

# Stochastic Supply Chain Optimization with Risk Management

Narut Suchartsunthorn, Kitipat Siemanond\*

The Petroleum and Petrochemical College, Chulalongkorn University, Bangkok, Thailand  
 kitipat.s@chula.ac.th

This paper presents the hypothetical case study of supply-chain management of the carbon dioxide-utilization products where its feedstock is sewage gas containing mostly methane and CO<sub>2</sub>. This supply chain begins with three suppliers sending sewage gas of different average CO<sub>2</sub> compositions to two acid gas removal units (AGRUs) for CO<sub>2</sub> removal. CO<sub>2</sub> from AGRUs is transferred to two CO<sub>2</sub>-utilization plants; producing products. The products are transferred to two distribution centers (DCs) from where they are sent to customers at three market places. The uncertainties occur in the compositions of sewage-gas from three suppliers and customer demands from three markets. The stochastic supply chain optimization under uncertainties with maximum expected profit is applied to find the optimal mass flow rates of sewage gas and CO<sub>2</sub> from suppliers to AGRUs and optimal mass flow rate of products between CO<sub>2</sub>-utilization plants, DCs, and markets. The stochastic supply chain model is compared to deterministic one using fifteen random sets of the sewage-gas compositions and customer demands for thirty daily scenarios. For the validation part, the results show that the optimal stochastic supply chain mostly gives a higher profit than deterministic one about eight out of ten sets of random data. To be more practical, the supply-chain optimization using the risk model is developed to design a stochastic supply chain with high chance to achieve profit larger than the targeted profit.

## 1. Introduction

Nowadays, the excessive greenhouse gases in the atmosphere are the serious concern due to the rapid growth of population, industry and agriculture. CO<sub>2</sub> as the main source is 64 % of the enhanced greenhouse effect (Halmann and Steinberg, 1999). AGRU is an interesting technology for removing CO<sub>2</sub> and H<sub>2</sub>S from raw natural gas; like sewage gas. Methyl Diethanolamine (MDEA) is used as a commercial absorbent for CO<sub>2</sub> removal. Captured CO<sub>2</sub> is sent to the CO<sub>2</sub> utilization for value-added aspect. CO<sub>2</sub> can be used as a feed stock for the plastic plant of polypropylene carbonate process and also methanol production as well. In this work, stochastic programming is applied to optimize the CO<sub>2</sub> supply chain. Stochastic programming is a modelling optimization problem that involves uncertainties where probability distributions governing the data are known (Shapiro and Philpott, 2007). There are a large number of articles that address supply chain modelling under uncertainties by stochastic programming against deterministic programming. Among them, Marufuzzaman et.al. (2014) focus on modelling the supply chain of biodiesel produced by the wastewater treatment unit. The model applied two-stage stochastic programming with uncertainty in term of scenarios, where all scenarios are based on historical data and predictable incident. Some articles account for uncertainty by using mean and standard deviation as a continuous uncertainty, like Rodriguez et.al. (2014) who propose an optimization to redesign supply chain of spare-part delivery where strategic and tactical perspectives are concerned for long term and short term decisions under demand uncertainty assuming it as a random continuous parameter with Poisson distribution. Shaw et. al. (2016) design a green supply chain model addressing carbon emissions and carbon trading issues using mean and standard deviation represent uncertainty. Li et. al. (2018) study supply chain network design using a sample average approximation (SAA) with a scenario decomposition algorithm to speed up the algorithm. In our work of modelling sewage-gas and CO<sub>2</sub> supply chain, there are two uncertain factors, which are compositions of sewage gas and product demands. Supply chain model is done by a mixed-integer linear programming (MILP) and its objective function is profit maximization. The results from the GAMS program are compared between the stochastic model and the deterministic model in the profit aspect. For the simulation of AGRU, Aspen HYSYS is used to predict the performance of the process.

## 2. Problem Statement

Supply chain model consists of five nodes; sewage gas suppliers (i), AGRU (j), CO<sub>2</sub>-utilization plants (k), DCs (l), and customers or markets (m) as shown in Figure. 1. Suppliers supply sewage gas containing methane, CO<sub>2</sub> and contaminants to AGRUs for removing CO<sub>2</sub>. After that, CO<sub>2</sub> from AGRUs is transferred to CO<sub>2</sub>-utilization plants for converting CO<sub>2</sub> to CO<sub>2</sub>-utilization products. Products are sent to and stored at DCs, where they are transferred to markets each day. There are uncertainties in sewage gas compositions at suppliers and product demands at markets; as shown in Table 1. If DCs do not store enough number of products to satisfy demands, the penalty cost will occur.

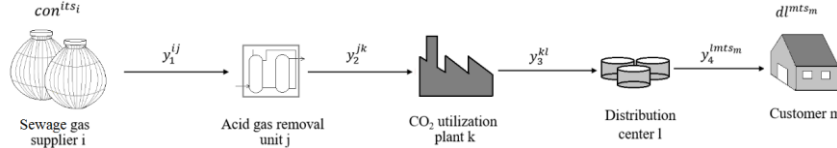


Figure 1: Supply chain diagram

Table 1: Average and standard deviation (SD) of uncertainties in suppliers and market demands

Node (i)	Average % CO <sub>2</sub> in sewage gas	SD(i)	Node (m)	Average products demand	SD(m)
Supplier (i1)	10 % wt	3	Demand (m1)	2,000 kg/d	200
Supplier (i2)	15 % wt	3	Demand (m2)	1,500 kg/d	150
Supplier (i3)	20 % wt	3	Demand (m3)	1,000 kg/d	100

## 3. Mathematical Model

Mixed-integer linear programming (MILP) model for designing supply chain is proposed to solve the optimal amount of mass flow between nodes; sewage-gas suppliers, AGRUs, CO<sub>2</sub> -utilization plants, DCs and customers. In this work, the mass flow balances between nodes of the supply chain are shown in Figure 2, and AGRU is simulated using commercial simulation software; ASPEN HYSYS.

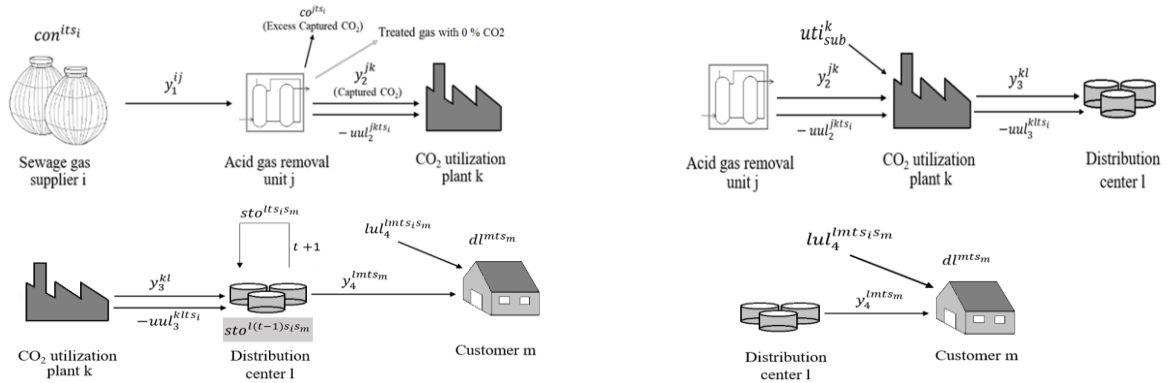


Figure 2: Mass flow balances of supply chain model

### 3.1 Stochastic model

The objective of this model is to design the supply chain under uncertainties with the maximum expected profit by solving for the optimal amount of mass flow. The uncertainties are from CO<sub>2</sub> composition in sewage-gas at suppliers and customer demands of CO<sub>2</sub>-utilization products. The following equations; Eq. (1), is the objective function to maximize expected profit (\$/30 d) for designing the optimal supply chain.

Maximize Expected profit

$$\begin{aligned}
 &= \sum_{s_i \in S_I} \sum_{s_m \in S_M} \frac{1}{S_I S_M} \sum_{t \in T} \{SOLD^{t s_m} \\
 &\quad - [TC^{t s_i s_m} + COC^{t s_i} + AGRUOC^{t s_i} + UOC^{t s_i} + SUBC^{t s_i} + PC^{t s_i s_m} + STOC^{t s_i s_m}]\}
 \end{aligned} \tag{1}$$

The objective function with maximum expected profit and its optimal amount of mass flow between the nodes of the supply chain are solved by MILP. Expected profit is the summation of profit at each daily scenario multiplied by each daily-scenario probability. For a set of 30 random daily data, each daily-scenario probability is the products between daily probability ( $\frac{1}{S_I}$ ) of ( $\frac{1}{30}$ ) from CO<sub>2</sub> composition in sewage-gas suppliers and daily probability ( $\frac{1}{S_M}$ ) of ( $\frac{1}{30}$ ) from customer demands. Profit at each daily scenario is calculated from daily revenue ( $SOLD^{ts_m}$ ) from selling products minus overall daily costs. Overall daily cost of the supply chain consist of transportation cost ( $TC^{ts_i s_m}$ ) in dollar per day, CO<sub>2</sub> releasing/capturing cost ( $COCT^{ts_i}$ ) in dollar per day, AGRU operating cost ( $AGRUOC^{ts_i}$ ) in dollar per day, CO<sub>2</sub>-utilization plant operating cost ( $UOC^{ts_i}$ ) in dollar per day, reactant-substance cost ( $SUBC^{ts_i}$ ) in dollar per day, penalty cost ( $PC^{ts_i s_m}$ ) in dollar per day and storage cost ( $STOC^{ts_i s_m}$ ) in dollar per day. Eq. (2) is used to calculate daily product revenues.

$$SOLD^{ts_m} = \sum_t \sum_m y_4^{lmt s_m} p_{pp} \quad (2)$$

The overall daily transportation costs ( $TC^{ts_i s_m}$ ) are calculated from transportations of suppliers-to-AGRUs, AGRUs-to-CO<sub>2</sub>-utilization plants, CO<sub>2</sub>-utilization plants-to-DCs and DCs-to-customers, as shown in Eq. (3), respectively.  $uul_2^{j k t s_i}$  and  $uul_3^{k l t s_i}$  represent the amount of mass not satisfying targets of CO<sub>2</sub>-utilization plants and distribution centres. On the other side,  $lul_4^{l m t s_i s_m}$  represents the amount of products unsatisfying target of customer demands. These amounts of products not satisfying targets are not in transportation-cost terms.

$$TC^{ts_i s_m} = \sum_{i \in I} \sum_{j \in J} y_1^{ij} d_1^{ij} t_{pc} + \sum_{j \in J} \sum_{k \in K} (y_2^{jk} - uul_2^{j k t s_i}) d_2^{jk} t_{pc} + \sum_{k \in K} \sum_{l \in L} (y_3^{kl} - uul_3^{k l t s_i}) d_3^{kl} t_{pc} + \sum_t \sum_m (y_4^{l m t s_m} - lul_4^{l m t s_i s_m}) d_4^{lm} t_{pc} \quad (3)$$

### 3.2 Risk model

The objective function of stochastic and deterministic supply-chain models is to maximize expected profit. In the real situation, each design of the supply chain has a low or high risk to achieve profit less than targeted profit. Or it has high or low chance to achieve profit larger than the targeted one. Therefore, this section is to develop a risk or chance model to design an optimal supply chain under maximum chance or cumulative probability to achieve profit larger than the target profit ( $\omega$ ). The objective of the risk model in this work is to maximize the cumulative probability of chance ( $\frac{1}{S_I S_M} x_z^{s_i s_m}$ ) multiplied by profit ( $profit^{s_i s_m}$ ) to achieve profit larger than targeted profit ( $\omega$ ); as shown in Eq. (4).

$$Maximize z = \sum_{s_i \in S_I} \sum_{s_m \in S_M} \frac{1}{S_I S_M} profit^{s_i s_m} x_z^{s_i s_m} \quad (4)$$

Maximum-chance probability model gives supply chain design with optimal mass flow rates between nodes, solved by Eq. (4) – (6) along with equations from the stochastic model. To find probability having profit greater than the targeted profit ( $\omega$ ), the logical constraint; Eq. (5), is introduced.

$$M_z^{up} - M_z^{lo} \leq [profit^{s_i s_m} - \omega] - M_z^{up} x_z^{s_i s_m} \leq 0 \quad \forall s_i \in S_I, s_m \in S_M \quad (5)$$

For the above logical constraint in each scenario  $S_I$  and  $S_M$ , if the profit ( $profit^{s_i s_m}$ ) is greater than or equal to  $\omega$ , the binary indicator;  $x_z^{s_i s_m}$ , will be 1. But if the profit ( $profit^{s_i s_m}$ ) is lower than  $\omega$ , binary indicator;  $x_z^{s_i s_m}$ , will be 0. Daily profit in each daily scenario  $S_I$  and  $S_M$  from Eq. (4) and (5) is calculated by Eq. (6).

$$profit^{s_i s_m} = \sum_{t \in T} \{SOLD^{ts_m} - [TC^{ts_i s_m} + COCT^{ts_i} + AGRUOC^{ts_i} + UOC^{ts_i} + SUBC^{ts_i} + PC^{ts_i s_m} + STOC^{ts_i s_m}]\} \quad (6)$$

## 4. An Illustrative Example

The illustrative example of supply chain; as shown in Figure 3, consists of three sewage-gas suppliers (i1, i2, and i3), two AGRUs (j1 and j2), two CO<sub>2</sub>-utilization plants (k1 and k2), two DCs (l1 and l2) and three customers or markets (m1, m2, and m3) which are operated for 30 days. The supply-chain design from the stochastic model is compared to one from the deterministic model using all average data in the calculation.

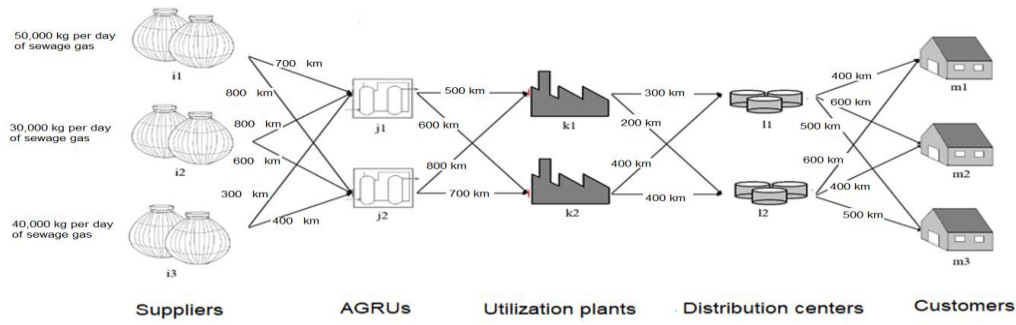


Figure 3: Supply chain diagram

This study applies stochastic and deterministic models to synthesize optimal supply chains with the maximum expected profit. The optimal stochastic and deterministic supply chains are shown in Table 2 and 3. To compare the profits between stochastic and deterministic designs, they were applied with ten sets of random daily data with uncertainties of sewage-gas and customer demands for 30 days, as shown in Table 4. The results show that the optimal stochastic supply chains mostly give higher profits than ones from the deterministic model about eight out of ten sets.

Table 2: Mass flow in stochastic supply chain

Nodes (i, j)	Sewage gas (kg/d)	Nodes (j, k)	CO <sub>2</sub> (kg/d)	Nodes (k,l)	Products (kg/d)
i1-j1	30,000	j1-k1	-	k1-l1	-
i1-j2	20,000	j1-k2	5,388	k1-l2	-
i2-j1	-	j2-k1	-	k2-l1	2,494.722
i2-j2	40,000	j2-k2	2,252.178	k2-l2	1,999.500
i3-j1	30,000				
i3-j2	-				

Table 3: Mass flow in deterministic supply chain

Nodes (i, j)	Sewage gas (kg/d)	Nodes (j, k)	CO <sub>2</sub> (kg/d)	Nodes (k,l)	Products (kg/d)
i1-j1	30,000	j1-k1	-	k1-l1	-
i1-j2	20,000	j1-k2	7,650	k1-l2	-
i2-j1	-	j2-k1	-	k2-l1	3,000
i2-j2	40,000	j2-k2	-	k2-l2	1,500
i3-j1	30,000				
i3-j2	-				

Table 4: The validation of stochastic and deterministic supply-chains under ten sets of uncertainties

Set of random data	Overall profit (\$/30 d) (Stochastic model)	Overall profit (\$/30 d) (Deterministic model)
Experimental set 1	2,859,830	2,841,679
Experimental set 2	2,857,662	2,786,313
Experimental set 3	2,833,551	2,782,292
Experimental set 4	2,805,861	2,754,062
Experimental set 5	2,814,914	2,788,697
Experimental set 6	2,788,172	2,766,162
Experimental set 7	2,809,599	2,816,511
Experimental set 8	2,762,902	2,770,802
Experimental set 9	2,775,988	2,748,948
Experimental set 10	2,812,379	2,790,757

#### 4.1 Stochastic supply-chain optimization with risk management

This study was to develop a risk model for the optimal supply chain with maximum products between profit and cumulative probability of chance to achieve profit larger than targeted profit. There were two targeted profits; \$ 2,790,074 and \$ 2,706,372. The risk model was solved for 2 supply chains. A supply chain with a targeted profit of \$ 2,790,074 was shown in Table 5. And the other one with a targeted profit of \$ 2,706,372 was shown in Table 6. The chance to achieve profit larger than the target profit for each supply chain was shown in Table 7.

Table 5: Mass flow in the stochastic supply chain at a target profit of \$ 2,790,074

Nodes (i, j)	Sewage gas (kg per d)	Nodes (j, k)	CO <sub>2</sub> (kg per d)	Nodes (k,l)	Products (kg per d)
i1-j1	30,000	j1-k1	-	k1-l1	-
i1-j2	20,000	j1-k2	5,388	k1-l2	-
i2-j1	-	j2-k1	-	k2-l1	2,370.527
i2-j2	40,000	j2-k2	2,246.93	k2-l2	2,120.608
i3-j1	30,000				
i3-j2	-				

Table 6: Mass flow in the stochastic supply chain at a target profit of \$ 2,706,372

Nodes (i, j)	Sewage gas (kg per d)	Nodes (j, k)	CO <sub>2</sub> (kg per d)	Nodes (k,l)	Products (kg per d)
i1-j1	30,000	j1-k1	-	k1-l1	-
i1-j2	20,000	j1-k2	5,388	k1-l2	-
i2-j1	-	j2-k1	-	k2-l1	2,661
i2-j2	40,000	j2-k2	2,195.124	k2-l2	1,799.661
i3-j1	30,000				
i3-j2	-				

Table 7: Chances/ risk of a model for optimal supply chains using different targeted profit

Targeted profit ( $\omega$ ), (\$ in 30 d)	Chance of profit higher than the target	Risk of profit less than target	Solution time (CPU s)
2,790,074	0.6	0.4	4,734
2,706,372	0.978	0.022	4,988

A supply chain with lower targeted profit; Figure 4(b), gives a higher chance of 0.978 to have profit larger than target than supply chain with higher targeted profit; Figure 4(a), does. Or supply chain with lower targeted profit; Figure 4(b), gives the lower risk of 0.022 than supply chain with higher targeted profit; Figure 4(a).

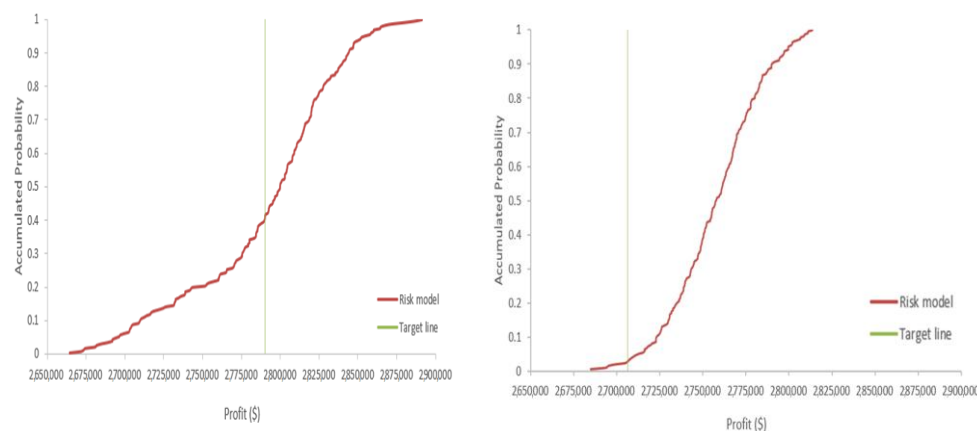


Figure 4: (a) Risk curve with target profit of \$ 2,790,074; (b) Risk curve with target profit of \$ 2,706,372

## 5. Conclusions

The optimal supply chain from the stochastic model is compared to one from the deterministic model, which uses average values of the sewage-gas compositions and the customer demands. For the comparison part, both optimal supply chains are applied to the same uncertainties of ten sets of thirty daily random data of the sewage-gas compositions and the customer demands. The results show that the stochastic supply chain mostly gives a higher profit than deterministic one about eight out of ten sets. For the supply-chain optimization using risk model, it generates two supply chains for different targeted profits of \$ 2,790,074 and \$ 2,706,372 with chances to achieve profit larger than the target profit of 0.6 and 0.978 Risk model can help logistics people make a decision on supply chain design more practically by selecting supply chain with higher chance giving profit larger than the target. The limitation of our model is to design only small scale supply chain; like 3-2-2-3 supply chain. In the future, it may need classical decomposition techniques; Bender's decomposition algorithm (Shaw et al., 2016) or Lagrangian relaxation to design a large scale supply chain; like 25-5-5-5-5 supply chain for saving computational time.

## Acknowledgements

Authors would like to express our gratitude to Ratchadapisek Sompoch Endowment Fund (2016), Chulalongkorn University (CU-59-003-IC) for funding support. And we also thank PTT public company limited for supporting ASPEN HYSYS software.

## References

- Li X., Zhang K., 2018, A sample average approximation approach for supply chain network design with facility disruptions, *Computers and Industrial Engineering* 126, 243-251.
- Marufuzzaman M., Eksioğlu S. D., Huang Y. E., 2014, Two-stage stochastic programming supply chain model for biodiesel production via wastewater treatment, *Computers and Operations Research* 49 (2014), 1-17.
- Nunes P., Oliveira F., Hamacher S., Almansoori A., 2015, Design of a hydrogen supply chain with uncertainty, *International Journal of Hydrogen Energy* 40, 16408-16418.
- Rodríguez M. A., Vecchiotti A. R., Harjunkoski I., Grossmann I. E., 2014, Optimal supply chain design and management over a multi-period horizon under demand uncertainty. Part I: MINLP and MILP models, *Computers and Chemical Engineering* 62, 194-210.
- Shapiro A., Philpott A., "A Tutorial on Stochastic Programming" School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, Georgia, USA (2007).
- Shaw K., Irfan M., Shankar R., Yadav S. S., 2016, Low carbon chance constrained supply chain network design problem: A Benders decomposition based approach, *Computers and Industrial Engineering* 98, 438-497.