observed that energy and investment are conflicting goals as with an increase in energy requirement, investment decreases. The limits of energy and investment assumed to be the same (i.e 79,551.17 kJ/s to 174,679.92 kJ/s for energy and \$ 84,000 to \$ 90,000 for investment budget). The analogous level of satisfaction of the fuzzy limits is calculated 0.695. The optimal value may be explained as follows.

Table 3: Simulated Result for Gas Allocation Network

Source Compressor Station	Pressure (kPa)	Flow rate for Demand 1 (Sm ³ /s)	Flow rate for Demand 2 (Sm ³ /s)
Y ₁	132.97	478.17	325
Y ₂	155.82	575	625
Y ₃	167.13	575	225
Y ₄	174.43	671.83	425

The minimum and maximum pressure index for the first compressor station is 17.14 kPa and 32.76 kPa while 25.25 kPa are at conservative pressure. The model consists of 23 single equations, 20 single variables and execution time is within a fraction of seconds. The optimal value of ϵ is calculated to be 0.695. It follows that the equivalent pressure index for this compressor station is 27.54 kPa which corresponds to the pressure is 132.97 kPa. The results are tabulated in Table 3. From the corresponding allocation network, the investment budget is calculated \$ 85,904.84 while the energy requirement is 108,977.42 kJ/s. It is found that the increase of 2.26 % in the least value of the investment budget provides a 37.6 % decrease in energy requirement which is a good amount of energy saving. This model provides the optimum allocation network and provides the compromised solution while minimizing investment and energy simultaneously. Table 3 summarizes the result of the example.

5. Conclusion

In this paper, for the design of GANs, with fuzzy parameters, a non-linear fuzzy programming model has been introduced. The model gives the compromised solution for GAN and also permits the resource management goals to be balanced with fuzziness in compressor stations data. The decision-maker can utilize the result to choose the suitable functioning point. The value of a suitable functioning point provides the exact value of pressure at which CS is working which provides an optimal balance between the energy budget and investment budget. From the illustrated example, it is found that there is almost 38 % decrease in energy budget while a 2.26 % increase in investment budget and also from example it can be observed utilizing this model, the decision-maker can reduce the energy requirement which also reduces the carbon emission due to less use of fossil fuel. The proposed model also capable to utilize fuzziness in supply pressures while optimization and determine the optimum values of pressures within its limit. In the illustrative example, if mean values of pressures are considered then the energy requirement rises by 11 %. The proposed model provides the optimal balance between the conflicting goals. In future work, the model formulation may be readily expanded to other constraints such as quality and carbon emissions in the GAN.

Nomenclature

Sets

I = { i / i = Installing CS }

J = { j / j = Demand CS }

Variables

ϵ = overall degree of fuzzy constraint satisfaction

f_{ij} = flow from supply compression station (i) to demand station (j), (Sm³/s)

E_m, C_m = actual energy budget and investment budget

Parameters

P_i^L = Lower limit of the pressure of installing ith CS corresponding pressure index (μ_i^L), (kPa)

P_i^U = Upper limit of the pressure of installing ith CS corresponding pressure index (μ_i^U), (kPa)

$\mu_{(i/j)}$ = Pressure index of supply and demand CS, (kPa)

μ_i^{MAV} = Most acceptable value of pressure index of installing ith CS, (kPa)

E^L = Lower energy budget, (kJ/s)
 E^U = Upper energy budget, (kJ/s)
 C^U = Upper investment budget, (\$/Sm³)
 C^L = Lower investment budget, (\$/Sm³)
 E = Energy requirement, (kJ/s)
 P_j, P_i = Demand and supply pressure, (kPa)
 n = Polytropic index
 F_{dj} = Flow requirement of demand, (Sm³/s)
 F_{si} = Maximum limit of supply flow from CS, (Sm³/s)

Subscript

i = supply CS
 j = demand

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