

# Application of Flexibility Index Approach for Sustainable Operation of Heat-integrated Autothermal Thermophilic Aerobic Digestion System

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The present study proposes a decision-making approach based on flexibility index calculation of already obtained solutions of a stochastic multi-scenario optimization model for redesign of heat-integrated two-stage Autothermal Thermophilic Aerobic Digestion (ATAD) bioreactors system for municipal wastewater treatment operating under uncertainties. Obtained solutions with minimal costs for redesign were presented in Vladova et al. (2018). Aiming to find the solution which provides sustainable operation of the heat-integrated ATAD system within the widest range of variation of its uncertain parameters, the Flexibility Index (FI) approach should be applied. A fast approach for evaluation of the flexibility index by presenting it as an inscribed a hyper-rectangle in the scaled hyper-space of variation of the uncertain parameters so as to ensure no violation of the process feasibility constraints is proposed. An optimization problem for determining the maximal flexibility index is formulated and solved for each solution obtained in Vladova et al. (2018). Decision making is carried out to choose most sustainable solutions for redesigned ATAD system. Analyzing the results obtained, it can be concluded that not always the solutions with the minimal annual costs for redesigning provide the sustainable operation of the ATAD system.

## 1. Introduction

The flexibility is related with insuring feasible steady-state operation over a variety of operating uncertainties of production systems. Swaney and Grossmann (1985) firstly introduced a general framework for flexibility analysis in the design process of chemical production systems. They have offered a quantitative measure called the flexibility Index, which measures the dimension of stochastic space on which a steady state of operation of the production systems can be achieved at fixed values -  $\bar{D}$  of the design variables. The main assumption in the proposed approach is that the uncertain parameters are independent each other. The index defines the boundaries of the uncertainty parameters in which process feasibility is provided and allows identification of the "worst case scenarios" of conditions that limit the flexibility of the process. This concept is the basis of a large number of studies in this area. Straub and Grossmann (1990) have expanded this approach and introduced the so-called stochastic flexibility expressed in terms of the probability of the process being carried out.

Ierapetritou (2001) has introduced an alternative to the Flexibility Measure called Feasible Convex Hull Ratio (FCHR). It represents the ratio of the "volume" of the region determined by the achievable convex envelope to the "volume" of the full space of variance of the stochastic variables. This measure is characterized by great computational difficulty and requires the use of special computational programs in the area of spatial geometry. Similar to the above approach, but less labor intensive is the approach proposed by Lai and Hui (2007) who offered a new process flexibility metric. According to that flexibility is reckoned as the size of the feasible space in which the uncertain parameters can be feasibly handled.

Sahinidis (2004) discussed the issues of identification or maximization of flexibility considering the developed system tools for measuring flexibility and analyzing the compromise between price and design process flexibility.

Qin et al. (2017) have proposed a method to quantify the intra-hour flexibility region. Yibin et al. (2018) have proposed a flexibility evaluation index and scenario typicality evaluation index.

The present study proposes a suitable decision-making approach based on Flexibility Index calculation of already obtained solutions of a stochastic problem. It is applied on the already obtained solutions for redesigning the heat-integrated two-stage Autothermal Thermophilic Aerobic Digestion (ATAD) bioreactors system for municipal wastewater treatment operating under uncertainties.

## 2. Flexibility Index determination

The purpose of the method is: in the scaled hyper-space of the uncertainty parameters without violating the implementation constraints to inscribe a hyper-rectangle whose volume, related to the total volume of the stochastic space to have a greatest value.

The first step of the proposed method of evaluating the Flexibility Index (FI) is the centering of the stochastic space at some Basic Point, where by  $N$  the number of stochastic parameters is denoted. In the stochastic space, each stochastic parameter  $\theta_n$  is changed within the boundaries  $\theta_n^{LB} \leq \theta_n \leq \theta_n^{UB}, \forall n, n \in N$ . Let with  $BP (\theta_1^{BP}, \dots, \theta_n^{BP}, \dots, \theta_N^{BP})$  the Basic Point is denoted in the stochastic space, for which it is known that stochastic problem has a solution.

This point is considered as a center of the coordinate system. Uncertain hyper-space is scalable in relation to it, so in any direction  $n, n \in N$  the lower and upper boundaries of stochastic parameters  $\theta_n^{LB}, \theta_n^{UB}$  take the values -1 and 1.

Then, the volume of the scaled hyper-space of the uncertain parameters is a correct hyper-rectangle (hyper-cube) for which the length of its sides in each direction  $n$  is determined by two single vectors - positive and negative. If the module of the negative vector is taken, the volume of the hyper-cube is:

$$V = \prod_{n=1}^{n=N} (|-1|_n + 1_n) = 2^N \quad (1)$$

Each uncertain parameter  $\theta_n$  depending on the place it occupied with respect to  $\theta_n^{BP}$  in the scaled space accepts either positive values from  $0 \leq \eta_{n,1} \leq 1$  if  $\theta_n^{BP} \leq \theta_n \leq \theta_n^{UB}$  or negative values from  $-1 \leq \eta_{n,2} \leq 0$  if  $\theta_n^{LB} \leq \theta_n \leq \theta_n^{BP}$ .

The second step relates to the presentation of the flexibility index in the new scaled space. Starting from the center of the coordinate system, in a scaled space, a hyper-rectangle is inscribed, the sides of which stretch towards the corresponding borders. The process continues until either the system performance constraints are violated or the boundaries of a scaled hyper-space are reached. For a two-dimensional stochastic space, the hyper-rectangle entry process is conditionally presented in Figure 1. As can be seen from the Figure 1, the hyper-cube stretching process continues until all the equality-type constraints (h1 and h2) and the type of inequality (g1) are satisfying. At the end of the process, on the coordinate axes the coordinates of the apex points of the inscribed hyper-rectangle defining the process feasibility boundaries in the scaled space  $\eta_{i,1}, \eta_{i,2}, \eta_{2,1}, \eta_{2,2}$  are determined. For two-dimensional space the related coordinates are two and number of quadrants is  $2^2$ , for the three-dimensional respectively they are three and number of quadrants is  $2^3$ , for the four-dimensional space they are four and the number of quadrants is  $2^4$ , etc. Therefore, the number of bound coordinates in each quadrant of the scaled space is equal to the number of stochastic variables  $N$ , and the size of the set they are produced is equal to the number of quadrants  $K = 2^N$ .

The volume of the so-recorded hyper-rectangle is determined by the lengths of the respective vectors  $\eta_{n,i}$  depending on their location, are positive or negative:

$$V \cdot = \prod_{n=1}^N \sum_{i=1}^2 |\eta_{n,i}| \quad (2)$$

where  $i=1$  at  $0 \leq \eta_{n,i} \leq 1$  and  $i=2$  at  $1 \leq \eta_{n,i} \leq 0$ .

The Flexibility Index is defined as a ratio of the volume of the entered hyper-rectangle to the volume of the entire scaled stochastic space (i.e. hyper-cube):

$$FI = \frac{\prod_{n=1}^N \sum_{i=1}^2 |\eta_{n,i}|}{2^N} \quad (3)$$

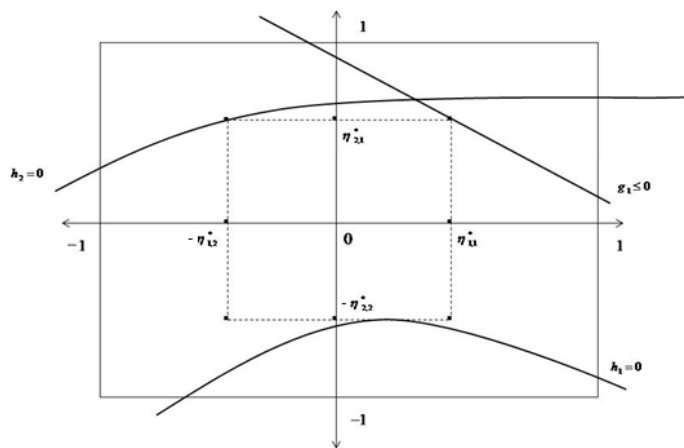


Figure 1: Determination of the feasibility of a process in a two-dimensionally scaled stochastic space

### 3. Defining the Flexibility Index for the ATAD system

The proposed above flexibility index is applied to assess space which is sustainable of the impact of the stochastic parameters in the heat-integrated ATAD system. Improving the energy efficiency of the ATAD system through heat reusing is presented in details in Vladova et al. (2018). The work proposes a common framework for heat integration of flows in the ATAD system. The proposed superstructure is shown in Figure 2. It is obtained by using the concepts of storing the "heat" and "cold" in batch production systems by the help of one and two heat storages - HS. Heat transfer is carried out by means of two heat exchangers where HE-c is used to heat the cold raw sludge entering the ATAD system and HE-h - to cool the hot product stream outgoing the second bioreactor to the product tank, such as transporting the respective streams through the heat exchangers and heat storages are carried out via pumps. The proposed energy integration framework reduces the impact of daily stochastic parameters, which leads to an increase in the operating temperatures in the two bioreactors. Defining the redesign problem in the terms of the two-stage stochastic programming the capital costs for redesign by using different sets of scenarios are assessed. In the paper the solutions with the lowest capital costs for redesign and operation are shown but some others with higher investment costs are obtained.

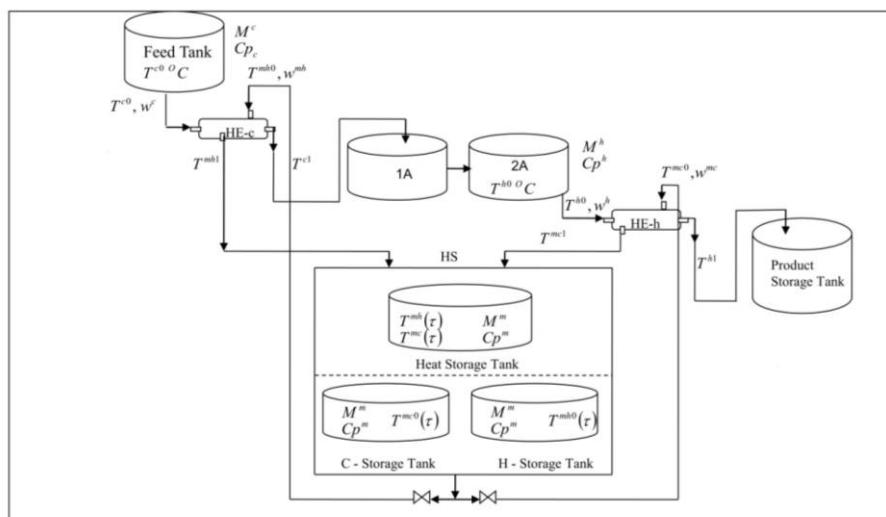


Figure 2. A common framework for heat integration of flows in the ATAD system

By using the above defined Flexibility Index (FI) the present study aims to find those from the already obtained solutions that would guarantee the system's sustainable operation in the widest range of uncertainty parameters. For the purpose, the obtained as the solutions of the first stage of the stochastic problem design parameters  $D^*$

are assumed to be constants. In our case study these are the areas of both heat exchangers and the volume(s) of the heat storage(s). After that, for each solution an optimization problem is formulated to determine the maximum value of FI which general form could be written as follows:

$$FI = \text{MAX} \frac{\prod_{n=1}^N \sum_{i=1}^2 |\eta_{n,i}|}{2^N} \quad (4)$$

subject to the following constraints:

$$0 \leq \eta_{n,i} \leq 1 \text{ for } i = 1, \forall n, n \in N; \quad (5)$$

$$1 \leq \eta_{n,i} \leq 0 \text{ for } i = 2, \forall n, n \in N; \quad (6)$$

$$\theta_{n,i} = \begin{cases} (\theta_n^{UB} - \theta_n^{BP})\eta_{n,i} + \theta_n^{BP} & \text{for } i = 1; \\ (\theta_n^{BP} - \theta_n^{LB})\eta_{n,i} + \theta_n^{BP} & \text{for } i = 2; \end{cases} \quad \forall n, n \in N \quad (7)$$

$$\overline{\eta_{n,i}}_k = \{k | \forall k \in K\} \text{ for } i \in 1,2; n \in N; \quad (8)$$

$$\overline{\theta_{n,i}}_k = \{k | \forall k \in K\} \text{ for } i \in 1,2; n \in N; \quad (9)$$

$$\overline{w}_k = \{k | \forall k \in K\}; \quad (10)$$

$$h_{ii}(D^*, \overline{w}_k, \overline{\theta_{n,i}}_k) = 0, \quad \forall ii \in I, \quad \forall k, k \in K; \quad (11)$$

$$g_j(D^*, \overline{w}_k, \overline{\theta_{n,i}}_k) \leq 0, \quad \forall j \in J, \quad \forall k, k \in K; \quad (12)$$

$$\overline{W}^{min} \leq \overline{w}_k \leq \overline{W}^{max}, \quad \forall k, k \in K \quad (13)$$

where the independent variables:  $\eta_{n,i}$  are the scaled values of stochastic parameters  $\theta_{n,i}$  and the variables  $\overline{w}_k$  are the same as the variables for the second stage of the stochastic problem - the times of loading the raw sludge and discharging the stabilized one. The investigated stochastic parameters  $\theta_{n,i}$  for the regarded ATAD system are the volumes of the daily loaded raw and drained stabilized sludge, the temperatures of the raw sludge loaded, and the temperatures of the discharged stabilized sludge. The optimization problem is completed with the set of equality constraints -  $h_{ii}$  (Eq(11)) describing the heat integration models for the cases with one and two heat storages and also with the sets of inequality constraints  $g_j$  (Eq(12)) representing technical, technological and temperatures constraints, Vakkieva-Bancheva et al. (2015; 2017).

The boundaries in which the relevant stochastic parameters change are also given and the coordinates of the Basic point are known. They are used to scale the stochastic space and to make transition from the scaled to real space, Eq(7).

#### 4. Results

Solving the optimization problem (4-13) the maximum values for the Flexibility Index could be determined and used to evaluate the obtained solutions for redesign of the heat integrated ATAD system in the conditions of the varying the stochastic parameters. Thus, the Flexibility Indices are determined not only for the best capital cost solutions presented in Vladova et al. (2018) but also to all other solutions obtained using different set of scenarios leading to different sizes of main and auxiliary equipment.

The stochastic parameters that have the greatest impact on the sustainable operation of the ATAD system are the volume of the loaded raw sludge, the temperature of the raw sludge and the temperature of the stabilized sludge that is discharged from the second bioreactor stage. The boundaries of the stochastic space are respectively  $12 \leq V \leq 20$ ;  $5,6 \leq T^{c0} \leq 20,2$ ;  $54,5 \leq T^{h0} \leq 68,1$ . The chosen Basic Point  $BP\{V^{BP}, T^{c0BP}, T^{h0BP}\}$  is  $\{18.9; 18.2; 66.3\}$ .

Table 1 shows the values of the Flexibility Index for the cases with minimal redesign costs shown in Vladova et al. (2018) for the heat integrated ATAD system with the use of one and two heat storages (HS). The boundaries of the uncertainty parameters within which the system is sustainable are also shown.

*Table 1: Flexibility indices and range of the change of the stochastic parameters for the solutions with minimum cost for redesign.*

Case	Costs for redesign (CU)	FI	Obtained threshold values of the stochastic parameters					
			$V^{LB}$ ( $m^3$ )	$V^{UB}$ ( $m^3$ )	$T_{c_0}^{LB}$ ( $^{\circ}C$ )	$T_{c_0}^{UB}$ ( $^{\circ}C$ )	$T_{h_0}^{LB}$ ( $^{\circ}C$ )	$T_{h_0}^{UB}$ ( $^{\circ}C$ )
One HS	14329	0.337	13.88	20.0	18.2	20.2	61.6	68.1
Two HS	16357	0.05	14.90	18.9	18.2	20.2	58.2	66.3

As can be seen from Table 1, the FI value for the case Two HS is very low. This means that the size of the main and auxiliary equipment, which determines the cost for redesign, ensures stable operation of the system in a rather narrow range of variation of the stochastic parameters, especially for the volume and temperature of the loaded raw sludge.

Table 2 shows some of solutions having average and maximum values the Flexibility Indices for the cases with one and two heat storages. They are obtained for the cases with greater capital costs, determined by the larger sizes of the equipment which values are shown in Table 3. It is seen from Table 2 that in three of four cases the upper boundaries of the range of stochastic parameters are reached. There are very large variations in the values of the lower thresholds of the stochastic variables, especially for the volumes and temperatures of the loaded raw sludge. The minimum thresholds for stochastic parameters to which the system is sustainable to the impact of stochastic parameters differ significantly from their low boundaries for the cases of two heat storages.

*Table 2: Solutions with average and maximum values of Flexibility indices and range of the change of the stochastic parameters.*

Case	Costs for redesign (CU)	FI	Obtained threshold values of the stochastic parameters					
			$V^{LB}$ ( $m^3$ )	$V^{UB}$ ( $m^3$ )	$T_{c_0}^{LB}$ ( $^{\circ}C$ )	$T_{c_0}^{UB}$ ( $^{\circ}C$ )	$T_{h_0}^{LB}$ ( $^{\circ}C$ )	$T_{h_0}^{UB}$ ( $^{\circ}C$ )
One HS-1	16474	0.431	12.4	19.5	9.7	20.2	61.4	68.1
One HS-2	18131	0.547	14.94	20.0	7.6	20.2	60.3	68.1
Two HS-1	18764	0.364	15.9	20.0	13.4	20.2	60.9	68.1
Two HS-2	19240	0.459	15.20	20.0	12.4	20.2	58.9	68.1

*Table 3: Sizes of main equipment in solutions with average and maximum values of Flexibility indices.*

Case	Sizes of main equipment		
	$V$ ( $m^3$ )	HE-c ( $m^2$ )	HE-h ( $m^2$ )
One HS	30	46	23
Two HS	2 x 33	59	30
One HS-1	40	68	29
One HS-2	40	62	42
Two HS-1	2 x 40	80	40
Two HS-2	2 x 42	80	40

It is obvious that the redesign options using two heat storages are not so preferable both because of their higher cost and because the thresholds of variation of the stochastic parameters in which the ATAD system operates sustainably are not wide enough. The good choice for improving the energy efficiency of the ATAD system is by using some of options with one heat storage either with Minimal Capital cost and the Low Flexibility Index or with a higher Flexibility Index and higher redesign cost. It is necessary to analyze the frequency of appearance of stochastic events below the determined low thresholds so as to make a choice for the most appropriate redesign project in the terms of the cost and sustainability. Which of these options have to be chosen is a matter of a decision-maker.

## 5. Conclusions

The present study provides an approximate and fast method for assessment the Flexibility Index of already obtained solutions of stochastic multi-scenario optimization problem. The Flexibility Index is defined as the ratio

between the volumes of the entered hyper-rectangle to the volume of the hyper-cube which envelope the entire scaled stochastic space. The method is applied for redesign of heat-integrated two-stage Autothermal Thermophilic Aerobic Digestion (ATAD) bioreactors system for municipal wastewater treatment operating under uncertainties. For the purpose, an optimization problem is formulated to obtain the maximum Flexibility index for each one of the already obtained solutions of the two-stage stochastic problem. A part of the solutions obtained for the cases of one and two heat storages has been shown and analyzed. It has been revealed that the redesign of ATAD system with the use of one heat storage provides a wider boundaries of the system sustainability from the impact of the stochastic parameters. The analysis carried out shows that not always the lowest capital and operating costs of the redesign determine the best solution in terms of the sustainable operation of the ATAD system. The final decision-making is a prerogative of the management team. The proposed method for assessing the Flexibility Index, although approximate, is very informative and easily can be applied for flexibility evaluation of designs and redesigns of other industrial systems.

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