

General Formulation for the Resilience of Processing Systems

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Resilience is one of the key indicators of processing systems, it expresses the behaviour of the system as a result of expected or unexpected failures. This indicator can be essential during systems design and operation, especially, when the system is part of or related to a critical infrastructure. The numerous contributions on systems resilience are related to a wide range of applications, however, there is no general uniform framework for resilience evaluation. For instance, most studies examine resilience as a function of the continuous parameters of the system, usually avoiding the influence of its structure. In the current work, a general framework for determining the structural resilience of processing systems is presented. This framework derives on formulas that satisfy the requirements of the original definition of resilience. The formerly developed P-graph framework is the mathematical basis of the procedure for determining the indicator. The resilience of the system is calculated as a function of the operative subprocesses for all possible failures and is a normalized indicator on [0, 1]. The examination of two industrial case studies shows that the proposed resilience can be an appropriate indicator to be considered in process design.

1. Introduction

The analysis of the general behaviour of complex processing systems has become crucial because of its major influence on the society and the economy. Several aspects are to be considered in designing or evaluating a system, e.g. profitability, reliability, and sustainability. Since all or at least most of the engineering systems are connected directly to other systems and indirectly to practically everything, general structural indicators are also important. Recently, three important indicators appeared, these are agility, supportability, and resilience. All of these indicators highly depend on the system's structure. Agility means that the system can be adapted for changing requirements. A system is supportable if it can be sustained effectively and efficiently during its whole life cycle. Resilience expresses the capability of the system to recover from difficulties, this has been examined in the current work.

Any system can be subject to several sources of failures during its operation. The failures can be internal (e.g. reactor leak) or external (e.g. earthquake), as well as predictable (e.g. equipment unit failure due to wear) or unpredictable (e.g. lightning strike). The first known definition of resilience was given by Holling (1973) for ecological systems, where it expresses the persistence of a system and also its ability to absorb disturbance while still remaining in an acceptable state. There are two main aspects of a system's resilience: (i) its ability to resist to change and (ii) its capability of recovering from change (Dalziel and McManus, 2004). In most cases, resilience is derived from other metrics of the system. The technical report of Trimintzios (2011) demonstrates that over 20 different metrics can be considered when determining the resilience of a communication system. Hosseini et al. (2016) give a thorough review of resilience measures, it covers organizational, social, economic, and engineering resiliences.

For engineering systems, the main considerations for resilience are controllability (Morari, 1983) and flexibility (Grossmann et al, 2014). Another possible metric for resilience is the system performance level after a disturbance (Bhamra et al., 2011). The repair or replacement times of the system's components are also taken

into account by Gong and You (2018). A recent book gives a general overview of resilience in theory and practice (Linkov and Trump, 2019).

Since systems resilience has no general definition, there are a large number of different specific definitions for the different areas of applications. Most of the resilience metrics are not normalized, so the different indicators cannot be compared. Besides, it is common that only certain types of failures are considered, and some of the potential issues are missing from consideration. A new philosophy is to be considered for developing a general framework.

Traditionally, in designing and evaluating systems, the predictable, known aspects are taken into account. On the basis of that, exact indicators, such as the system reliability, can be determined. The key point for such analyses is to enumerate all cause-effect options. This type of evaluation is not appropriate for determining the resilience of processing systems since resilience is based on unexpected events in addition to the expected events. Consequently, the cause part of the cause-effect relation is not known or not effective. To overcome this philosophical issue, the possible solution is to enumerate all effects. Since enumeration can conveniently be based on the P-graph framework, it can be a firm foundation of a new formulation of resilience. It is important to emphasize that most disasters behave as a process, it is beneficial to consider both the original desirable process and the undesirable disaster in a common framework.

2. P-graph

P-graph framework is a combinatorial tool for process synthesis that represents the superstructure (termed as maximal structure) as a bipartite graph of materials and operating units. The framework is based on the fundamental structural properties of feasible process structures. These properties are general and independent of the types of mathematical models of the operating units, and the framework is also general. The P-graph framework has been selected to evaluate structural resilience for its high capability of enumerating the feasible sub-networks.

The combinatorial properties of the networks of feasible processes are collected and described as a set of five axioms (Friedler et al., 1992). Networks of operating units satisfying these axioms are called combinatorially feasible structures. The search for the optimal network can be reduced to the set of combinatorially feasible networks without the risk of losing the optimality (Peters et al., 2003), providing a significant acceleration in searching for the optimal processing network. The P-graph framework consists of general algorithms including algorithm MSG for generating the maximal structure (Friedler et al., 1993), algorithm SSG (Friedler et al., 1995) for generating the whole set of combinatorially feasible networks, and algorithm ABB, an accelerated branch-and-bound algorithm (Friedler et al., 1996). These algorithms are considered as building blocks for the current work.

3. Structural resilience and resilience of processing systems

A processing system is considered here resilient if after any expected or unexpected failures it is

- able to perform its designated job in full or on a pre-specified partial level;
- able to return to its original state.

First, assume that a processing system has only a single product. Let α be the maximal production flowrate, β be the normal production flowrate, γ be the minimum accepted production flowrate in terms of resilience, and δ be the minimum production flowrate where the system is operable. For example, a processing system has $\alpha = 120$ kg/h maximum production flowrate, its normal operation is $\beta = 105$ kg/h. If the production system is part of a critical infrastructure, in any catastrophic failure, any production flowrate above $\gamma = 85$ kg/h is sufficient in terms of resilience. Due to technical reasons, the processing system cannot operate anything below $\delta = 30$ kg/h. Note that $\alpha \geq \beta \geq \gamma \geq \delta \geq 0$, and $\gamma > 0$. In most applications, γ is considered to be identical with β , i.e. the return to the original state is assumed in determining resilience.

Any operating unit of the normal operation of the production system can be subject to failure. Let S_n^f denote the set of all combinations of at most n simultaneous failures in the normal operations mode. S_n denotes the set of all combinations of at most n failures that do not prevent the processing system to achieve a production level of at least δ ($S_n \subseteq S_n^f$). Let N_n^f be the cardinality of set S_n^f and N_n be the cardinality of set S_n . The structural resilience for at most n failures is given by Eq(1).

$$R_n^s = \frac{N_n}{N_n^f} \quad (1)$$

Note that

- $R_n^s \geq R_m^s$ if $n < m$

- R_n^s highly depends on both the values of γ and δ

If K is the number of the operating units of the normal operations mode of the processing system, its *structural resilience* is specified by Eq(2).

$$R^s = R_K^s \quad (2)$$

Let h be the time horizon when the recovering capability of the processing system is estimated, also let s be an element of S_n^f . The function $F(s,t)$ denotes the production flowrate of the system at time t ($t \in [0, h]$) as a consequence of failures in s at time 0. The normalized production level at time t in case of failures in s is $p(s,t)$, it is defined by Eq(3). It takes into account that there is no production below δ at any time and the accepted production level is γ . Note that $0 \leq p(s,t) \leq 1$ for any set of failures given by s at any time t . In determining $p(s,t)$, the production level is changing with time as operating units are returning from failure operation to normal operation ($0 \leq t \leq h$).

$$p(s,t) = \begin{cases} 1, & \gamma < F(s,t) \\ \frac{F(s,t)}{\gamma}, & \delta \leq F(s,t) \leq \gamma \\ 0, & F(s,t) < \delta \end{cases} \quad (3)$$

Function $f(y)$, penalty function, is for expressing the importance of the deviation from the required production level in partial operation. It is assumed that $f(y)$ is a monotonically increasing function from interval $[0,1]$ to interval $[0,1]$, i.e. $f(y): [0,1] \mapsto [0,1]$, where $f(0)=0$ and $f(1)=1$. In practice, the penalty function can simply be $f(y)=y$. The relative production rate of the system during time horizon h as a consequence of failures in s started at time 0 is given by $q(s)$ as defined by Eq(4).

$$q(s) = \frac{\int_0^h f(p(s,t)) dt}{h} \quad (4)$$

The resilience of the processing system of at most n failures is given by Eq(5) and the resilience of the processing system is given Eq(6). Both R_n and R depend on the time horizon (h), the minimum accepted production flowrate (γ), and the minimum production flowrate (δ) through Eqs(3) and (4).

$$R_n = \frac{\sum_{s \in S_n^f} q(s)}{N_n^f} \quad (5)$$

$$R = R_K \quad (6)$$

Based on the definition of resilience R , it can be proven that $0 < R \leq 1$. Resilience R is 1 if and only if the system's production flowrate is at least γ on the whole time horizon $[0, h]$ for any failures. This can be the case especially when for each operating unit of the normal operation, the system has an equivalent redundant operating unit. The resilience of a system can be close to zero if only the normal operation's production flowrate is above δ (i.e. for any failure, the production is below δ) and the repair time of any failed operating unit is at least h .

If the processing system produces k different products simultaneously, all of the previously introduced α, β, γ , and δ of a single product are defined separately for each product; $\alpha_i, \beta_i, \gamma_i$, and δ_i belong to product i ($i = 1, 2, \dots, k$). For the resilience formula of multiproduct systems, $p(s,t)$ is defined as a function of the individual $p_i(s,t)$ production levels as shown by Eq(7). Naturally, it is supposed that all arguments and the values of function g are in the interval $[0,1]$, i.e. $[0,1]^n \mapsto [0,1]$. For example, $p(s,t)$ can be defined as the average, weighted average, or the minimum of the production levels of the products.

$$p(s,t) = g(p_1(s,t), p_2(s,t), \dots, p_k(s,t)) \quad (7)$$

4. Case studies

Two case studies are presented in this section. The first one demonstrates the evaluation of resilience of a single product system, and the other illustrates the application to a multiproduct facility for three instances: no redundant units, redundancies in 4 particular operations, and redundancies in all units.

4.1 Case study 1

Figure 1 illustrates a process of adipic acid production by nitric acid oxidation of KA oil (mixture cyclohexanol/cyclohexanone). Oxidation is carried out in a first reactor at low temperature (A) and a second at

high temperature (B). Nitrogen oxides are removed from the effluent in the bleacher and recovered as nitric acid in the absorber. Water generated in the reaction step is removed in the concentrator. The crude adipic acid is removed from the concentrated solution by means of crystallization and filtering, subsequently, it is purified by resorting to crystallization and drying. The liquid effluent of the filter is divided; the first part is recycled to the reaction step, and the second is treated to recover the catalyst and to remove by-products from the process. The process includes five redundant operating units, one for each of the operations Pump, Compressor, Concentrator, Filter, and Centrifuge. The capacities of the redundant operating units are lower than that of the operating units in the normal operation. The relative capacities of the pump, the compressor, the concentrator, the filter, and the centrifuge are 0.7, 0.7, 0.6, 0.8, and 0.8. The process can remain operational if either the Catalyst recovery or the byproducts treatment fails, because the related materials can be transferred to storage units for later processing. The maximal, the normal, and the accepted production flowrates are 4,375 kg/h, i.e. $\alpha = \beta = \gamma = 4,375$ kg/h. Minimum production level is 2,000 kg/h, i.e. $\delta = 2,000$ kg/h. The time horizon is 720 h.

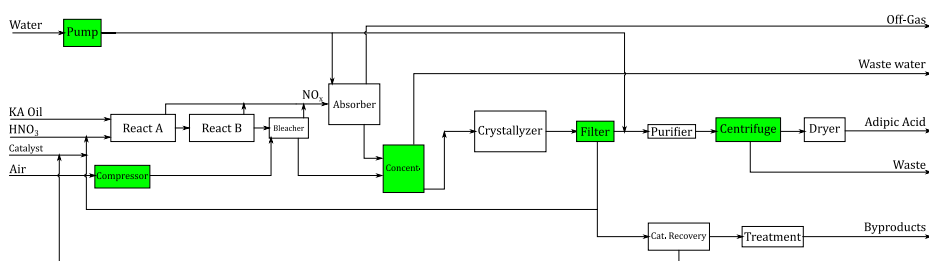


Figure 1: Process network of Adipic Acid production

The system resilience is visualized in Figure 2. Unit-level redundancy is only installed for a few operating units, most failures render the system non-operational. Figure 2a shows the resilience as the function of the minimal production flowrate (δ) while the accepted production flowrate is constant, ($\gamma = 4,375$ kg/h). On Figure 2b, the resilience is given as the function of the accepted production (γ) for fixed β and δ ($\beta = 4,375$ kg/h, $\delta = 2,000$ kg/h). Most of the failures make the system non-operational resulting in low level resilience. The resilience of the process can be improved by considering parallel units in some of the main steps of the process. This would enable these operations to remain operational, while the failed parallel operation is repaired.

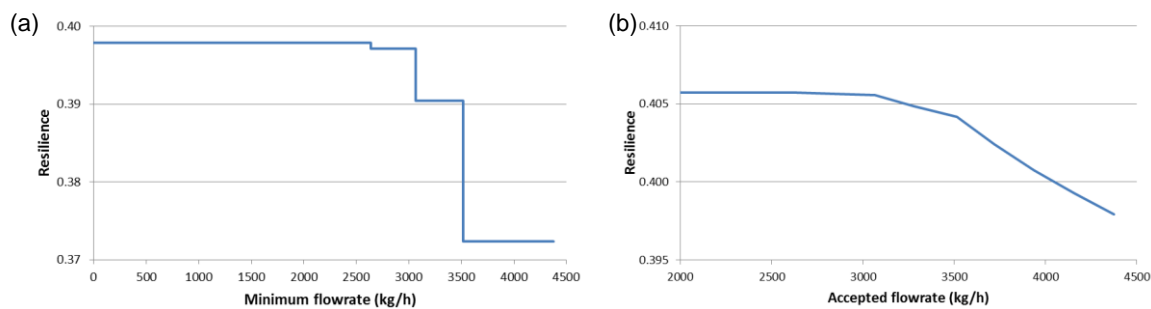


Figure 2: (a) Resilience as a function of δ (case study 1) (b) Resilience as a function of γ (case study 1)

4.2 Case study 2

The processing system of case study 2 is a multiproduct plant consisting of six individual processes. It was originally introduced by Iyer and Grossmann (1998) for process synthesis and was adapted by Gong and You (2018) for their resilience examination. Figure 3a shows the processing system, as presented by Gong and You (2018). Processes 1 and 2 involve addition reactions to acetylene, producing acrylonitrile and acetaldehyde, Process 3 generates acrylonitrile by means of the ammoxidation of propylene. Processes 4 and 5 are the steps for manufacturing phenol from benzene and propylene; process 4 comprises the formation of cumene and process 5 the oxidation step. Process 6 generates isopropanol through the addition of water to propylene. In their work, Gong and You determined the cost-resilience relation for different redundancy policies by taking into account only prespecified failures. In the current work, the resilience is determined globally by considering all possible failures that can occur expectedly or unexpectedly. The import of raw materials can also be subject to failures, they are represented by additional operating units. The extended system is given as a P-graph in

Figure 3b showing all raw materials. There is a process-level redundancy built in the system. Operating units O1 and O3 provide the system with 4 possible modes of operations, i.e. four different subprocesses that may generate all products. The flowrates of the products at normal operation (β_i) are given in Table 1. $\gamma_i = \beta_i$ and $\delta_i = 0$ for $i=1,2,\dots,6$. The system's resilience can be determined for all products.

To do so, function g of Eq(7) is defined as the minimum of the production levels of the products, i.e. $p(s, t) = \min(p_1(s, t), p_2(s, t), \dots, p_6(s, t))$. The operating units of the normal operation are listed in Table 2 along with their time to repair; the time horizon h is 720 h.

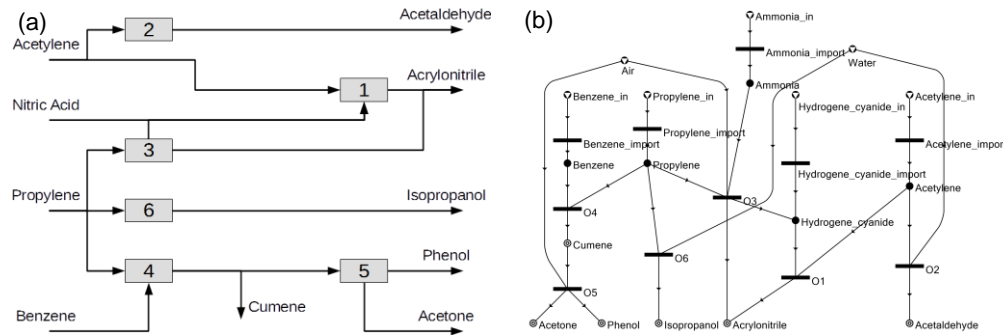


Figure 3:(a) Process network of case study 2 (b) Extended network of case study 2 as a P-graph

Table 1: Flowrates of the products in normal operation (β_i)

Product	β_i (kg/h)	Product	β_i (kg/h)
Acetaldehyde	5,000	Cumene	3,750
Acetone	8,750	Isopropanol	5,875
Acrylonitrile	15,000	Phenol	15,000

Table 2: Repair times of the operating units of the normal operating mode

Unit	Repair time (h)	Unit	Repair time (h)
O1	240	O6	528
O2	360	Ammonia_import	360
O3	480	Benzene_import	360
O4	432	Propylene_import	360
O5	600	Acetylene_import	360

Case study 2.1: no redundant units are built into the system.

Case study 2.2: There are four redundant units, units O2, O4, O5, and O6. The relative capacities of the redundant units compared to the original units are 0.5, 0.7, 0.7, and 0.8.

Case study 2.3: There are 6 redundant units, units O1, O2, O3, O4, O5, and O6. The relative capacities of the redundant units are 0.8, 0.5, 0.8, 0.7, 0.7, and 0.9.

Table 3 shows the system resilience for the three cases. In accordance with the engineering practice, system resilience is higher for a higher level of redundancy.

Table 3: System resilience for the three redundancy options

Case study	Resilience
case study 2.1	0.265
case study 2.2	0.463
case study 2.3	0.468

5. Conclusions

A novel methodology has been introduced for analysing the resilience of process systems. The capability of exhaustive enumeration of the P-graph framework is employed for determining all expected and unexpected failures and their effects on the operability of the system. The formula for determining the resilience of a production system in general, it can consider a processing system of any structure, any number of products, and any number of failures. The penalty function is introduced for the opportunity of expressing the importance

of the deviation from the required production level in partial operation. The resilience indicator is normalized for the interval $[0, 1]$, which the indicators of different processing systems can be compared. Two case studies illustrate the application of the proposed formula for a single and a multiproduct processing system. The future work will address the evaluation of partial operating levels for operating units, cascading effects of failures, and the consideration of resilience indicator in process synthesis.

Notations

α	maximal production flowrate	β	normal production flowrate
γ	minimum accepted production flowrate	δ	minimum production flowrate
t	time	h	time horizon
$q(s)$	relative production rate	$p(s,t)$	normalized production level
g	function to determine production level of a system of multiple products	s	combinations of failures
$F(s, t)$	production flowrate of the system at time t as a consequence of failures in s	f	penalty function
N_n^f	cardinality of set S_n^f	S_n^f	set of all combinations of at most n simultaneous failures
N_n	cardinality of set S_n	S_n	$\subseteq S_n^f$, their production flowrate is at least δ
R_n^s	structural resilience for at most n failures	K	number of operating units in normal operation
R_n	resilience for at most n failures	R^s	structural resilience
		R	resilience

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