

## A Study on Meat Drying Process Using a Porous Media Approach. Part 2: Analysis and Mathematical Modeling

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This paper delivers a contribution aimed at improving available mathematical models of salami ripening by switching to a description of the sausage as a heterogeneous material, obviously made of fat and minced lean meat, and to the adoption of the porous medium approach for water transport from the lean meat phase to outside, which represents a novelty.

The literature data as well as mathematical correlations able of describing or approximating water diffusion in fresh lean meat have been reviewed. Two semi-empirical correlations for water diffusivity as a function of the local moisture content were selected, i.e., from Saravacos and Datta. Then, the optimal parameters were determined for them by coupling a distributed-parameter mathematical model of minced meat drying (adopting the porous media approach) and an optimization algorithm with available experimental data. A code was implemented and successfully run by invoking the “transport in porous media” and “optimization” physics in the Comsol Multiphysics® 4.3 platform.

The mathematical modeling work allowed discussion and comparison of the predictions against independently available experimental results, in terms of weight loss as well as space and time profiles of moisture in a reference lean meat specimen.

### 1. Introduction

Since many centuries, salami products have raised a great attention in the western world, first for consumers satisfaction, then for production technique (initially at family and artisanal level, more recently industrial), with a log rich of historical, nutritional and economic key aspects. Nowadays, salami production is carried out in properly ventilated and conditioned industrial chambers rather than under uncontrolled natural conditions (Burgio et al., 1999). Hence, the same product quality can be obtained regardless of local, environmental (Kottke et al., 1996) and climatic conditions (Katz et al., 1987).

Of course, there have been (and still are) many types of dry sausages differing in composition (e.g., type of meat, cut size of lean and fat, ingredients and additives, nature of the casing, microbial starters) or in process and technique for drying and curing (e.g., long-maturing vs short-maturing salamis). In all cases, they are originated from minced lean meat and fat; therefore, the fresh sausages can be undoubtedly described as a heterogeneous, irregular mixture of lean meat and fat globules. Such a “discrete” granular structure is retained during fermentation, drying and ripening, and can be still recognized in the final product.

The acquisition of scientific knowledge about the mechanisms involved in the maturation of the meats is fairly recent (Pirone et al., 2007). The ability to describe the drying process depends to a greater extent on the knowledge of the phenomena related to water transport inside the product and, to a lesser extent, of evaporation from the outside surface. It is well known that a water concentration gradient establishes between the inner core of the sausage and its external surface. Owing to salami granular structure, water actually moves in a complex way and the main transfer mechanisms (Okos et al., 1992) are usually considered to be: 1) capillary flow of free water; 2) bound water movement; 3) diffusion in liquid phase; 4) condensation-evaporation. These mechanisms are often competitive (Baldini et al., 2001) and the drying kinetics is the consequence of both their interaction and the external conditions applied (Bessadok et al., 2013).

In today's industrial production of salamis, well-planned and carefully monitored process conditions are necessary to achieve the targeted weight loss, quality and safety of dry fermented sausages. Therefore, the availability of reliable mathematical models and manageable software codes for sausage drying, maturation, optimal production and chamber control are highly welcome.

Initially, mathematical models of salami drying were addressing the sausage as it were a homogeneous material: this can be found in Imre and Korney (1990), Grassi and Montanari (2005), Diaferia et al. (2011), Cascone et al. (2015a) and Cascone et al. (2015b). Then, the Authors engaged themselves in developing models in which the sausage mixture was heterogeneous but isotropic, still with no distinction between the inner part and the casing: therefore, the sausage is a two-phase material consisting of a fatty part regularly dispersed in a lean meat matrix in which the moisture transport phenomena take place (Cascone et al., 2017). The water is considered present in the lean meat only, at each time during sausage processing.

A further step is the switch to the porous medium approach, which makes it possible to consider the fat as inert and the lean meat as a porous medium in which the actual water transport takes place. A mathematical model of sausage drying according to the porous medium approach requires the assumption of small pores ( $\approx 100 \mu\text{m}$ ), weak evaporation and a capillary pressure formulation (Datta, 2007) for the initially very wet lean meat. However, a model of this kind, which describes water concentration as a distributed parameter in the lean meat phase, requires as a constitutive equation a quantitative relationship giving the capillary diffusivity or, in a simplified way, the effective diffusion coefficient  $D_e$  as a function of the local water content. There is a clear lack of such relationships in the current literature (Cascone, 2018).

The present paper represents an attempt to overcome the above drawback. Hence, two semi-empirical relationships for the effective diffusivity in the lean meat as a porous medium were obtained by taking advantage of the experimental activity reported in a companion paper (Cascone et al., 2019) and of the mathematical modeling work discussed here. To this end, the "transport in porous media" and "optimization" physics were invoked and successfully run in a code developed under the Comsol Multiphysics®4.3 platform.

## 2. Materials and Methods

### 2.1 Mathematical model

The porous media approach is adopted in this work with reference to lean meat. This latter is considered as a homogeneous porous system with small pores ( $\approx 100 \mu\text{m}$ ), weak evaporation and a capillary pressure formulation (Datta, 2007). The resulting mathematical model is the classical water transport partial differential equation:

$$\frac{\partial c_w}{\partial t} - \nabla(D_w \nabla c_w) = 0 \quad (1)$$

$$I.C.: @t = 0, c_w = c_0$$

$$B.C.: \begin{cases} @ x \in \Gamma_{NO \text{ flux}}, N_w = 0, & \forall t > 0 \\ @ x \in \Gamma_{flux}, N_w = N_R, & \forall t > 0 \end{cases}$$

where:

$c_w$ = water concentration in the capillary system ( $\text{kg}/\text{m}^3$ ),  $c_w=f(X)$ ;

$\Gamma_{flux}$ = boundary where the mass transfer takes place;

$\Gamma_{NO \text{ flux}}$ = boundary where the mass transfer does not take place;

$N_w$ = water interface flux;

$D_w$ = capillary diffusivity ( $\text{m}^2/\text{s}$ );

$X=f(c_w)$ , moisture content on dry basis;

$t$ = time;

$N_R$ = convective water mass flux at the interface (Cascone, 2018).

The reference specimen for the application of the above Eq(1), the control volume, the geometrical domain and the relevant physic-chemical properties are described in the companion paper (Cascone et al., 2019).

The mathematical model implementation requires choosing 1 or 2 boundary surfaces  $\Gamma_{flux}$  in Eq(1), according to the two types of experimental configurations: "A" with one surface transferring moisture to outside, "B" with two exchanging surfaces (Cascone et al., 2019).

The corresponding energy balance is not required because of the negligible heat effects associated to the quite slow drying phenomena, as confirmed by the companion experiments (Cascone et al., 2019).

### 2.2 Optimization procedure for diffusivity parameter estimation

Based on the available experimental data, i.e., moisture concentration and weight loss time and special profiles in the companion paper (Cascone et al., 2019), and the above mathematical model, a parameter optimization idea was conceived and set up for two literature correlations providing diffusivity for foods other

than meat. This was intended to yield more reliable correlations able to express moisture diffusivity in lean meat during drying.

A code was implemented in COMSOL Multiphysics® adopting the above mathematical model coupled with a constitutive equation representing a quantitative relationship between the diffusivity and the local water content. The code adopted the “Transport in porous media” physics to solve the above mathematical model and the “Optimization” physics, which is built in COMSOL, to determine the optimal values of the parameters appearing as coefficients in a water diffusivity correlation.

As regards the solution of the mathematical model, i.e., Eq(1), the shape of the sample holder, i.e., a length L (60 mm) much larger than the height h (22 mm), allowed having appreciable concentration variations only in the longitudinal direction (x). In this way, it was possible to choose a 1D geometry in the L direction for the internal mass transfer phenomena and, hence, to make the model computations less time-consuming in COMSOL by means of the Finite Element Methods (FEM).

The mesh used for the simulation is of the type Free Tetrahedral with an “extremely fine” calibrated dimension for General physics.

The first selected constitutive equation was the correlation for effective diffusivity proposed by Saravacos (2001) for food:

$$D = \frac{1}{1+X} D_0 \exp \left[ -\frac{E_0}{R} \left( \frac{1}{T} - \frac{1}{T_r} \right) \right] + \frac{X}{1+X} D_i \exp \left[ -\frac{E_i}{R} \left( \frac{1}{T} - \frac{1}{T_r} \right) \right] \quad (2)$$

where:

D= effective moisture diffusivity (m<sup>2</sup>/s);

T= temperature (K);

T<sub>r</sub>= 60°C, reference temperature;

R= 0.0083143 (kJ/mol K), ideal gas constant;

X= moisture content of lean meat (kg/kg<sub>db</sub>);

The parameters, the optimal values of which were to be determined, D<sub>0</sub>, D<sub>i</sub>, E<sub>0</sub>, E<sub>i</sub>, were:

D<sub>0</sub>= diffusivity @ X=0, T=T (m<sup>2</sup>/s);

D<sub>i</sub>= diffusivity @ X=∞, T=T<sub>r</sub> (m<sup>2</sup>/s);

E<sub>0</sub>= activation energy for diffusion @ X=0 (kJ/mol);

E<sub>i</sub>= activation energy for diffusion @ X=∞ (kJ/mol).

The second constitutive equation here taken into consideration was the semi-empirical formulation of capillary diffusivity by Datta (2007):

$$D_w = A \exp[-B + C \cdot X] \quad (3)$$

where:

D<sub>w</sub>= capillary water diffusivity (m<sup>2</sup>/s);

where the optimal parameters to be determined were A (m<sup>2</sup>/s), B (-), C (kg<sub>db</sub>/kg).

The implementation of the “Optimization” physics involved the use of two types of data:

- global experimental data sets (weight loss) on the domain as a function of time until the end of the test, in the option “Global Least-Squares Objective”.
- local experimental data sets (water concentration profile) on the domain at fixed times, i.e., 0 h, 24 h, 48 h, 72 h during the drying test, in the option “Least-Squares Objective”.

Actually, the “Optimization” physics required the inclusion of an “Optimization” item in the “Study” option in which the user chooses the method to solve the optimization calculation with the experimental data: in this work, the Levenberg–Marquardt Algorithm is chosen. This latter concerned the minimization of an objective function defined as the sum over time of the squared errors between the simulated and experimental results, firstly for a macroscopic variable such as the sample weight loss, secondly for a variable changing in the spatial domain such as the water concentration.

Properties of the built-in materials in the COMSOL software were not used; vice versa, all the characteristics of the lean meat were implemented in “Parameters in Global Definitions” (if parameters) or in “Variables in Definitions” (if variables depending on the moisture content).

Running the code required as input data the initial properties of the fresh minced lean meat (i.e., commercial *longissimus dorsi* pork meat) and the drying air conditions. So, as input to the model, the data related to experimental drying tests from Cascone et al. (2019) were used:

Table 1: Code data input with porous media model for the 1D experimental drying case

Name	Value	Description
$T_0=T_{air}=T$	303 K	Air and meat temperature
$X_{r0}$	0.75	Initial meat moisture fraction
$m_t$	31.997 g	Initial meat weight
$L_{sample}$	60 mm	Specimen length
RH	0.15	Air relative humidity
$k_{oswin}$	10.8e-2	K Oswin constant
$n_{oswin}$	0.43	n Oswin constant
$u$	0.2 m/s	Air velocity
Duration	3 d	Drying duration
Step	0.1 d	Calculation step

As far as the starting values of the parameters to be optimized, the calculation code aimed at optimizing the effective diffusivity expressed by Eq(2) had the parameters of the potato listed in Table 2a (Saravacos, 2001) as starting values. Similarly, the calculation code aimed at optimizing the capillary diffusivity expressed by Eq(3) had the parameters of potato listed in Table 2b (Datta, 2007) as starting values.

Table 2: Starting parameters in optimization of a) effective diffusivity and b) capillary diffusivity

Name	Value	Name	Value (b)
<b>(a)</b>		<b>(b)</b>	
$D_0$	$4.31 \cdot 10^{-10} \text{ m}^2/\text{s}$	A	$10^{-9} \text{ m}^2/\text{s}$
$D_i$	$1.57 \cdot 10^{-9} \text{ m}^2/\text{s}$	B	2.8
$E_0$	76.9 kJ/mol	C	2 $\text{kg}_{db}/\text{kg}$
$E_i$	44.7 kJ/mol		

Other conventional parameters with dependencies by temperature and water concentration were literature correlations (Perry et al., 1998).

The development of the codes and the simulation runs have been carried out on a MS Windows workstation based with the Intel® Core™ i7-6700HQ processor, 2.60 GHz clock rate and 32 GB RAM.

### 3. Results and Discussion

First, the results are discussed when coupling modeling and optimization by adopting the effective diffusivity correlation in Eq(2) with the start-up parameters shown in Table 2a.

Through a repeated calculation cycle, the COMSOL code evaluated the mass transfer physics simulation results, compared them with the experimental data selected for optimization, i.e., weight loss and moisture concentration profiles in the experimental test with one moisture surface exchange to outside (Cascone et al., 2019), changed the current  $D_0$ ,  $D_i$ ,  $E_0$ ,  $E_i$  parameter values and started a new simulation. In this way, the software identified a local minimum of the least squares errors function (selected as optimization goal) between predicted results and experimental data, and provided the set of parameters corresponding to this minimum.

Such a procedure yielded the parameters reported in Table 3a.

Table 3: a) Effective diffusivity parameters and b) Capillary diffusivity parameters as optimization result

Name	Value	Name	Value
<b>(a)</b>		<b>(b)</b>	
$D_0$	$4.6217 \cdot 10^{-10} \text{ m}^2/\text{s}$	A	$1.4653 \cdot 10^{-9} \text{ m}^2/\text{s}$
$D_i$	$3.6622 \cdot 10^{-9} \text{ m}^2/\text{s}$	B	2.3347
$E_0$	69.319 kJ/mol	C	0.56064 $\text{kg}_{db}/\text{kg}$
$E_i$	31.953 kJ/mol		

It can be immediately noted that the optimal parameters reported in Table 3a remained of the same order of magnitude as those published for potato (Saravacos, 2001) and here used just as start-up values.

The simulation results just achieved with the optimal parameters reported in Table 3a were compared to the experimental data obtained in the experiments with one exchange surface (test A in Cascone et al., 2019). Both the simulated weight loss (Figure 1a) and the simulated water concentration profile (Figure 1b) turned out

reasonably close to the experimental data, after taking into consideration the uncertainty associated to these latter and expressed by the standard deviation bars in Figure 1. It is to be noted here that the water concentration profile simulated as a function of the specimen length (see Figure 1b) is also a function of time in batch experiments: for sake of simplicity, here the results have been reported in a non-dimensional shape just at a fixed drying time, i.e., an intermediate value corresponding to  $t=48$  h.

In order to have a further and independent validation of the mathematical model provided with the parameters obtained by the optimization procedure, a simulation was run under the conditions of the experiments with two exchange surfaces (test B in Cascone et al., 2019). The weight loss predicted by such a simulation as a function of the drying time is reported in Figure 1c and compared to the experimental data, yielding a very good closeness of the simulation results to the experimental ones and thus confirming the soundness of the approach adopted in this work.

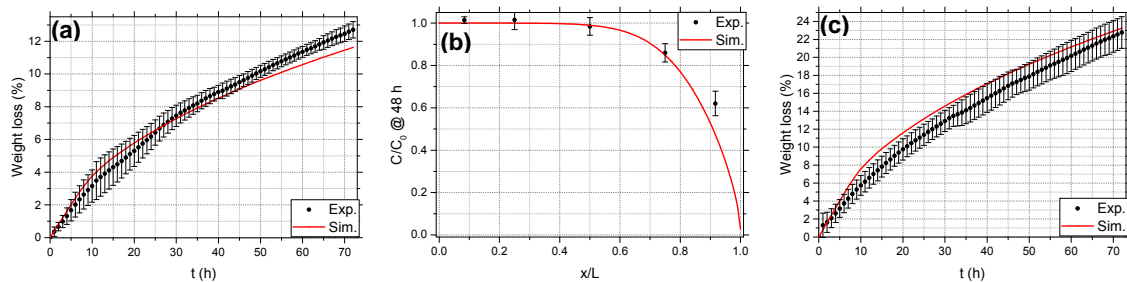


Figure 1: a) Effective diffusivity simulation and experimental weight loss comparison for one surface experiment, b) Effective diffusivity simulation and experimental concentration profile comparison @ $t=48$  h for 1 surface experiment, c) Effective diffusivity simulation and experimental weight loss comparison for two exchange surfaces experiment

Second, a similar job was done by switching to the calculation code provided with the capillary diffusivity correlation Eq(3) and running it with the start-up parameters shown in Table 2b.

Such a procedure yielded the optimal parameters reported in Table 3b.

Again, it is remarked that the optimal parameters reported in Table 3b remained of the same order of magnitude as those published for potato (Datta, 2007) and here used just as start-up values (see Table 2b).

The simulation results just achieved with the optimal parameters reported in Table 3b were compared to the experimental data obtained in the above-mentioned experiments with one exchange surface (test A in Cascone et al., 2019). The comparison turned out qualitatively and quantitatively similar and, all in all, satisfactory for both weight loss (see Figure 2a) and moisture concentration profile (see Figure 2b).

Again, with the parameters obtained by the optimization procedure, an independent simulation was run under the conditions of the experiments performed with two exchange surfaces (test B in Cascone et al., 2019). The comparison (see Figure 2c) was equally agreeable.

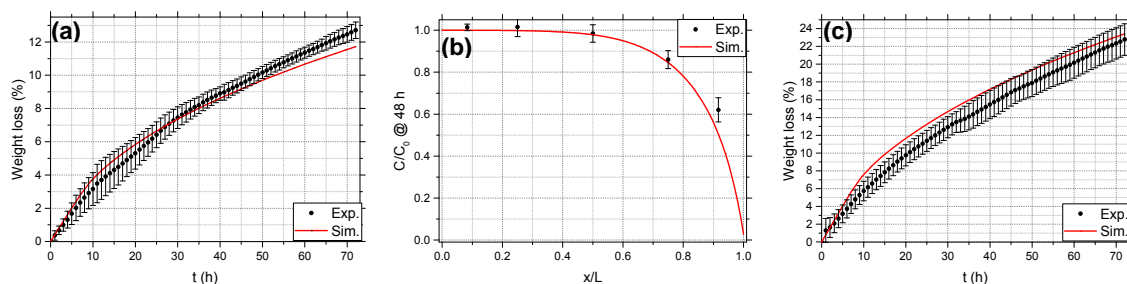


Figure 2: a) Capillary diffusivity simulation and experimental weight loss comparison for one surface experiment, b) Capillary diffusivity simulation and experimental concentration profile comparison @ $t=48$  h for one surface experiment, c) Capillary diffusivity simulation and experimental weight loss comparison for two surfaces experiment

#### 4. Conclusions

A mathematical model was developed, and a related calculation code was implemented in Comsol Multiphysics®4.3 platform, which considers salami as a heterogeneous material, separately made of fat and

lean meat, and reasonably describes this latter according to a porous medium approach. Such a work lends itself to attempt, even more in general, the simulation of the drying process of a particular food for which the internal water diffusion characteristics are unavailable or unrealistic in the literature.

The idea of overcoming the lack of literature correlations for the water diffusion in a porous meat matrix by adopting and adapting relationships published for foods other than lean meat was pursued in a reasonably successful way. By joining modeling with porous media approach, an optimization algorithm for parameter estimation and the experimental data provided by an experimental companion work carried out by the authors themselves, it was possible to determine new specific parameters for two different correlations taken from literature and aimed at predicting moisture diffusivity as a function of the “local” water content.

The goodness of the adopted approach has been further confirmed by an independent validation of the mathematical model (provided with the parameters obtained by the optimization procedure) against different additional experiments, the results of which had not been used for parameter estimation.

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