

# Circulating Cooling Water System Optimisation under Uncertainty

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Circulating cooling water system (CCWS) is widely used in industry. Usually the relative optimisation problems are solved under determined values of input parameters. Once some parameters become uncertain, the system may not be optimal or stable. In a CCWS, the existence of these uncertainties will make the original optimal cooling network unreasonable, the design of pump is not suitable, and thus the entire system may no longer be safe. However, there is limited research on the synthesis of CCWS involving uncertainty, although they have been considered for some other process systems. This work presents a mathematical optimisation model for the design of CCWS under uncertainties. Uncertain parameters are described in the form of probability distribution functions. The models are formulated by chance constrained (CCP) methods, and they are solved by GAMS software. The objective is to determine the optimal network configuration that achieves the minimum total annualized cost. Meanwhile, the total circulating water flowrate and the heat transfer efficiency are optimised. To a certain extent, the system has operational stability and reliability. A modified industrial case study is solved to illustrate the proposed approach.

## 1. Introduction

Circulating cooling water system (CCWS) has a profound influence on the development of the chemical industry. Many researchers are devoted to the research of the CCWS, and their work mainly focuses on the configuration of the cooling water network, the design of the pump system and the optimisation of the cooling tower. Instead of the common used parallel configure, Ma et al. (2018a) constructed the cooling water network in a series structure, and considered the constraint of return water temperature to avoid fouling. Liu et al. (2018) optimised water cooler network and CCWS simultaneously by setting the cooling water as a new source of unknown variables, a stage-wise superstructure is used to integrate the system.

However, the uncertainty problems of CCWS are not mentioned in the research all above, which have big impact on the whole system. The uncertain parameters exist in all aspects of the CCWS, which may have extremely adverse consequences. Khulaifi and Mutairi (2016) takes the variability of stream properties as functions of temperature to attain the optimal heat exchanger network configuration, and a change cost is gained compared to traditional models. So it is of vital importance to consider uncertain problems in the CCWS.

During the past decades, several methods are proposed to solve uncertainty problems. Sahinidis (2004) reviewed the theory and methodology coped with uncertainty, and introduced some methods including stochastic programming, fuzzy mathematical programming and stochastic dynamic programming. Grossmann et al. (2016) described recent advances and important barriers to the application of optimisation methods under uncertainty. Chen et al. (2018) reviewed the development of process optimisation under uncertainty over the past ten years and made a new classification of these techniques. Among these methods, chance-constrained programming (CCP) is an effective way to solve uncertainty problems. Li et al. (2008) summarised the applications of CCP on linear, nonlinear, nonlinear dynamic chance-constrained systems. Jiao et al. (2012) used the CCP method to solve the optimisation problem of the hydrogen network. Lee et al. (2018) applied the CCP

model to handle uncertain problems of renewable energy resources in the hybrid power system. CCP approach is used for CCWS optimisation under uncertainty in this paper.

Typical circulating cooling water system optimisation problems are solved under determined values of input parameters. Once some of these values become uncertain, the system may not be optimal or stable; the optimised structure may become suboptimal or even unfeasible. Attention should be paid to the optimisation problems of CCWS under uncertain conditions. This paper proposed a chance-constrained programming model to cope with uncertainty. The CCP method is stochastic programming under known probability distribution functions. Due to the decision may not satisfy the constraints, a predefined confidence level is introduced to describe the degree of constraint violation, allowing normal operation of the system to a certain extent. Equivalent deterministic equations are employed to reduce computing costs.

## 2. Problem statement

In a CCWS, a set of hot streams with known inlet and outlet temperature and flow rate are cooled by air cooler or water cooler or both of them separately. Meanwhile, the low-temperature cooling water flows through the cooler, heats by the heat stream, rises to a certain temperature, and after that, it enters the cooling tower for cooling and prepares for the next heat exchange process and the pump is used to transport cooling water. The structure of the CCWS can be described in Figure 1.

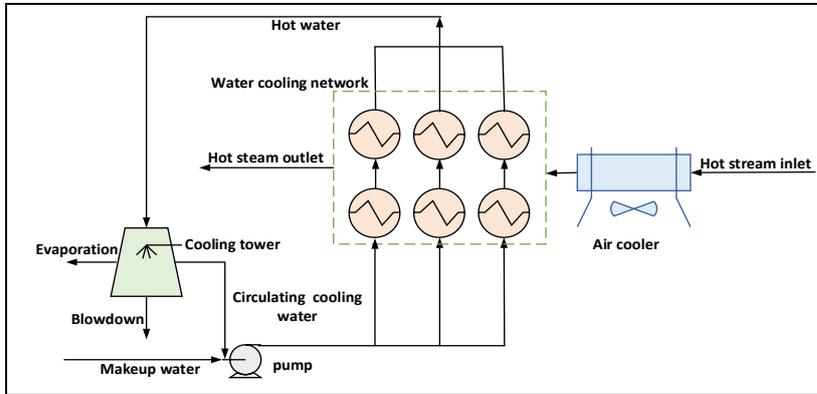


Figure 1: The structure of cooling water system with air coolers

In the process of system operation, uncertain parameters exist all the time, which influence the system greatly. In this paper, the electricity prices and freshwater prices are considered as uncertain variables and assume that they conform to a normal distribution. The optimisation objective is not only to minimise the total cost of the system, including the operating cost and capital cost, but also the best configuration of the cooling water network under uncertainty. CCP method is employed to formulate the system model, and transformed to corresponding equivalent deterministic forms.

## 3. Model formulation

### 3.1 Objective function

The objective function is to minimise the total cost (TAC) of all the components, including the operating cost and capital cost. The TAC is written as:

$$\min TAC = E(\tilde{C}_A) + C_W + E(\tilde{C}_P) + E(O\tilde{C}_T) + CC_T \quad (1)$$

Where  $\tilde{C}_A$ ,  $\tilde{C}_P$ ,  $O\tilde{C}_T$  are the uncertain cost of air coolers, the uncertain cost of the pump, and the operation cost of the tower.  $C_W$  and  $CC_T$  are water coolers cost and tower capital cost, and are independent of uncertain parameters.

These costs can be expressed as Eq(2) to Eq(6):

$$\tilde{C}_A = Af \cdot \sum_{i=1}^n \left[ c_A \cdot AA(i)^{0.525} \right] + Pf_A(i) \cdot \tilde{P}e \cdot h \quad (2)$$

$$C_W = Af \cdot \sum_{i=1}^n [a + b \cdot A_w(i)] \quad (3)$$

$$\tilde{C}_P = Af \cdot \left[ c_{fP} + c_P \cdot \left( \frac{f_i \cdot \Delta p_i}{\rho} \right)^\gamma \right] + \frac{f_i \cdot \Delta p_i}{\rho \cdot \eta_P} \cdot \tilde{P}_e \cdot h \quad (4)$$

$$O\tilde{C}_T = OC_{fT} + 110 f_i + \tilde{P}_w \cdot h \cdot M + 1138B \quad (5)$$

$$CC_T = Af \cdot \left[ 746.74 f_i^{0.79} \cdot R^{0.57} \cdot A_p^{-0.9924} + (0.022 T_{wb} + 0.39)^{2.447} \right] \quad (6)$$

Where  $Af$  is the annualising factor,  $c_A$  is the constant of air cooler capital costs,  $A_A(i)$  is the area of air cooler  $i$ ,  $P_{fa}(i)$  is the fan power consumption of air cooler  $i$ ,  $\tilde{P}_e$  is the uncertain electricity price,  $h$  is the annualised operation time.  $a$  and  $b$  denote the constants of water cooler capital cost,  $A_w(i)$  is the area of water cooler  $i$ ,  $c_{fP}$  is fixed charge for pump,  $c_P$  is the pump pressure cost coefficient,  $f_i$  is the total flowrate of cooling water,  $\Delta p_i$  is the total pressure drop of pump,  $\rho$  is the density of cooling water,  $\eta_P$  is the pump efficiency,  $OC_{fT}$  is tower fan operational cost,  $\tilde{P}_w$  is the uncertain water price,  $M$  is the flowrate of makeup water,  $B$  is the flowrate of water blowdown.  $R$  is the temperature difference between cooling tower inlet and outlet temperature,  $A_p$  is the temperature difference between cooling tower outlet temperature and air wet bulb temperature  $T_{wb}$ . The models of the cooler, cooling water, air cooler, pump and other constraints can be found in Ma et al. (2018b).

### 3.2 Chance-constrained programming

As the above optimisation objective functions conclude uncertain parameters, the model can be redefined with chance constraints. Consequently, the objective function can be rewritten as:

$$\min TAC = \bar{C}_A + C_W + \bar{C}_P + O\bar{C}_T + CC_T \quad (7)$$

The Eq (2), Eq (4), Eq (5) involved in the CCP model can be redefined as:

$$\Pr \left\{ Af \cdot \sum_{i=1}^n [c_A \cdot A_A(i)^{0.525}] + P_{fa}(i) \cdot \tilde{P}_e \cdot h \leq \bar{C}_A \right\} \geq \alpha_A \quad (8)$$

$$\Pr \left\{ Af \cdot \left[ c_{fP} + c_P \cdot \left( \frac{f_i \cdot \Delta p_i}{\rho} \right)^\gamma \right] + \frac{f_i \cdot \Delta p_i}{\rho \cdot \eta_P} \cdot \tilde{P}_e \cdot h \leq \bar{C}_P \right\} \geq \alpha_P \quad (9)$$

$$\Pr \{ OC_{fT} + 110 f_i + \tilde{P}_w \cdot h \cdot M + 1138B \leq O\bar{C}_T \} \geq \alpha_T \quad (10)$$

$\alpha$  is the confidence level which represents the violation degree of constraint. The greater its value, the more reliable the system is and the higher the cost, and vice versa. So this is a trade-off, and its value ultimately depends on the needs of the decision makers.

Since Eq(8) and Eq(9) contain the same variables, they can be rewritten as:

$$\Pr \left\{ Af \cdot \sum_{i=1}^n [c_A \cdot A_A(i)] + P_{fa}(i) \cdot \tilde{P}_e \cdot h + Af \cdot \left[ c_{fP} + c_P \cdot \left( \frac{f_i \cdot \Delta p_i}{\rho} \right)^\gamma \right] + \frac{f_i \cdot \Delta p_i}{\rho \cdot \eta_P} \cdot \tilde{P}_e \cdot h \leq \bar{C}_A + \bar{C}_P \right\} \geq \alpha_{A+P} \quad (11)$$

In order to solve the chance constrained problem of the circulating cooling water system, the probability constraints should be transformed to the corresponding equivalent deterministic form, which needed the probability distribution functions of uncertain variables and then the constraints Eq(10) and Eq(11) can be reorganised as:

$$\frac{O\bar{C}_T - OC_{fT} - 110 f_i - 1138B}{h \cdot M} \geq \Phi^{-1}(\alpha_T) \quad (12)$$

$$\frac{\bar{C}_A + \bar{C}_P - Af \cdot \sum_{i=1}^n [c_A \cdot AA(i)] - Af \cdot \left[ c_{fP} + c_P \cdot \left( \frac{f_i \cdot \Delta p_i}{\rho} \right)^\gamma \right]}{P_{fA(i)} \cdot h + \frac{f_i \cdot \Delta p_i}{\rho \cdot \eta_P} \cdot h} \geq \Phi^{-1}(\alpha_{A+P}) \quad (13)$$

Where  $\Phi^{-1}$  represents the inverse function of the probability distribution function. Then the constraints contained uncertainties are transformed into deterministic forms. As long as the distribution functions are known, the model can be rewritten in the form of deterministic MINLP problems and solved by GAMS software.

#### 4. Case study

To classify the effectiveness of the proposed model, a case study taken from Ma et al. (2018b) is employed. The results of the case study explore the influence brought by uncertain parameters. In this paper, we assume uncertain parameters are conformed to normal distribution. The distribution of the electricity price and the water price are expressed as the form of Eq(14) and Eq(15), according to the historical data showed in Figure 2.

$$\tilde{P}_e \square N(0.0811, 0.0365^2) \quad (14)$$

$$\tilde{P}_w \square N(0.636, 0.219^2) \quad (15)$$

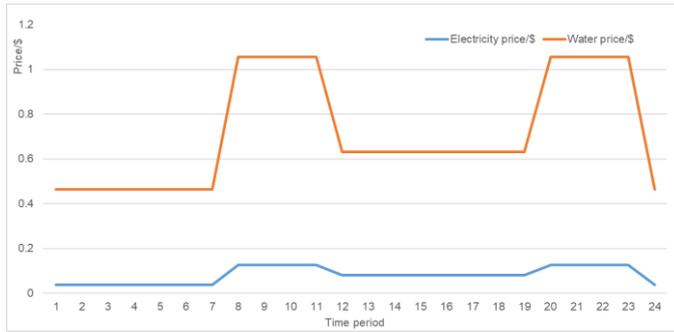


Figure 2: Electricity price and water price

The objective is a trade-off between cost and reliability, instead of conventional pursuits. Physical and economic parameters of case study can be found in Ma et al. (2018b). The case study involved ten hot streams, the relative data is shown in Table 1.

Table 1: Hot steam parameters

Streams	1	2	3	4	5	6	7	8	9	10
Th <sub>in</sub> (°C)	150	80	105	185	90	135	75	150	195	80
Th <sub>out</sub> (°C)	85	55	40	65	55	65	50	85	50	65
F <sub>cp</sub> (kW /°C)	200	150	60	100	80	120	100	65	90	55
H (W/m <sup>2</sup> · °C)	854	1743	720	1352	750	785	542	782	121	974

When the confidence level of the two chance constraints are both 95%, we got a better solution of the system than that of the original design. Compared to original cost 900,350 \$, the TAC is 632,870 \$ in this new model and generates 29.7% reduction. Table 2 shows the calculation results.

The original network and optimized configuration can be seen in Figure 3. We can draw the following conclusion by comparing the data in the two figures. Compared with Figure 3 (left), the system in Figure 3 (right) has increased demand for air coolers, both in number and area, while the number of water coolers is decreased and the heat load of the cooling tower is reduced. Therefore, the original series structure has changed to the optimal structure that the series and parallel forms existed simultaneously.

Table 2: Optimized results

Stream	$\alpha_{A+P} = 99\%$	$\alpha_{A+P} = 97\%$	$\alpha_{A+P} = 95\%$	$\alpha_{A+P} = 93\%$	Original case
	$\alpha_T = 99\%$	$\alpha_T = 97\%$	$\alpha_T = 95\%$	$\alpha_T = 93\%$	
TAC/\$	651,865	638,752	632,870	636,462	900,530
$C_W$ /\$	134,589	135,056	132,084	136,372	119,556
$OC_T$ /\$	97,978	9,3201	90,942	99,037	429,958
$CC_T$ /\$	7,957	7,615	7,454	8,033	47,711
$C_A + C_P$ /\$	410,820	402,280	402,390	393,020	303,302

The reason for these changes is the occurrence of uncertain parameters. Since the electricity price and the water price fluctuate all the time, the original optimal structure becomes sub-optimal, and the total cost is no longer the smallest. In order to achieve minimum cost and new optimal structure, it is necessary to reduce the consumption of electricity and water. Power consumption mainly occurs in the air cooler, the fan of the cooling tower, and the pump. The water consumption mainly occurs in the water network of the cooling tower. Therefore, the resulting impact on cost of the water network is greater than that on the air coolers. In order to achieve new balance, the system tries to increase the use of air coolers and reduce the demand for water network. Finally we get a safer and more reliable system.

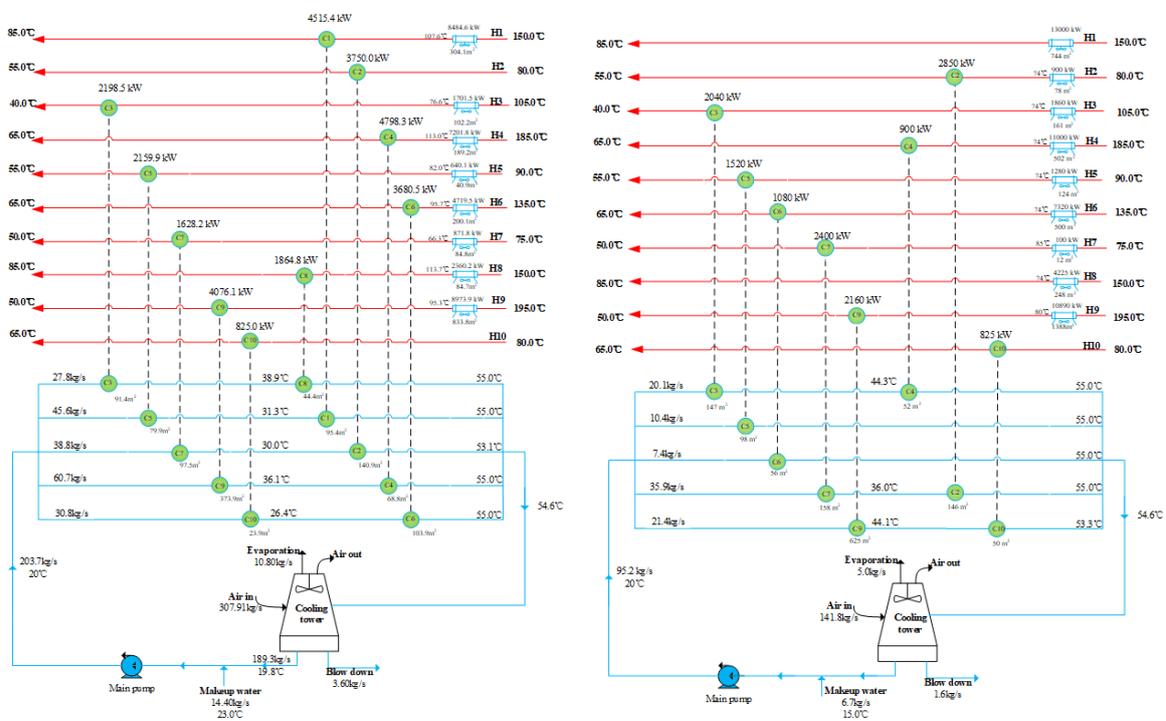


Figure 3: The network of case study (left: original network, right: optimal network)

And the reason of costs reduction is shown in the two figures, the uncertain prices vary all the time, which caused to an increase on the cost of the cooling network, the increased use of air coolers is in need, so that the flowrate of cooling water is reduced as well as the heat load of cooling tower. To obtain a minimised cost, Table 2 shows the optimized results under different confidence level. According to Table 2, the total cost of air coolers and pump are increased as the existence of uncertain parameters. At the same time, the cost of cooling tower is reduced, the cost of the water network is not changed much. The final result is a reduction in total cost. It is worth noting that the costs under different confidence levels are different, but they are all less than the cost of the original network. And the total cost increases as confidence increases, showed in Figure 4. A larger value of confidence level means that the higher confidence level, the more cost input, and a more reliability system is obtained. But at the same time, operating cost is also increased. As can be seen in Table 2 and Figure 4, when the confidence level is gradually changed from 99 % to 93 %, the operating cost is reduced.

And this trend exposes the balance between the total cost and the operation stability. When there are different needs, decision makers can choose proper confidence level to get the trade-off.

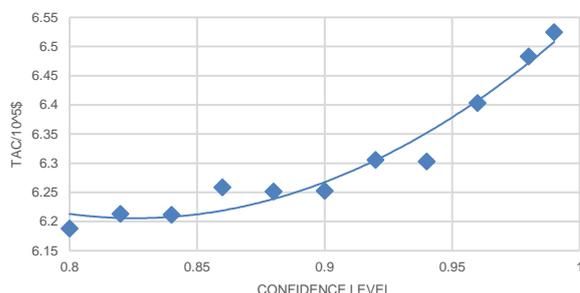


Figure 4: TAC changes with confidence level

## 5. Conclusion

In this paper, we proposed a chance constrained programming method to optimize the circulating cooling water system under uncertain circumstance. Price disturbance is taken into consideration, normal probability distribution functions of uncertain variables are introduced to transform the uncertain model to corresponding equivalent deterministic forms. The transformed model is a MINLP problem, solved by GAMS software. A better configuration of cooling water network and a minimised total cost are obtained compared to original case study. The total annual cost of the optimized network using the CCP method is about \$650,000/y, which can save \$250,000/y approximately under different confidence levels. Compared with the original network, the total annual cost is reduced by around 28 %. In addition, the total cost fell from \$651,865 to \$632,870 as the confidence level decreased from 99 % to 93 %, revealing the balance between costs and system stability. And the cost of the cooling water network, the pump system and the cooling tower have changed accordingly. The proposed model provides an opportunity to address uncertainties in real world, and is applied to solve the system power and water source uncertainty problems. Therefore, the uncertainty should be taken into consideration when designing or retrofitting a CCWS, and some margin should be maintained for the reliable operation of the system. The impact of streams nature and environmental parameters will be taken into account in future work.

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