One-dimensional Aerothermodynamics Modelling for Gas Turbine Design Considering Effect of Thermal Barrier Coating

Yuanzhe Zhang, Pei Liu*, Zheng Li

State Key Lab of Power Systems, Department of Energy and Power Engineering, Tsinghua University, Beijing 100084, China
liu_pei@tsinghua.edu.cn

Gas turbines are important equipment in power grid peak shaving and distributed energy systems, and performance of a gas turbine depends largely on its turbine inlet temperature. Cooling has become one of the most remarkable characteristics of modern gas turbines due to the temperature limitation of materials. Large quantity of cooling air makes the actual cycle of a gas turbine seriously deviating from an ideal Brayton Cycle. Furthermore, use of thermal barrier coating (TBC) adds new characteristic to the heat transfer process, making impact of air cooling on gas turbine performances more complex. However, it still remains a challenge to quantify this impact via a first-principle gas turbine model. In this paper, considering the impact of TBC on the basis of aerothermodynamics calculation, a mathematical model for gas turbine design is proposed. Comparing with performance of a design model without considering TBC impact, results indicate that the proposed modelling approach can improve calculation accuracy of internal energy conversion of a gas turbine.

1. Introduction

Advanced gas turbines play an important role in the energy conservation and emission reduction (Wang D. H., 2006). The ability of independent research and design of gas turbine is the key to develop advanced gas turbines. The turbine design is one of the most important work for gas turbine design. Okajima Y. and Kyle J. (2019) propose a unique design of radial turbine blades for a portable micro gas turbine engine with double curvatures, which demonstrates the considerable advantage in terms of efficiency and power output. There are many aspects involved in the design of multistage turbines, such as aerothermodynamics calculation, strength and vibration checking, structural and technological considerations, etc. (Shu S. Z. et al., 1991). The accuracy of preliminary turbine design has a big impact on the whole design process. Large quantity of cooling air makes the actual cycle of a gas turbine seriously deviating from an ideal Brayton Cycle, which has a significant impact on gas turbine performance. Mithilesh K. S. and Sanjay (2017) report comparative analysis of basic and complex cooled gas turbine cycle and show intercooled recuperated GT cycle offers higher efficiencies over basic GT cycle. Christina S. et al. (2018) show that both the thermal efficiency and the specific fuel consumption of recuperative gas turbines cycles are affected when turbine blade cooling is taken into account. The cooling air will impact the design of turbine as well. Furthermore, the use of thermal barrier coating (TBC) adds new characteristic to the heat transfer process. Okajima Y. et al. (2014) discuss the TBC development and verification utilizing the MHI’s actual power plant. Sahith M. S. et al. (2018) review the analysis of thermal barrier coating done according to various criteria and conditions. The contribution of this paper is combining the aerothermodynamics calculation and the turbine blade cooling with TBC, and proposing a mathematical model for turbine design considering cooling with TBC on the basis of aerothermodynamics calculation. The model is validated to be more accurate than the turbine design model without considering turbine blade cooling.
2. Model development

2.1 Aerothermodynamics of turbine

In general, the aerothermodynamics calculation of axial-flow multistage turbines design contains 5 parts, as shown in Figure 1 (Shu S. Z. et al., 1991). The following items are included, inlet temperature, inlet pressure, mass flow, outlet pressure, revolving speed and some thermal properties of the working medium. With these conditions, analysis and calculation can be carried out.

Figure 1: Calculation process of turbine aerothermodynamics

(1) In parameter estimation, the number of element stage, the total enthalpy drop, the outlet temperature, the average diameter and blade length of the last stage, etc. will be obtained. In this paper, take a 4-stages turbine as an example.
(2) In the second part, the meridian plane air flow channel pattern and flow pattern need to be chosen by designer. In this paper, uniform internal diameter scheme and constant circulation flow pattern are used. This step is to calculate the velocity diagram and the actual enthalpy drop at each radius. Several radii are selected between the rotor radius and the radius of the last stage blade tip.
(3) Thermodynamic calculation is the main part of this process, which need to be calculated stage by stage. This step is to calculate all the thermal parameters at all the radii selected in flow pattern calculation on the characteristic section 1-1 and 2-2 in all the stages. As shown in Figure 2, the parameters of 2-2 cross-section will be taken as the parameters of 0-0 cross-section of the next stage. The inlet parameters of the first stage are given, then the outlet parameters of the last stage can be obtained. In the calculation at different radii, the mainstream mass flow between two adjacent radii will be calculated. The conservation of mass is used to determine the length of each stage blade.
(4) After that, the geometry parameters can be calculated and the verifying calculation can be carried out.

Figure 2: Schematic diagram of an element stage

2.2 The heat transfer model considering thermal barrier coating

Thermal barrier coating is ceramic coating with low thermal conductivity. They are deposited on the surface of high temperature resistant metals or super-alloys, and can improve the working conditions of the substrate parts at high temperature (Zhong Y. H., 2015). They can withstand chemical or physical decomposition or corrosion damage at high temperature, and can withstand the corrosion of molten metals. The use of thermal barrier coating is one of the key factors to improve the intake temperature of gas turbine. Figure 3a shows the structure of a typical TBC. The effect of TBC on the blade with the introduction of cooling air is shown in Figure 3b.

Figure 3: Schematic diagram of the TBC on blade
The high-temperature gas in a high temperature passes through the static blade, impacts the vane on the rotor, and converts the internal energy of the gas into mechanical energy of rotor. In the heat transfer process, blades absorb energy from high temperature flue gas, mainly by means of radiation heat transfer and forced convection heat transfer; while the energy of blades is transferred to cylinders in the form of radiation, and the blades is cooled by forced convection of cooling air. Finally, a small amount of heat is transferred to axles or cylinders by conduction and to flue gas in the form of radiation.

To calculate the heat transfer with TBC, the following assumptions are made in this model:

- The heat absorption and release of gas turbine blades is a balanced process under normal working conditions, which can be regarded as a steady-state process.
- Because the blade is hollow and the cooling air flows through the blade, the curvature of the blade is not very large, and the thickness of the blade is very small relative to the length and width of the blade, so it can be regarded as one-dimensional heat transfer (Zhu J. et al., 2003).
- In the convective heat transfer between blade outer surface and high-temperature gas flow, as well as between blade inner surface and cooling air flow, the boundary condition is constant blade surface temperature.
- The ratio of inner and outer surface area of blade can be adjusted in the turbine design, which is simplified in this paper because this study is mainly focused on the effect of TBC. The inner and outer surface areas of blades are considered equal.
- The blade can be regarded as grey body.

Newton cooling formula, Fourier’s law and Stephen Boltzmann’s Law are used to describe the convection heat transfer, heat conduction and radiation heat transfer respectively (Zhu H. R. et al., 2017).

The equations of the endothermic process of turbine blade are described as follows:

\[
q_1 = \alpha_1 \Delta T_{\text{int}} = \alpha_1 \frac{(T_{w1} - T_{g2}) - (T_{w1} - T_{g1})}{\ln \frac{T_{w1} - T_{g2}}{T_{w1} - T_{g1}}}
\]

(1)

\[
q_2 = \alpha_2 \Delta T_{\text{meq}} = \alpha_2 \frac{(T_{w2} - T_{c2}) - (T_{w2} - T_{c1})}{\ln \frac{T_{w2} - T_{c2}}{T_{w2} - T_{c1}}}
\]

(2)

where \(q_1\) is the convection heat transfer flux between the high-temperature gas flow and blade outer surface, \(\alpha_1\) is the convective heat transfer coefficient that ranges from 100 to 150 W/(m² · K) according to the design experience, \(\Delta T_{\text{int}}\) is the log mean temperature difference (LMTD) between the high-temperature gas flow and blade outer surface, \(T_{w1}\) is the temperature of blade outer surface, \(T_{g1}\) and \(T_{g2}\) are the temperature of high-temperature gas at the element stage inlet and outlet respectively, \(q_2\) is the radiative heat transfer flux between the high-temperature gas flow and blade outer surface, \(\epsilon_g\) and \(\epsilon_1\) are the blackness of the high-temperature gas and the blade respectively, \(\alpha_1\) is the absorptivity of the blade, whose value is the same as the blackness of the blade at the same temperature.

The equations of the heat dissipation process of turbine blade are described as follows:

\[
q_3 = \alpha_3 \Delta T_{\text{meq}} = \alpha_3 \frac{(T_{w2} - T_{c2}) - (T_{w2} - T_{c1})}{\ln \frac{T_{w2} - T_{c2}}{T_{w2} - T_{c1}}}
\]

(3)

\[
q_4 = 5.67 \epsilon_c \left[ \left( \frac{T_{c1}}{100} \right)^4 - \left( \frac{T_{c2}}{100} \right)^4 \right]
\]

(4)

where \(q_3\) is the convection heat transfer flux between the cooling air flow and blade inner surface, \(\alpha_3\) is the convective heat transfer coefficient that ranges from 50 to 100 W/(m² · K) according to the design experience, \(\Delta T_{\text{meq}}\) is the log mean temperature difference (LMTD) between the cooling air flow and blade inner surface, \(T_{w2}\) is the temperature of blade inner surface, \(T_{c1}\) and \(T_{c2}\) are the temperature of cooling air at the cooling channel inlet and outlet respectively, \(q_4\) is the radiative heat transfer flux between the turbine cylinder and blade outer surface, \(T_{c}\) is the temperature of the turbine cylinder.

The equations of the heat conduction between inner and outer surface of blade are described as follows:

\[
q_5 = \lambda_b \frac{T_{w1} - T_b}{\delta_b} = \lambda_b \frac{T_b - T_{w2}}{\delta_b}
\]

(5)
where \( q_5 \) is the heat conductive flux, \( \lambda \) is the heat conductivity coefficient, \( \delta \) is the thickness, the subscripts \( c \) and \( s \) denote TBC and substrate respectively, \( T_b \) is the temperature of the coating-substrate interface, where the temperature is highest in the substrate.

After get all the heat transfer flux, the heat balance can be described as follows:

\[
q_1 + q_2 = q_3 + q_4
\]

\[
q_5 = q_6
\]

There are three independent equations in total. The heat transfer calculation can be finished with the above formulas and some known conditions.

### 2.3 Aerothermodynamics of turbine considering cooling with TBC

TBC adds new characteristic to the heat transfer process, making impact of air cooling on gas turbine performances more complex and turbine design more different.

As shown in Figure 3c, there’s cooling air and TBC in the stage. With the existence of cooling air, the parameter of the upper stage outlet cannot be taken as the parameters of the input of next stage any more. The cooling air will mix with the mainstream at the upper stage outlet. Therefore, the mass flow will be lager and temperature will be lower than the original. In this section, the heat transfer model considering TBC is added to aerothermodynamics calculations of turbine, a mathematical model for turbine design is proposed. The equations are described as follows:

\[
m_{\text{out}} = m_{\text{n}} + m_c
\]

\[
m_{\text{out}} \cdot h(T_{g2}) = m_{\text{n}} \cdot h(T_{g1}) + m_c \cdot h(T_{c2})
\]

Where \( m_{\text{out}} \) is the outlet mass flow, \( m_{\text{n}} \) is the inlet mass flow, \( m_c \) is the cooling air mass flow, \( h(T) \) is the enthalpy at the temperature \( T \), \( T_{g2} \) is the outlet temperature when there’s no cooling air. In the model without turbine cooling, \( m_c = 0 \). Figure 4 shows the new calculation process.

![Figure 4: Calculation process of turbine aerothermodynamics considering cooling with TBC](image)

There are three independent equations in the heat transfer model mentioned in Section 2.2. Taking \( T_{c1}, T_{g1}, T_{g2}, T_b, T_w, \delta_c \) and \( \delta_s \) as the inputs of the heat transfer model, \( T_{w1}, T_{w2} \) and \( T_{c2} \) can be calculated. The cooling air is bled from compressor, \( T_{c1} \) can be obtained by compressor. \( T_{g1} \) and \( T_{g2} \) can be obtained from the original thermodynamic calculation. \( T_b, T_w, \delta_c \) and \( \delta_s \) can be obtained by design experience. After \( T_{w1}, T_{w2} \) and \( T_{c2} \) are obtained, the cooling air mass flow can be calculated.

\[
m_c = \frac{a_p P L}{c_p \ln \left( \frac{T_{w2} - T_{c1}}{T_{w2} - T_{c2}} \right)}
\]

Where \( m_c \) is the cooling air mass flow, \( P \) is perimeter of the cooling channel cross section, \( L \) is blade length.

### 3. Model comparison

Effect of the above mathematical model needs to be tested by comparing with the original model. In this work, a series of parameters for turbine design are selected for calculation for the two models. The information of the turbine design requirement is listed in Table 1. In the proposed mathematical model, some parameters of heat transfer are needed. The information of the heat transfer parameters is listed in Table 2. Table 3 lists some parameters in each stage thermodynamic calculation. And there’s no cooling in the last stage.
Table 1: Main parameters for turbine design

<table>
<thead>
<tr>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet temperature</td>
<td>°C</td>
</tr>
<tr>
<td>Inlet pressure</td>
<td>atm</td>
</tr>
<tr>
<td>Mass flow</td>
<td>kg/s</td>
</tr>
<tr>
<td>Outlet pressure</td>
<td>atm</td>
</tr>
<tr>
<td>Revolving speed</td>
<td>r/min</td>
</tr>
<tr>
<td>Series</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2: Main parameters of heat transfer

<table>
<thead>
<tr>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝛼₁</td>
<td>W/(m²·K)</td>
</tr>
<tr>
<td>𝛼₂</td>
<td>W/(m²·K)</td>
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<td>𝛼₄</td>
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</tbody>
</table>

Table 3: Main parameters in each stage thermodynamic calculation

<table>
<thead>
<tr>
<th>Unit</th>
<th>1st stage</th>
<th>2nd stage</th>
<th>3rd stage</th>
<th>4th stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tᵇ</td>
<td>°C</td>
<td>827</td>
<td>727</td>
<td>667</td>
</tr>
<tr>
<td>Tʷ</td>
<td>°C</td>
<td>600</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>Tᶜ₁</td>
<td>°C</td>
<td>377</td>
<td>277</td>
<td>227</td>
</tr>
</tbody>
</table>

In a multistage turbine, the loss of the upper stage will lead to the increase of the temperature of the next stage, so that the sum of the ideal enthalpy drop of each stage is larger than the ideal enthalpy drop of the whole turbine. Reheat factor α is used to describe this phenomenon. Turbine efficiency η describe the loss. Turbine outlet pressure and turbine outlet temperature are the outlet pressure and outlet temperature of the last stage. θ can be obtained in the geometry calculation.

\[
\alpha = \frac{\sum_{i=1}^{4} \Delta h_{ii}}{\Delta H_{s}} - 1
\]

\[
\eta = \left( \frac{\sum_{i=1}^{4} \Delta h_{i}}{\sum_{i=1}^{4} \Delta h_{ii}} \right)^{\frac{1}{2}}
\]

Where \( \Delta h_{1}, \Delta h_{2}, \Delta h_{3}, \Delta h_{4} \) and \( \Delta H_{s} \) are the ideal enthalpy drop of the 1st, 2nd, 3rd, 4th stage and turbine respectively, \( \Delta h_{1}, \Delta h_{2}, \Delta h_{3}, \Delta h_{4} \) and \( \Delta H_{s} \) are the actual enthalpy drop of the 1st, 2nd, 3rd, 4th stage respectively.

The results are listed in Table 4. The relative error is the error between the calculated value and the given value. They're large because these two models are used in the preliminary design of turbine. In this paper, the main purpose is to compare the two models. The reheat factor error, the turbine efficiency error of the proposed model are nearly equal to the original model. This indicate that the turbine cooling doesn’t affect the accuracy of reheat factor and turbine efficiency in the preliminary design. For turbine outlet pressure, the last stage blade diameter length ratio, the relative errors decrease from greater than 10% to less than 1%. The relative error of turbine outlet temperature also decreased to less than 1%. The proposed model in this paper is more precise than the original model, which can improve the accuracy of preliminary design and shorten design period.

As shown in Figure 5, the blade length calculated by the proposed model is larger than the original model except the first stage. The reason is that the cooling air is considered in the proposed model and the mainstream mass flow become larger. As a result, the flow area needs to be larger and the blade become longer.
Table 4: Relative error of results

<table>
<thead>
<tr>
<th></th>
<th>Original model</th>
<th>Proposed model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reheat factor $\alpha$</td>
<td>69.2%</td>
<td>69.1%</td>
</tr>
<tr>
<td>Turbine efficiency $\eta$</td>
<td>12.1%</td>
<td>12.1%</td>
</tr>
<tr>
<td>Turbine outlet pressure $p_4$</td>
<td>11.7%</td>
<td>0.9%</td>
</tr>
<tr>
<td>Turbine outlet temperature $T_4$</td>
<td>1.5%</td>
<td>0.4%</td>
</tr>
<tr>
<td>Blade diameter length ratio of the last stage $\theta$</td>
<td>11.3%</td>
<td>0.8%</td>
</tr>
</tbody>
</table>

Figure 5: Geometry calculated by the two models

4. Conclusions

This paper developed a model for turbine design considering cooling with TBC based on the physical mechanism method, which is closer to the actual gas turbine. The heat transfer processes in turbine mainly consist of convection heat transfer between the high-temperature gas flow and blade outer surface, radiation between the high-temperature gas flow and blade outer surface, convection heat transfer between the cooling air flow and blade inner surface, radiation between the turbine cylinder and blade outer surface, heat conductive inside the blade. By the comparison to the original model, the model considering cooling with TBC is validated to be more accurate for turbine design. This work provides a foundation for further turbine design model development with an agreeable cooling performance.

Acknowledgments

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References