

Fuzzy Mixed Integer Linear Program for Planning Enhanced Weathering

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Silicate minerals, which are naturally present in rocks, react with CO₂ in the atmosphere to produce solid carbonate minerals or bicarbonate ions dissolved in oceans. In both cases, CO₂ is trapped almost indefinitely. More commonly known as weathering, this mechanism is the natural way of moderating the CO₂ levels in the atmosphere. In this regard, enhanced weathering which aims to hasten this natural process in order to sequester CO₂ within human relevant time scales, has been proposed as a potential strategy towards moderating climate change. This process involves grinding the silicates and applying them on land surfaces to facilitate dissolution. Most experts estimate that the process has the capability of sequestering gigatons of CO₂ annually. However, implementing this strategy on an industrial scale will require systematic methodologies to maximize its potential benefits. The property of silicate rocks affects its potential for CO₂ sequestration while soil properties can impose limits on its application. Furthermore, estimates for its economic feasibility remain uncertain. A fuzzy mixed integer linear program is developed in this work for planning enhanced weathering networks in consideration of system uncertainties. Results suggest that sequestration potential and application rate largely influences the network structure.

1. Introduction

Climate change is a critical environmental issue that raises the challenge of managing greenhouse gas (GHG) emissions to levels that will avert catastrophic temperature rise. In addition to measures such as improving energy efficiency and using more renewables, mass deployment of CO₂ capture and storage (CCS) and negative emissions technologies (NETs) will be needed in order to cut GHG emissions to zero by mid-century (Haszeldine et al., 2018). NETs are technological systems that achieve net removal of atmospheric CO₂ through different physical, chemical or biological pathways. Examples include direct air capture (DAC), bioenergy with CCS (BECCS), biochar application, ocean fertilization, and enhanced weathering (EW). McLaren (2012) gave a comparative assessment of different NETs considering CO₂ abatement potential, cost, and technology maturity. A series of review papers on NETs was published recently, focusing on research landscape (Minx et al., 2018), risk-factored abatement potential (Fuss et al., 2018), and commercialization challenges (Nemet et al., 2018). The potential of NETs to curb GHG emissions has been done at various scales, with results being reported for Scotland (Alcalde et al., 2018), the UK (Smith et al., 2016b), and the entire world (Smith et al., 2016a). These assessments considered constraints such as land footprint, water footprint, energy footprint, and nutrient footprints as limiting factors for carbon sequestration.

The potential to capture CO₂ via EW of silicate-bearing rocks was first proposed by Seifritz (1990). The reaction of dissolved CO₂ in rainwater with silicate minerals forms bicarbonate ions which are eventually carried into the oceans via the natural water cycle. This is a naturally occurring process which can be accelerated by reducing the silicate-bearing rocks into a fine powder with large specific surface area. This powder can then be applied to soil at a rate based on land area, rainfall and expected dissolution speed (Moosdorf et al., 2014). The theoretical CO₂ sequestration rate per unit mass of rock is determined stoichiometrically by its composition, but in practice, the potential may be limited by site conditions such as average ambient temperature and rainfall level. Dissolution rate can be increased by reducing particle size, at the expense of increased specific energy consumption and CO₂ emissions for rock crushing (Strefler et al., 2018). Edwards et al. (2017) discussed the

possible co-benefits of EW in agricultural land, such as favourable pH adjustment of acidic soils and introduction of phosphorus to increase fertility. Renforth (2012) estimated that up to 430 Gt CO₂ can be offset by EW in the UK alone based on mineral resources; however, in practice, actual potential is limited by application rates on land due to the risk of unwanted changes in soil chemistry.

Process Integration (PI) plays a key role in the design and planning of sustainable, low-carbon systems (Klemeš et al., 2019). The core tool of Pinch Analysis (PA) has diversified from the initial application of industrial heat recovery towards a broad range of sustainability applications (Klemeš et al., 2018). The area of Carbon Management Networks (CMNs) has recently emerged as a specialized application of PI tools (Tan and Foo, 2018). The potential of PI, including both PA and complementary Mathematical Programming (MP) approaches, for planning large-scale use of NETs was suggested by Tan et al. (2020).

In this work, a fuzzy mixed integer linear programming (FMILP) model is developed for planning EW-CMNs. The formulation is based on a simple linear programming (LP) model (Tan and Aviso, 2019) which is extended by introducing binary variables, a fuzzy goal (objective), and fuzzy constraints. These new features allow important problem features to be represented in the model. The binary variables allow different user-specified characteristics to be introduced (Poplewski et al., 2010), while the fuzzy goals and constraints enable uncertainties to be explicitly captured by the model. The rest of the paper is organized as follows. Section 2 gives the formal problem statement. Section 3 describes the FMILP model formulation. Section 4 shows its application to an illustrative case study. Finally, Section 5 gives the conclusions and prospects for future work.

2. Problem statement

The formal problem statement can be stated as follows:

- Given m sources of silicate-bearing rocks of known quantity and annual capacity;
- Given n application sites with pre-determined fuzzy application rate limits;
- Given that the silicate-bearing rocks have fuzzy CO₂ sequestration potential;
- Given other user-specified constraints on the CMN.

The aim of the problem is to determine the optimal allocation of silicate-bearing rocks to sinks which achieves the greatest CO₂ potential in consideration of uncertainties in application rates and sequestration potential characteristics. The problem structure resembles a conventional supply chain network optimization (Tan and Aviso, 2019). The superstructure of the network is illustrated in Figure 1.

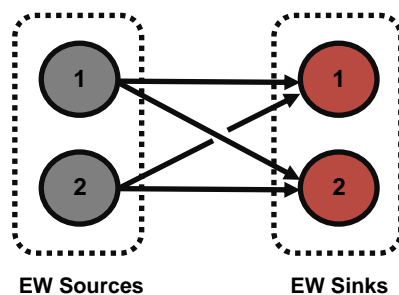


Figure 1: Superstructure of Enhanced Weathering Management Network

3. Optimization model

Fuzzy set theory was proposed by Zadeh (1965) to represent sets which allow partial membership of objects. Bellman and Zadeh (1970) discussed the use of this theory as basis for fuzzy decision making, which entails seeking the intersection of fuzzy goals (or objectives) with fuzzy constraints. Zimmermann (1978) then developed a generic fuzzy MP formulation which is the basis for the FMILP model described here. In the formulation, the optimal solution is one that has the highest degree of aggregate membership in the fuzzy goals and fuzzy constraints. The degree of membership of each fuzzy goal or constraint linearly increases from 0 to 1, where 0 represents the least degree of satisfaction and 1 indicates full satisfaction. Different types of fuzzy membership functions exist depending on the desired objective, trapezoidal membership for example is used for equality constraints. The relevant fuzzy membership functions are illustrated in Figure 2. In Figure 2a, a piecewise maximizing linear membership function for CO₂ sequestration rate is shown. In this case, higher values are considered more desirable. In Figure 2b, a piecewise minimizing linear membership function is shown for rock application rate, taking into account the risk of adverse effects of soil chemistry. In this case, lower

values will incur lower risk and be seen as more desirable by a conservative decision-maker. Both types of membership functions can be considered simultaneously within the same FMILP model.

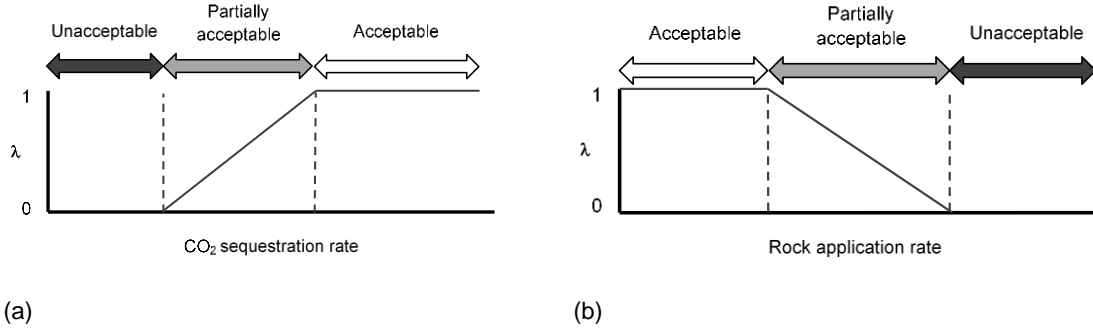


Figure 2: Fuzzy membership function for a) CO_2 sequestration rate and b) rock application rate

The objective function is to maximize the over-all degree of satisfaction, λ , as indicated in Eq(1). It is subject to production capacity constraints as indicated in Eq(2) where r_{ij} is the amount of ultramafic rock from source i applied to sink j and S_i is the annual capacity of source i . Eq(3) accounts for sink application limits where D_j is application limit for sink j which is subject to the fuzzy upper and lower limits as indicated in Eq(4). Eq(4) indicates that it is more desirable to minimize the application rate. Eq(5) accounts for the over-all capacity of sinks, C_j , to accommodate the application of powdered rock, wherein the total amount of powdered rock delivered to sink j is equal to the application rate (r_{ij}) multiplied by the total operating life of the source (P_i). Eq(6) accounts for the total carbon footprint (CF) of the network where α corresponds to the CO_2 sequestration potential of ultramafic rocks, β_{ij} is the carbon footprint associated with the transport of powdered rock from source i to sink j and γ is the carbon footprint associated with the production of powdered rock (i.e. includes emission from mining, crushing and transport). It is desired that CF be as low as possible, such that the degree of satisfaction λ approaches the value of 1.00 as CF approaches the lower limit CF^L as indicated in Eq(7). Eq(8a) indicates whether a link between source i and sink j is activated where M is an arbitrary large number and b_{ij} is a binary variable which takes a value of 1 when the link is activated. Eq(8a) can also take the form of Eq(8b) wherein minimum (F_{ij}^L) and maximum limits (F_{ij}^U) to flowrates are imposed for connection between source i and sink j . Furthermore, other topological conditions might be required such as defining limits on the number of sinks that sources can deliver to as given by Z (Eq(9a)), forbidden source-sink matches (Eq(9b)) and necessary links (Eq(9c)). Eq(10) defines that b_{ij} should be a binary variable.

$$\max \lambda \quad (1)$$

$$\sum_{j=1}^n r_{ij} \leq S_i \quad \forall i \quad (2)$$

$$\sum_{i=1}^m r_{ij} \leq D_j \quad \forall i \quad (3)$$

$$D_j \leq D_j^U - \lambda(D_j^U - D_j^L) \quad \forall j \quad (4)$$

$$\sum_{i=1}^m P_i r_{ij} \leq C_j \quad \forall j \quad (5)$$

$$CF = \sum_{j=1}^n \sum_{i=1}^m (\alpha + \beta_{ij} + \gamma) P_i r_{ij} \quad (6)$$

$$CF \leq CF^U - \lambda(CF^U - CF^L) \quad (7)$$

$$r_{ij} \leq b_{ij} M \quad \forall i, j \quad (8a)$$

$$b_{ij}F_{ij}^L \leq r_{ij} \leq b_{ij}F_{ij}^U \quad \forall i, j \quad (8b)$$

Topological constraints:

If there is a limit to the total number of sinks that can be connected to source i

$$\sum_{j=1}^n b_{ij} \leq Z \quad \forall j \quad (9a)$$

If the connection between source i and sink j is not allowed:

$$b_{ij} = 0 \quad (9b)$$

If a connection between source i and sink j should be present in the network:

$$b_{ij} = 1 \quad (9c)$$

$$b_{ij} \in \{0,1\} \quad (10)$$

4. Case study

The limiting data for the sources are indicated in Table 1 while those for the sink are in Table 2. Data from the sources are considered crisp because information regarding the available capacity of a plant to crush rocks can easily be obtained and determined in contrast to knowing the actual limit of rock application to soil. The CO₂ sequestration potential for the soil application of powdered rocks is $\alpha = -0.3$ kt·CO₂/kt of rock. The emission factor associated with the transport of powdered rock from source to sink (β_{ij}) are shown in Table 3, these were obtained using the assumption that the carbon footprint associated with the transport of materials will generate 0.1 kg·CO₂/t/km (Tan, 2016). The emission factor associated with powder production is $\gamma = 0.05$ kt·CO₂/kt rock (Strefler et al., 2018).

Table 1: Limiting Data for Sources (Tan and Aviso, 2019)

Sources	Rock Quantity (kt)	Rock Flowrate (kt/y)	Operating life (y)
1	25	1.00	25
2	40	2.00	20
3	75	2.50	30

Table 2: Limiting Data for Sinks (adapted from Tan and Aviso, 2019)

Sinks	Rock Quantity (kt)	Fuzzy limits for Rock Flowrate (kt/y)		Operating life at maximum operation
		Lower limit	Upper limit	
D1	4	0.02	0.40	10
D2	16	0.45	0.80	20
D3	20	0.70	1.00	20
D4	15	0.12	0.60	25
D5	160	1.00	4.00	40

Table 3: Emission factor for the transport of rocks from source to sink (β_{ij}) in kt·CO₂/kt of rock

Sources	D1	D2	D3	D4	D5
1	0.0025	0.0160	0.0095	0.0045	0.0075
2	0.0170	0.0015	0.0070	0.0080	0.0063
3	0.0030	0.0150	0.0180	0.0175	0.0190

The lower limit for the carbon footprint of the entire network was obtained by minimizing the CF using the largest possible application rate and results in $CF^L = -33.34 \text{ kt}\cdot\text{CO}_2$ while the upper limit is taken as $CF^U = 0 \text{ kt}\cdot\text{CO}_2$ which represents the case of not utilizing the network.

Two different scenarios are considered for this case study. Scenario 1 considers that there is no topological constraint in establishing the network, while Scenario 2 considers topological constraints. For Scenario 1, Eq(1) is solved subject to the constraints defined by Eq(2) to Eq(7). The resulting over-all degree of satisfaction is $\lambda = 0.7156$ which corresponds to $CF = -23.86 \text{ kt}\cdot\text{CO}_2$. This indicates that the fuzzy constraints were satisfied to a degree of at least 71 %. The optimal network is then summarized in Table 4 with the application rates indicated at the last row. It can be seen that the application rate for each sink and the optimal CF are closer to the fuzzy lower limits. Results show that the total available powdered rock from Sources 1 and 3 are completely used up while those from Source 2 is only partially utilized. With regards to the distribution of crushed rock, the sink with the highest potential for carbon sequestration based on application rate limit is prioritized with D5 receiving the most amount of rock. The link between Source 3 and D5 for example has the highest amount of rock application despite having the highest associated transport emission. This indicates that the sequestration capability of powdered rock application offsets the CO_2 generated from processing and transport. Furthermore, since this scenario did not consider any topological constraints, Source 3 is linked to 4 out of 5 sinks.

Table 4: Optimal allocation of crushed rock in kt/y (r_{ij}) for Scenario 1

Sources	D1	D2	D3	D4	D5	Total
1	0.0000	0.0000	0.6637	0.2565	0.0798	1.0000
2	0.0000	0.0487	0.0241	0.0000	0.0000	0.0728
3	0.1281	0.5009	0.0975	0.0000	1.7735	2.5000
Application rate	0.1281	0.5496	0.7853	0.2565	1.8533	

Scenario 2 is then considered with the constraint that sources can only be linked to a maximum of 2 sinks ($Z = 2$). Eq(1) is solved subject to Eq(2) to Eq(8a), Eq(9a) and Eq(10). The resulting network is shown in Table 5. This network has a slightly higher carbon footprint of $CF = -23.63 \text{ kt}\cdot\text{CO}_2$ with an over-all degree of satisfaction of $\lambda = 0.7091$. Only Source 1 was completely utilized and similar to Scenario 1, sink D5 still receives the most amount of powdered rock. When compared against the network of Scenario 1, it can be seen that for Source 1 and Source 3, the links which transported the higher amounts of powdered rocks were maintained. However, for Source 2, the link was changed to transport crushed rock to D1 and D3 instead of D2 and D3 in Scenario 1. Such alternative solutions provide options for a decision-maker such that different designs can be considered when deciding on the final carbon management network.

Table 5: Optimal allocation of crushed rock in kt/y (r_{ij}) for Scenario 2

Sources	D1	D2	D3	D4	D5	Total
1	0.0000	0.0000	0.7402	0.2598	0.0000	1.0000
2	0.1307	0.0000	0.0472	0.0000	0.0000	0.1779
3	0.0000	0.5333	0.0000	0.0000	1.8738	2.4071
Total	0.1307	0.5333	0.7874	0.2598	1.8738	

5. Conclusions

A fuzzy mixed integer linear model for planning enhanced weathering networks has been developed in this work. This model accounts for uncertainties such as allowable application rate limits and carbon footprint targets in determining the optimal network solution. Two different scenarios were considered in this work where the first Scenario did not account for topological constraints and the second accounted for constraints and limitations on the number of connections that can be made with the sources. For both instances, the model is able to identify a compromising solution which results in net negative carbon footprint without maximizing the soil application limits. The integration of topological constraints resulted in a slight increase in carbon footprint. Enhanced weathering is just one of many negative emission technologies and future work can be done on the integration of enhanced weathering networks with biomass co-firing networks which also generate byproducts for soil application. Exploring other types of topological constraints on these networks, investigating near optimal networks and the considering economic feasibility and social acceptability to represent other sustainability concerns, can also be considered for future studies.

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