

Topology Optimization of Pipe Network in a Distributed Energy System

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Under the stress of energy saving and environmental conservation, distributed energy system becomes more promising due to its high energy efficiency. Previous research on the network of distributed energy system mainly emphasize shortest pipe length, but ignore the heat and pressure loss during transportation in different topologies. This work proposes two new topologies for the pipe network of distributed energy system, e.g. Euclidean Steiner minimum tree and Rectilinear Steiner minimum tree, to reduce the investment and energy loss. The objective of this work is to minimize total annual cost involving capital cost, pressure drop and heat loss. Graphic theory such as GeoSteiner and Kruskal algorithm are used to solve the problem. Linear model is used to describe the calculation model of flow rate in the pipeline. Both graphic theory and linear programming are coupled in the optimization framework. To illustrate the effectiveness of the two topologies, this work compares them with conventional topologies, e.g. star style and Multiple Spanning Tree style. Based on the results, Euclidean Steiner minimum tree has better economic performance, its total annual cost is 12 % and 9.17 % lower than star style and Multiple Spanning Tree.

1. Introduction

Distributed energy systems are synthetical energy utilization systems located in user end. The system is easier to provide energy according to the different energy demand of users, so that compared with conventional energy systems, distributed energy systems are more efficiency and environment friendly.

A distribution network is an essential element of a distributed energy system. For a network, the layout of pipeline, the selection of diameter and Insulation layer thickness, and the hydraulic calculations all have great impact on the economic and efficiency performance of the system. Li et al. (2010) optimize the system considering supply variation of cooling and heating and the operation adjustment in different periods of a year by using genetic algorithm (GA). Khir and Haouari (2015) studied the control system of distributed network, and developed a Mixed Integer Nonlinear Programming based method to find the optimal cooling duty and storage capacity. Zeng et al. (2016) investigated conventional central circulating pump system and the distributed variable speed pumps system in a distributed energy system by using integer-coded genetic algorithm. Wu et al. (2016) established a multiple objective optimization framework to determine the optimal combination of cogeneration and renewable energy. Li et al. (2019) proposed a modelling and optimization framework for design of distributed energy systems in remote areas, featuring residential, small industrial, commercial and agricultural power loads, off-grid network, and solar, wind, and biomass as primary energy sources.

The above-mentioned works do not consider the optimization of pipe network topology. However, it affects the capital investment and operation cost significantly. For the research of pipe network optimization, the relative research is rare. Chan et al. (2007) considered the optimization of pipe network in a district cooling system, in

their work, GA incorporated with local search techniques was developed to find the optimal/near optimal configuration of the piping network in a hypothetical site. For the pipe network optimization of distributed energy system, most scholars aimed to minimize capitalized cost with user demand, velocity and pipe diameter as constraints. The layout optimization problem is turned to Minimum Spanning Tree (MST) problem in graphic theory and the pipe network is considered as branch style network. However, in these works, energy loss, including heat loss and pressure drop, are not involved and different pipe network styles are not considered.

Based on predecessors' work, this work considers the optimization of pipe network with Steiner Tree style. Minimizing total annual cost (TAC) is set to be the objective function. Energy loss are involved in the optimization. Different graphic theory algorithms are used to optimize different pipe network style and the results are compared and analyzed. This work can be used to find the best pipe network topology style and guide the practical design of distributed energy system.

2. Mathematical models

The objective function is shown in Eq(1) as follows.

$$\min C = C^{pipe} + C^{pressureloss} + C^{heatloss} \quad (1)$$

$$C^{pipe} = \sum_{k=1}^n C_k^{pipe} \quad (2)$$

$$C^{pressureloss} = \sum_{k=1}^n C_k^{pressureloss} \quad (3)$$

$$C^{heatloss} = \sum_{k=1}^n C_k^{heatloss} \quad (4)$$

In the equations, C (¥/y) is the total annual cost, C^{pipe} , $C^{pressureloss}$ and $C^{heatloss}$ are the capital cost, cost due to pressure drop and cost due to heat loss of steam supply network. n is the number of branches in the pipe network. It is noted that this work only considers one pipe network.

2.1 Model for pipe network capital cost

Capital cost can be calculated through Eq(5). In the equation, t_{year} (y) is the lifetime of the pipe network, I is the interests rate, a_k (¥/m) is the unit price of steam pipeline, L_k (m) is the length of each branch. a_k is related to the diameter of the pipeline, and the diameter can be calculated through Eq(6). In Eq(6), D_k^{in} (m) is the inner diameter of pipeline, W_k (kg/s) is the mass flow rate, ρ (kg/m³) is the density of steam, u (m/s) is velocity. It is noted that in different branch within the network, mass flow rate is different, so the mass flow rate is solved through a linear programming (LP) model, as shown in Eq(7). Eq(8-12) is the constraints of the LP model. In these equations, $w_{\alpha,\beta,\gamma}$ (kg/s) is the mass flow rate of branch connection between vertices α and β . b^{binary} is a binary variable indicating if a connection existed between the two vertices. W_γ is the mass flow rate of vertices γ . If a user produce steam, W_γ will be a negative value, otherwise, it is a positive value. In each branch, the fluid may flow from α to β , or from β to α , the term $(-1)^r$ is used to determine the flow direction.

$$C_k^{pipe} = \frac{I(1+I)^{t_{year}}}{(1+I)^{t_{year}} - 1} a_k L_k \quad (5)$$

$$D_k^{in} = \sqrt{\frac{4W_k}{\pi\rho u}} \quad (6)$$

$$\min W = \sum_{\gamma} \sum_{\alpha} \sum_{\beta} b^{binary} \times w_{\alpha,\beta,\gamma} \quad (7)$$

$$\sum_{\alpha} \sum_{\beta} ((-1)^r b^{binary} \times w_{\alpha,\beta,\gamma} + (-1)^r b^{binary} \times w_{\beta,\alpha,\gamma}) - W_\gamma = 0 \quad (8)$$

$$\left[\begin{array}{l} b^{binary} = 1 \\ [\alpha = \alpha_j] \wedge [\beta = \beta_j] \end{array} \right] \vee \left[\begin{array}{l} b^{binary} = 0 \\ [\alpha \neq \alpha_j] \vee [\beta \neq \beta_j] \end{array} \right] (\alpha_j, \beta_j) \in E_T \quad (9)$$

$$\begin{bmatrix} r=1 \\ \alpha < \beta \end{bmatrix} \vee \begin{bmatrix} r=2 \\ \alpha \geq \beta \end{bmatrix} \quad (10)$$

$$w_{\alpha,\beta,\gamma} \geq 0 \quad (11)$$

$$w_{\beta,\alpha,\gamma} \geq 0 \quad (12)$$

$$a_k = A_1 W t_k + A_2 (D_k^{out})^{0.48} + A_3 + A_4 D_k^{out} \quad (13)$$

$$W t_k = 644.3 (D_k^{in})^2 + 72.5 D_k^{in} + 0.4611 \quad (14)$$

$$D_i^{out} = 1.052 D_i^{in} + 0.005251 \quad (15)$$

In this work, only low-pressure steam is considered in the case. α_k can be calculated through Eq(13) (Stijepovic and Linke, 2011). In this equation, D_k^{out} (m) is the outer diameter of pipeline, $W t_k$ (kg/m) is the weight of unit length pipeline, A_1 (¥/kg) is the price of unit weight pipe, A_2 (¥/m^{0.48}) is the installation cost of pipeline, A_3 is the road usage cost, A_4 is the insulation cost. Eq(14) and(15) is used to calculated D_k^{out} and $W t_k$.

2.2 Model for pressure drop cost

Steam does not need a compressor to transport it but during the transportation, pressure drop occurs. The pressure of condensed water must be increased to meet the steam production requirement. Therefore, the pressure drop cost $C_k^{pressureloss}$ is calculated in terms of water pump cost in this work, as shown in Eq(16).

$$C_k^{pressureloss} = \frac{a^E t^{time} N_k}{1000} \quad (16)$$

$$N_k = \frac{N e_k}{\eta} \quad (17)$$

$$N e_k = H_k^f W_k g \quad (18)$$

$$H_k^f = \left(\sigma \frac{L_k}{D_k^{in}} + \sum \zeta^E + \zeta_k \right) \frac{u^2}{2g} \quad (19)$$

$$\sum \zeta^E = \frac{L_k}{25} \zeta^E \quad (20)$$

$$\begin{bmatrix} \zeta_k = 0 \\ j = 0 \end{bmatrix} \vee \begin{bmatrix} \zeta_k = \left(1 - \frac{S_{k-1}^{in}}{S_k^{in}} \right)^2 \\ [S_{k-1}^{in} < S_k^{in}] \end{bmatrix} \vee \begin{bmatrix} \zeta_k = 0.5 \left(1 - \frac{S_k^{in}}{S_{k-1}^{in}} \right)^2 \\ [S_{k-1}^{in} > S_k^{in}] \end{bmatrix} \quad (21)$$

$$S_k^{in} = \frac{\pi (D_k^{in})^2}{4} \quad (22)$$

In Eq(16), a^E (¥/kW·h) is the unit power cost, t^{time} (h) is the annual operation time, N_k (W) is the shaft power of power to overcome the flow resistance, and it can be calculated out through Eq(17). η is the efficiency of pump, $N e_k$ (W) is the available power of pump, and it can be calculated through Eq(18). H_k^f (m) is the pressure head loss in the pipeline and it can be calculated through Eq(19) to (22). In these equations, ζ_k is the local resistance coefficients when the flow area of pipeline is changed, ζ^E is the local resistance coefficients of elbow, S is the flow area of pipeline, σ is the friction factors.

2.3 Model for heat loss cost

Heat loss cost can be calculated through Eq(23). In the equation, a (¥/kg) is the unit price of steam and q (kJ/kg) is the latent heat of steam, Q (kJ/m·s) is the heat loss. When there is a isolation layer existed, the heat loss of pipeline can be calculated through Eq(24)

$$C_k^{heatloss} = \frac{a Q_k L_k t^{time}}{q} \quad (23)$$

$$Q_k = \frac{\varepsilon\pi(T - T_a)}{1000\left(\frac{1}{\lambda} \ln \frac{D_k^o}{D_k^n} + \frac{2}{\varepsilon D_k^o}\right)} \quad (24)$$

In the equations, T ($^{\circ}\text{C}$) is the temperature of pipeline outside surface, T_a (K) is the ambient temperature, D_k^o (m) and D_k^n (m) are the inner and outer diameter of isolation layer. λ (W/m·K) is the thermal conductivity of isolation layer, ε (W/m²·K) is the convective heat transfer coefficient between the layer surface and ambient.

3. Optimization algorithm

3.1 Kruskal algorithm

Kruskal algorithm belongs to graph theoretic approach, it can find the minimum spanning tree from a connected graph, and it is a well-known greedy strategy to calculate the undirected connected graph with weights. So Kruskal algorithm is selected.

3.2 GeoSteiner algorithm

GeoSteiner is the fastest program for computing exact solutions to Steiner tree problems, including the rectilinear Steiner minimal tree (RSMT) and Euclidean Steiner minimal tree (ESMT) problem. Therefore, it is selected to optimize the length of the pipe network. There are two phases for generating ESMT and RMST, e.g. the generation of full Steiner tree (FST) and FST connections. The first phase is full Steiner tree (FST) generation which is achieved by enumerate algorithm. The second phase is FST concatenation which is solved by branch and cut algorithm. FST is a non-degenerate minimum Steiner tree in which every terminal has degree.

4. Case studies

The case considers four types of network topologies, e.g. Star, MST, RSMT and ESMT. MST is solved by Kruskal algorithm, RSMT and ESMT are solved by GeoSteiner algorithm. The modelling and optimization processes are achieved through C++ coding. The model parameter used in this case is shown in Table 1.

Table 1: Model parameters in case study

Parameter	Number	unit	Parameter	Number	unit
T_{year}	10	y	T^{time}	8,760	h
l	0.02		η	0.8	
ρ	0.60	kg/m ³	σ	0.015	¥/kg
u	30.00	m/s	α_k	0.1945	
A_1	5.74	¥/y	Q	1,999.9	kJ/kg
A_2	1295	¥/m ^{0.48}	T_a	276.5	K
A_3	47.6	¥/m	λ	0.06	W/m ² ·K
A_4	2065	¥/m	v	3.50	m/s
α^E	0.21	¥/kW·h	ε	11.63	W/m·K

The calculation results are shown in Table 2 and Figure 1. In the figure, four types of topologies are shown and the thickness of the line indicating the diameter of the pipes.

Table 2: Results of the case study

Topology type of pipe networks	Star	MST	RSMT	ESMT
Total pipe length ($\times 10^4$ m)	3.65	1.19	1.19	1.03
Total pipe cost ($\times 10^7$ ¥/y)	0.869	1.03	1.28	0.939
Pressure drop cost ($\times 10^7$ ¥/y)	0.274	0.704	0.936	0.654
Heat loss cost ($\times 10^7$ ¥/y)	1.10	0.443	0.458	0.387
Total cost ($\times 10^7$ ¥/y)	2.25	2.18	2.67	1.98

As shown in Table 2, the total cost of ESMT is the lowest, and that of RSMT is the highest. From the first row in Table 2, it can be seen that the total pipe length of Star type is much longer than the others. The reason is that every user and energy hub is connected by an individual pipeline, no pipeline is shared. This topology is the safest connection way because when a pipeline is damaged, only one user is affected. It is noted that although

the connection is the longest for the star type, the pipe cost is not that high, the reason is that the diameter for all the pipes is small. However, this type is not practical when the users are far away due to the very long connections. Compared with star, MST RSMT ESMT is more suitable for practical design. Among them, the connection of MST is relatively simpler, but the length of pipe line is relatively longer, so it is not good for energy saving. The connection of RSMT is the neatest one, so it can have a good performance in practise to fit the road layout. The connection of ESMT is the shortest among all four types, and the cost is also the lowest. If there are no constraints for practical path, the connection of ESMT is the most economic design. Since ESMT considers the connection of Steiner point, the pipe length of ESMT will definitely shorter than MST. Meanwhile, the path way of RSMT is constrained, so its pipe length is longer than ESMT.

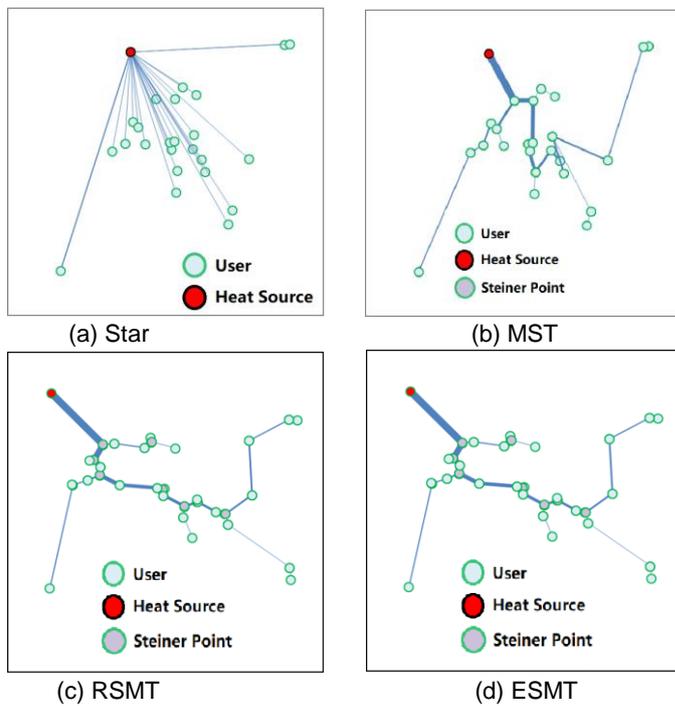


Figure 1: Results for different topology types

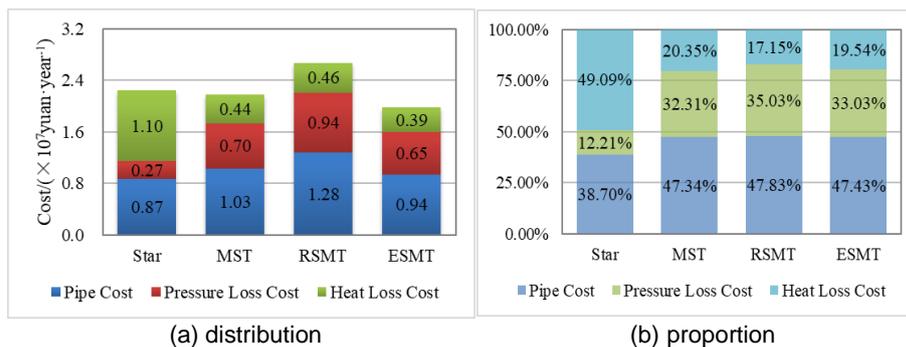
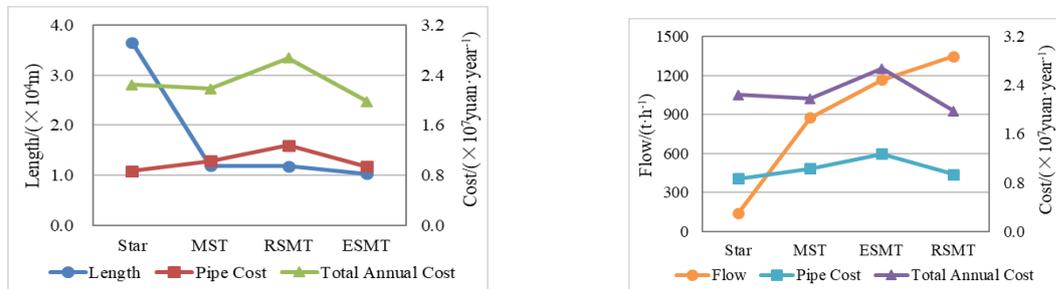


Figure 2: Cost distribution and proportion for the four topologies

Figure 2 shows the percentage of pipe cost, pressure drop cost and heat loss cost for the four topology types. The cost distribution is quite similar for MST, RSMT and ESMT topologies, but the star topology is very different from the others. For star topology, the percentage of heat loss cost is very high, the reason is that heat loss is highly related to the length of the pipes, and star topology is one with the longest pipe length. The same phenomenon can be found in ESMT topology because the pipe length for this topology is the shortest, heat loss cost account for lower percentage of the total cost. For pipe investment, ESMT is the shortest on followed by MST. From the model, the length of pipe network and flow rate in each branch both have a significant impact on pipe investment. This is again the reason for the quite low pipe cost from star topology due to the lower flow

rate in each connection. Contrarily, RSMT topology has large numbers in both pipe length the flow rate in branches, so that the pipe investment for this topology is the highest.

Figure 3 illustrates the variation trend between network length, flow rate, pipe cost and TAC. It can be found that, for different topology types, pipe length, the total flow rate cannot accurately reflect the variation of pipe cost separately. These results illustrate the effectiveness of the proposed method to comprehensively consider the factors involved in the pipe network design.



(a) Relation between length, pipe cost and TAC

(b) Relation between Flow rate, pipe cost and TAC

Figure 3: Relation between detailed elements for the four topologies

5. Conclusions

This work establishes a topology optimization model to optimize the pipe network of the distributed energy system with the objective of minimizing TAC. New topology types such as ESMT and RSMT are proposed and pressure drop and heat loss related cost are involved in the optimization. LP model is used to solve the optimal flow rate in each branch of pipe network. Based on the results of the case study, it is found that:

- (1) By compared the optimal structure and performance, ESMT is the most economic topology type.
- (2) RSMT topology is a quite expensive type but it fits well with real life road layout.
- (3) Star topology has the longest pipe connection and it is not practical for large scale problem with long distance with users and energy hub.
- (4) Pipe length and total flow rate cannot accurately reflect the economic performance of the network separately. The proposed method is effeteness for finding the best network topology for a distributed energy system.

In this work, the proposed work can be used to find the most economic pipe network. However, reliability of pipe network is not quantified. Therefore, it is not guaranteed that the network topology is the best. In future, this work will further extend to consider reliability.

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