CHEMICAL ENGINEERING TRANSACTIONS

VOL. 76, 2019

A publication of AIDIC

The Italian Association of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Petar S. Varbanov, Timothy G. Walmsley, Jiří J. Klemeš, Panos Seferlis

ISBN 978-88-95608-73-0; **ISSN** 2283-9216

DOI: 10.3303/CET1976123

Convex-lifting-based Robust Control of a Laboratory Plate Heat Exchanger

Juraj Oravec*, Monika Bakošová, Michaela Horváthová, Lenka Galčíková, Michal Slávik, Anna Vasičkaninová, Alajos Mészáros

Slovak University of Technology in Bratislava, Faculty of Chemical and Food Technology, Institute of Information Engineering, Automation, and Mathematics, Radlinskeho 9, SK-812 37 Bratislava, Slovak Republic
juraj.oravec@stuba.sk

The paper investigates the problem of advanced optimization-based robust control design for heat exchanger. To minimize the computational effort of real-time control, a convex-lifting-based robust control strategy was designed. The controlled device was a laboratory-scale plate heat exchanger. The mathematical model was derived using the set of experimentally measured data. The simulation of the closed-loop control was evaluated considering the uncertain model of the device. Both, reference tracking problem and disturbance rejection problem were investigated. The offset-free reference tracking control performance was ensured by introducing the integral action into the closed-loop system setup. The total energy consumption was evaluated.

1. Introduction

Heat exchangers are widely-utilized in all fields of industrial production. The increasing energy costs lead to the necessity to optimize the energy consumption in the industry. Control of heat exchangers is the challenging task as the behaviour of the device is non-linear, asymmetric, and affected by various uncertain and time-varying parameters (Klemeš and Varbanov, 2018).

Therefore, the development of advanced robust control strategies has attracted high interest of researchers in recent years. Advanced optimization-based robust control strategies are designed to face the impact of the parametric uncertainties and the noise measurement.

Model predictive control (MPC) and, particularly, robust MPC can overcome the above-mentioned obstacles. The simulation case study of the multivariable control of the temperature in a fermentation unit was designed by MPC in Violaro et al. (2018). In the paper Markowski and Trzcinski (2018), the online control maximizing the heat recovery in a heat exchanger network was designed. In a similar way, Trafczynski et al. (2018), investigated the control performance of the heat recovery maximization of the heat exchanger network coupled with a crude distillation unit in the presence of fouling. The advanced algorithm for a control-relevant design subject to the structural controllability and observability analysis of heat exchanger networks was proposed in Leitold et al. (2018).

The bottleneck of the implementation of the advanced optimization-based robust control strategies is determined by the necessity to solve the complex optimization problem in each control step. The solver-time limits the application of the advanced robust control methods for the system with fast dynamics, and the complexity/performance is bounded by the limits of the computational requirements of the embedded hardware. This paper directly extends the results of the paper by Nguyen et al., (2017), where the convex-lifting-based robust control design was introduced, and the paper by Oravec et al. (2019), where the original approach was improved by introducing the tunable robust positive invariant (RPI) sets. The optimized control action is computed either by linear control law or by solving the problem of linear programming (LP), see Boyd and Vandenberghe (2004). The proposed strategy aims to ensure further energy savings by minimizing the necessity to solve the optimization problem.

The main contribution of this project is to design advanced offset-free convex-lifting-based robust control for a laboratory plate heat exchanger. A novel approach of construction of the robust positive invariant set and the

Paper Received: 16/03/2019; Revised: 02/05/2019; Accepted: 03/05/2019
Please cite this article as: Oravec J., Bakosova M., Horvathova M., Galcikova L., Slavik M., Vasickaninova A., Meszaros A., 2019, Convex-lifting-based Robust Control of a Laboratory Plate Heat Exchanger, Chemical Engineering Transactions, 76, 733-738 DOI:10.3303/CET1976123

associated convex lifting in the set that is complementary subject to the feasibility set of the system initial conditions is proposed. The designed optimization problem of linear programming can be effectively solved. Moreover, the necessity to solve the optimization problem in real-time is minimized by the advanced construction of the robust positive invariant set. A case study investigates the control performance and the energy savings of the proposed strategy in various control conditions subject to both, the reference tracking problem and the disturbance rejection problem.

2. Controlled plate heat exchanger

2.1 Laboratory plate heat exchanger

A heat exchanger is a device used to provide heat transfer between two or more fluids. The aim of control is to ensure the desired temperature of the outlet fluid. The considered device is a plate heat exchanger, where the metal plates are used to transfer heat from one fluid to another. The advantage of this class of heat exchanger is that the fluids spread all around the plates and increase the heat transfer area.
Particularly, the considered controlled device is a laboratory plate heat exchanger manufactured by Armfield, see Armfield (2007). The device is a three-stage indirect liquid-liquid plate heat exchanger. For the controller design purposes, just the heating stage was utilized. The dimensions of the device are as follows: length = 103 mm, width = 90 mm, and height = 160 mm. Further technical details can be found in Armfield (2007). The plate heat exchanger is depicted in Figure 1, where two retention tanks serve to store the input cold fluid (Figure 1, device II), the input hot fluid is preheated in the retention tank (Figure 1, device III), two peristaltic pumps ensure dosing of the input cold fluid (Figure 1, device IV) and dosing input hot fluid (Figure 1, device V). The closed loop control setup considers feedback control, where the considered controlled variable is the output temperature *T* of the plate heat exchanger. The associated manipulated variable, i.e., the control input, was the volumetric flow rate *q* of the hot fluid dosing the heat exchanger. Both fluids are represented by water with different input temperatures. A retention tank for the hot fluid (Figure 1, device III) contains an additional support PID closed-loop control to ensure a constant temperature of input hot fluid. Further technical details are listed in Oravec et al. (2016).

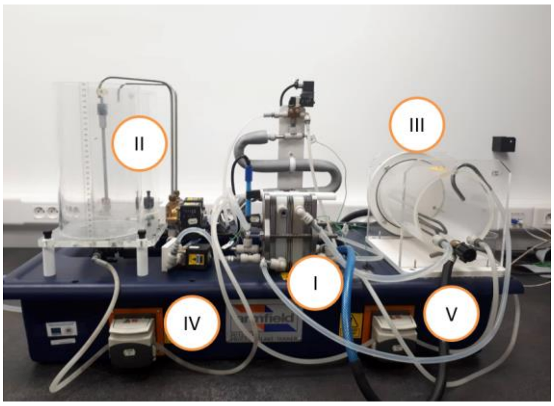

Figure 1: Laboratory plate heat exchanger of Armfield PCT23 (I), two retention tanks storing input cold fluid (II), retention tank for preheating input hot fluid (III), peristaltic pump dosing cold fluid (IV), and peristaltic pump dosing hot fluid (V).

2.2 Mathematical model of the plate heat exchanger

For the robust controller design purposes, it is necessary to evaluate the proper mathematical model. The step-response-based identification of the experimentally collected data was realized to determine the system parameters, mainly the system gain and system time constant. As the heat exchanger has complex non-linear and asymmetric behaviour, the set of increasing and decreasing input step changes were evaluated to generate the associated set of step responses. Then, the model of the heat exchanger was derived considering the form of the state space system in the discrete time domain subject to the interval uncertainties:

$$x(k+1) = A_v x(k) + B_v u(k) + w(k), \qquad y(k) = C_v x(k), \qquad x(0) = x_0, \tag{1}$$

where *k* is the discrete time sample subject to the considered sampling time $t_s = 5$ s, *x*(*k*) is the real-valued vector of system states, *u*(*k*) is the control input, and *y*(*k*) is the system output. The amplitude of the noise measurement, i.e., the additive disturbance, $w(k)$ is limited by w_{max}. The system matrices A_v, B_v, C_v have appropriate dimensions. The considered interval uncertainty of the controlled system in Eq(1) has the form

$$\mathbb{A} = \mathrm{convhull}([A_v, B_v, C_v], \forall v = 1, \ldots, 4) \tag{2}$$

where $\mathbb{A}$ is the convex hull of the system vertexes. The nominal system is represented by the set of matrices A_0, B_0, C_0 given by the mean values of the interval uncertainties. The manipulated variables and the system states need to respect the symmetric constraints in the form:

$$u_{\min} \leq u(k) \leq u_{\max}, \qquad x_{\min} \leq x(k) \leq x_{\max}, \tag{3}$$

where u_{min}, u_{max} and x_{min}, x_{max} are the considered limit values of the manipulated variables and the system states, respectively. Finally, Table 1 summarizes the boundary values of the identified model, i.e., the minimum and maximum values of the system matrices in Eq(2). These parameters were identified using the set of the experimentally measured step-responses. Further technical details related to the identification are explained in Oravec et al. (2016).

Table 1: Minimum and maximum parameters of the uncertain system model in the discrete-time domain.

Vertex matrix	Av	Bv	Cv
Minimum	0.5179	0.2236	1
Maximum	0.7967	3.7119	1

3. Convex-lifting-based robust control

The principles of the considered convex-lifting-based robust control design were introduced in the paper Oravec et al. (2019). The main idea is to preserve the advantages of the advanced optimization-based robust control design subject to the reduced computational effort of real-time control. Therefore, the robust controller design procedure was split into 2 main parts: (i) *offline phase* corresponds to the preparation phase and is evaluated in advance to the real-time control; and (ii) *online phase* is executed during the real-time control. The online phase serves to precompute the convex-lifting-based polytopic partition, to design 2 RPI sets, and to design the associated 2 controllers. The online phase in each control step optimizes the value of the manipulated variable.

In the offline phase, 2 designed RPI sets are: (i) *outer RPI set* that is designed to maximize its volume but has reduced aggressivity of the associated controller K_1; and (ii) *inner RPI set* that has reduced the volume in comparison to outer RPI set but has a more aggressive associated controller K_2. The considered linear control laws had the form:

$$u(k) = K_1 x(k), \qquad u(k) = K_2 x(k), \tag{4}$$

In the online phase, to compute the current value of the manipulated variable in the real-time, 3 scenarios are considered: (i) if system sates are located in the inner RPI set, then aggressive controller K_2 is implemented; (ii) if system states are located in the outer RPI set, then less aggressive controller K_1 is implemented; (iii) otherwise, i.e., if system sates are located in the polytopic partition of the convex lifting, then solve LP to find the optimal value of the manipulated variable.

The control performance was optimized subject to the minimization of the well-known LQR-based quality criterion in the form:

$$J = \sum_{k=0}^{N} \left(x(k)^{\mathrm{T}} Q_{\mathrm{P}} x(k) + \left(\sum_{i=0}^{k} e(i)\right)^{\mathrm{T}} Q_{\mathrm{I}} \left(\sum_{i=0}^{k} e(i)\right) + u(k)^{\mathrm{T}} R u(k) \right), \tag{5}$$

where $e(k) = T(k) - T_{ref}$ is the control error subject to the reference temperature T_{ref}, and $Q_P > 0$ is the weighting matrix of the proportional part, $Q_I > 0$ is the weighting matrix of the integral action, and R is the weighting of the manipulated variables. Then, $Q = \mathrm{diag}([Q_P, Q_I])$.

The technical details can be found in Oravec et al. (2019). To ensure the offset-free control performance, i.e., to remove the steady-state error, the integral action was introduced into the controller design procedure. The original uncertain system in Eq(1) and Eq(2) was modified subject to the extended vector of states. Due to the lack of space, the further technical details on how to design the convex lifting and the integral action were described, e.g., in Oravec et al. (2018).

4. Results and discussion

The numerical simulations of the closed-loop control were evaluated using a CPU i5-825OU 1.80 GHz, 8 GB RAM. The *MATLAB/Simulink* R2018b programming environment (Mathworks, 2019) was provided as the platform to generate the closed-loop system simulations. The construction of the convex lifting and the formulation of the optimization problems were handled by *MPT* (Herceg et al., 2013) and *YALMIP* toolbox (Lofberg, 2004). In the offline phase, the optimization problems of SDP class were solved by the solver *MOSEK* (Mosek, 2019) and multi-parametric LPs were solved by *MPT*. In the online phase, the LPs were solved by *linprog* (Mathworks, 2019).
For the closed-loop control simulation purposes, the controlled device, i.e., the plate heat exchanger in Figure 1, was represented by the state-space system in the discrete time domain in Eq(1) and Eq(2).
The considered constraints on the manipulated variable in Eq(3) were: $0 \leq q(k) \leq 12$ ml s^{-1}, and the constraints on the controlled variable were: $35 \leq T(k) \leq 55$ °C. The additive disturbance $w(k)$ was limited by $w_{\max} = 0.1$ °C. The pairs of the weighting matrices in Eq(5) were set to: $Q_1 = I \times 10^{-2}$, $R_1 = 1$, and $Q_2 = I \times 10^{-4}$, $R_2 = 1$.
Following the procedure for the construction of the convex-lifting-based robust control design for the plate heat exchanger, the polytopic partition depicted in Figure 2 was generated. The constructed convex lifting has 3 main parts: (i) lifted polytopic partition, (ii) outer RPI set with assign controller K_1, and (iii) the inner RPI set with assigned controller K_2. In Figure 2, the system state x_1 corresponds to a normalized controlled output and the system state x_2 is associated with the integral action. Finally, the feasible set of the system initial condition was lifted subject to the piece-wise affine function $l(x)$ denoting the lifted value assigned to the given system states. The RPI sets correspond to $l(x) = 0$.
Properties of the designed convex lifting are summarized in Table 2, where *volume* denotes the total volume of the constructed RPI set, and Proportional Gain and Integration Gain are the designed parameters of the associated controllers K_1, K_2, respectively.
The simulation results of the closed-loop control were investigated subject to (i) the disturbance rejection problem and (ii) the reference tracking problem.
The reference tracking problem was analysed subject to the increasing step changes of the reference value. Therefore, the reference temperature changed its value from the steady-state represented by the value $T_{\text{ref},1}$ = 45 °C to $T_{\text{ref},2}$ = 50 °C in time t_{step} = 500 s. The sampling time of the discrete time domain was t_s = 5 s. The disturbance rejection problem was analysed subject to the output step-change of the disturbance T_{dist} = +5 °C.

Table 2: Construction of convex-lifting-based robust control.

RPI set	volume of RPI set	Proportional Gain	Integration Gain
outer	2 443	0.262	0.015
inner	2 148	0.160	0.008

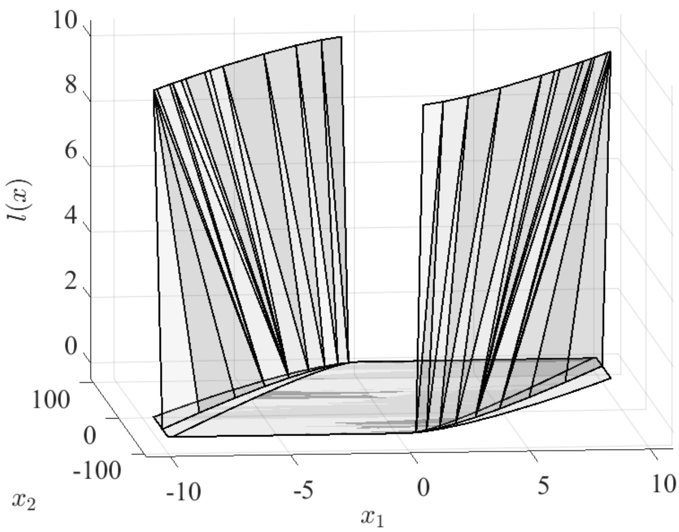

Figure 2: Constructed convex-lifting-based polytopic partition for robust control of heat exchanger.

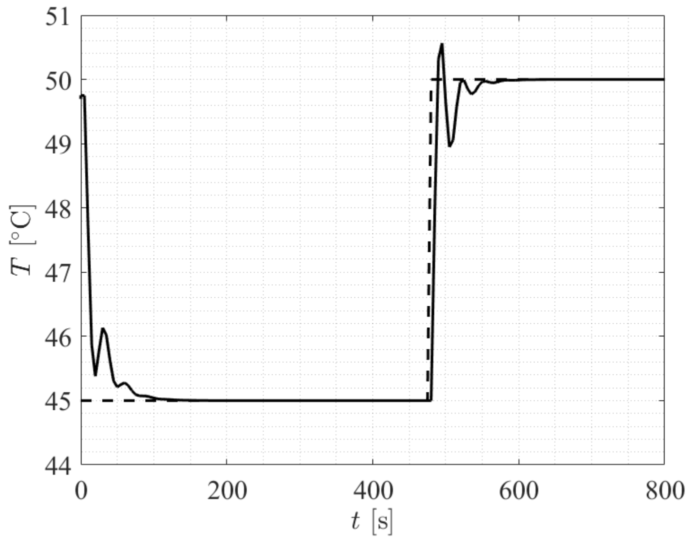

Figure 3: Closed-loop trajectories ensured by the convex-lifting-based robust control: controlled variable – temperature (solid) and reference (dashed).

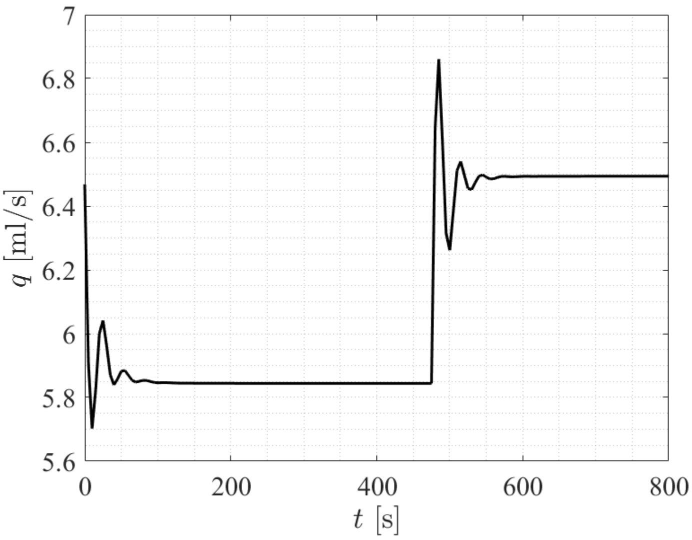

Figure 4: Control trajectories generated by the convex-lifting-based robust control: manipulated variable – volumetric flow rate of the hot fluid.

Figure 3 shows the closed-loop control trajectory of the controlled variable, i.e., the output temperature of the heat exchanger. In Figure 3, the controlled temperature is plotted by the solid curve and the reference is denoted by the dashed line.

As can be seen, the designed convex-lifting-based robust control with integral action was able to ensure the offset-free control trajectory. Particularly, the disturbance rejection problem was addressed first. The output temperature was affected by the output disturbance +5 °C that was eliminated by the designed controller. Then, the reference tracking problem was investigated considering increasing step change of the reference within +5 °C, see Figure 3, t > 500 s. Although the steady state error was removed, the overshoot of the controlled variable was observed.

The associated trajectories of the manipulated variables are shown in Figure4, where the flow rate of the hot fluid dosing the heat exchanger is depicted by a solid curve. As can be seen, the constraints on manipulated variable were satisfied. The generated control trajectories (Figure 3 and 4) may be further tuned to remove the

overshoot. Nevertheless, the primal goal was to investigate the possibilities to implement the proposed robust control strategy for the considered closed-loop control setup.
The total energy consumption was evaluated. The total consumption of hot fluid was measured. Then, the total energy necessary to preheat the hot fluid was computed:

$$E = \int_0^{\infty} \left(q \, \rho \, c_\mathrm{p} \, \Delta T \right) \mathrm{d}t = \int_0^{\infty} \left(\dot{m} \, c_\mathrm{p} \, \Delta T \right) \mathrm{d}t = m_\mathrm{total} \, c_\mathrm{p} \, \Delta T = \left(V_\mathrm{total} \, \rho \right) c_\mathrm{p} \, \Delta T = 1\,686 \times 10^3 \ \mathrm{kJ}, \quad (6)$$

where q is the volumetric flow rate of hot fluid, ρ is the density of hot fluid, $\dot{m}$ is the mass flow rate of hot fluid, c_p = 4.219 kJ K^{-1} kg^{-1} is thermal heat capacity of hot fluid, ΔT = 50 °C is the temperature difference, and m_total = 7.99 kg and V_total = 8 × 10^{-3} m^3 are the total mass and volume of the hot fluid, respectively.

5. Conclusions

The convex-lifting-based robust control of the plate heat exchanger was analysed using the simulation case study. The mathematical model was determined based on the set of experimentally measured step-responses. The control performance was investigated subject to both, i.e., the reference tracking problem and the disturbance rejection problem. The steady state error was removed by introducing the integral action. The generated control trajectories confirmed the possibility to successfully implement the proposed robust control strategy. The future research is focused on the further necessary tuning of the convex-lifting-based robust control design to minimize the overshoot in the control trajectories. Then, the implementation and the closed-loop control performance of the real laboratory plate heat exchanger is going to be investigated

Acknowledgements

The authors gratefully acknowledge the contribution of the Scientific Grant Agency of the Slovak Republic under the grants 1/0403/15, 1/0585/19, and the Slovak Research and Development Agency under the project APVV-15-0007.

References

Armfield, 2007, PCT 23: Process Plant Trainer, manual, Armfield Limited, Ringwood, UK.

Boyd, S., Vandenberghe, L., 2004, Convex Optimization, Cambridge University Press, UK.

Herceg, M., Kvasnica, M., Jones, C., Morari, M., 2013, Multi-Parametric Toolbox 3.0. European Control Conference, Zurich, Swtizerland, 502–510.

Klemeš, J.J., Varbanov, P., 2018, Heat transfer improvement, energy saving, management and pollution reduction, Energy, 162,267-271.

Leitold, D., Vathy-Fogarassy, A., Abonyi, J., 2018, Design-Oriented Structural Controllability and Observability Analysis of Heat Exchanger Networks, Chemical Engineering Transactions, 70, 595-600.

Löfberg, J., 2004, Yalmip: A Toolbox for Modelling and Optimization in MATLAB. Proc. of the CACSD Conference, Taipei, Taiwan, 284 – 289.

Mathworks, Inc., 2019, MATLAB R2018b <www.mathworks.com> accessed 15.03.2019.

Markowski, M., Trzcinski, P., 2018, On-line control of the heat exchanger network under industrial constraints, Chemical Engineering Transactions, 70, 517-522.

Mosek ApS, 2019, MOSEK v8.1 <www.mosek.com> accessed 15.03.2019.

Nguyen, N.A., Olaru, S., Rodríguez-Ayerbe, P., Kvasnica, M., 2017, Convex liftings based robust control design, Automatica, 77, 206-213.

Oravec J., Bakošová M., Mészáros A., 2018, Robust Model Predictive Control of a Plate Heat Exchanger. Chemical Engineering Transactions, 70, 25–30.

Oravec J., Bakošová M., Mészáros A., Míková N., 2016, Experimental Investigation of Alternative Robust Model Predictive Control of a Heat Exchanger, Applied Thermal Engineering, 105, 774 – 782.

Oravec, J., M. Bakošová, M., Trafczynski, M., Vasičkaninová, A., Mészáros, A., Markowski, M., 2018, Robust model predictive control and PID control of shell-and-tube heat exchangers. Energy, 159, 1–10, 2018.

Oravec, J., Holaza, J., Horváthová, M., Nguyen, N.A., Kvasnica, M. Bakošová, M., 2019, Convex-lifting-based robust control design using the tunable robust invariant sets. European Journal of Control, 2019.

Trafczynski, M., Markowski, M., Urbaniec, K., Grabarczyk, R., 2018, Energy saving potential and the eficacy of using different control strategies for the heat exchanger network operation, Chemical Engineering Transactions, 70, 823-828.