Evaluation on Fuel Cost Saving based on System Simulation

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From the perspective of system stimulation, this paper took estimation of fuel economy in system modification as the subject, to which regard, a correlation model was built based on the analysis of its influencing factors with corresponding simulation tests conducted toward this end, to provide future reference for economical evaluation of system modification. First of all, this paper divided the influence of refitted system into the fluctuation in fuel price and the changes in fuel consumption rate based on the estimation of fuel cost saving of the same. Simulation models for the fluctuation in fuel price and fuel consumption were established according to the stochastic process and aircraft operation & use. Finally, the author put forward an overall framework for fuel economy. Seen from the MATLAB simulation results, the model presented a sound evaluation over the expectation and variance risks of fuel cost saving under future environmental fluctuations to cover the shortage of current studies.

1. Introduction

Fuel cost is an important part of the economic analysis of civil aircraft. According to International Air Transport Association (IATA), fuel cost takes up 30\% of the average cost of airlines. The annual statements of the three major airlines (Air China, 2014; China Eastern Airlines, 2014; China Southern Airlines, 2014) reported that the fuel cost of Air China, China Eastern Airlines and China Southern Airlines accounted for 39.12\%, 40.83\% and 40.14\% of the total cost, respectively. Given fuel cost is the most influential item in the operating cost, this chapter started with the impact of system modification on fuel cost saving firstly.

The fuel cost of individual aircraft is affected by various factors (Zhu and Li, 2008), such as fuel price, aircraft’s fuel efficiency, engine fuel efficiency, aircraft’s service life, etc. The fuel cost of a single aircraft is determined by fuel price and fuel consumption, of which, the former is determined by the range of the fuel price vitality and the duration of the fluctuation, while the latter is a multi-factor variable that is subject to the aircraft itself, like its fuel efficiency, engine fuel efficiency, its service life, overall weight, flight altitude, flight speed, flight plan, flight assignment, flight duration, and the similar.

Although the fuel cost of each set is affected by many factors, for refitted system, the factors that influence the fuel economy most turn to be the fuel price and aircraft service life under other similar conditions. Fuel price is the most important item affecting fuel cost. According to annual report of China Eastern Airlines, should the average fuel price in 2015 increases or decreases by 5\%, the fuel cost would goes up or down by RMB 1.016 billion Yuan (Zheng, 2011), when taking no fuel surcharge and other factors into account. Longer service time brings higher probability of aging and failure, therefore the service life of the system may exert negative impact on the saving of weight type fuel consumption.

1.1 Prediction method of fuel cost fluctuation

In the economic simulation of cost, Qin et al. (2015) and Zhao et al. (2007) simulated the fuel economy of automobiles based on Monte Carlo method and analyzed the sensitivity of influencing parameters for fuel economy on this basis. Xu et al. (2005) conducted a computer simulation of power and fuel economy of an economical car by MATLAB programming using the mathematical model of the same regarding automobile power, and analyzed its parameter sensitivity. Xu et al. (2008) brought forward a fuel economy model under different air traffic control schemes and proved the accuracy of the model in order to improve the economy of
airspace operation. Jin (2012) simulated the navigation economy of the arctic route and analyzed China’s strategic choice of China for arctic route based on the simulation results. Through the analysis of above-mentioned research results, the simulation method is widely used in system reliability analysis and cost economy simulation, with reasonable results obtained wherefrom. With respect to the features of economy estimation of refitted system, the Monte Carlo Simulation can be introduced to predict and analyze the uncertain cost terms affected by random variables such as component life, maintenance time, fuel price and fuel saving amount after the system modification.

1.2 Assumption of fuel price volatility

The fluctuation of fuel oil is affected by many factors. According to such a fluctuation in international fuel price, we can make certain reasonable assumptions. According to the fluctuating trend pattern, we assume that the price volatility of aviation kerosene price $k = f(x)$ is a random value subject to multistage distribution and subject to different distribution under different possibilities. Where, through the statistical analysis of fuel prices, one can give a subjective probability distribution of fuel price volatility. For example, the fuel price volatility tends to be a normal distribution in a short term, where the expected value and variance are also a random value subject to a certain distribution. In the long run, the volatility of fuel price is violent, and the fluctuation ratio is usually a uniformly distributed random value.

2. Construction of Fuel Cost Simulation Model

2.1 Simulation of fuel price fluctuation

Since the fuel price is a random variable affected by many factors, it is necessary to make some reasonable assumptions about the fluctuation of fuel price to enable the quantitative simulation calculation of cost saving. Firstly, given the basic fact that the range of fuel price volatility is determined by the ratio and cycle of the fluctuation, the fuel price volatility of the modified system is set as a random value subject to multi-segment distribution, and the weight of each distribution function is given by experts combining their actual situation and personal experience. According to the value of weight given, the volatility could be determined by a random value generated based on the uniform distribution on U(0,1).

According to the statistical analysis of the fluctuation cycle of fuel price, we set Day $T_i$ as the average fluctuation cycle, and the fluctuation occurred at the end of Day $T_i$. Based on the fact that the system’s weight/flight-hour fuel consumption decreases with the increase of the age of the aircraft, the decrease period is set to change annually at the end of the year, and the change is a random value subject to U (a, b).

Through the above analysis, the simulation of fuel price fluctuation can be established as shown in equations (1), (2). In Eq. (1), the fuel price in phase $i$ equals to the sum of the fuel price in phase $i-1$ and the fluctuation of fuel price in phase $i$. The fuel saving of the system in phase $j$ equals to the difference between the fuel consumption saving in phase $j-1$ and phase $j$. Eq. (2) summed up the fuel cost saving in all cycles through the whole operation.

$$P_{i} = P_{i-1} \times (1 + k_i), \quad Q_{j} = Q_{j-1} \times (1 + l_j)$$

$$n = \frac{360 \times N}{T_i} - 1$$

$$CF = \sum_{i=1}^{n} P_i \times Q_i$$

Where, one year is set at 360 days and $N$ marks the modified system’s operating period; $n$ is the times of fluctuation in the operating period; $CF$ is the weight fuel cost saving; $P_i$ is the fuel price of fluctuation in phase $i$; $Q_i$ is the value of fuel consumption saving in phase $i$ in the year $j$; $k_i$ is the value of the fluctuation of fuel price in phase $i$; $l_j$ is the decrease rate of fuel consumption in weight in the year $j$.

2.2 Simulation model of fuel cost saving

Based on above-mentioned analysis, this paper developed a Monte Carlo simulation model for fuel cost saving by combining the fuel price and fuel consumption. The algorithm is described as follows:

- (1) Firstly, the initial simulation parameters should be determined; $T$ means the service time of refitted system (simulation time), $P_0$ means the initial fuel price, $Q_0$ means initial average fuel saving per period in weight and $N_T$ means the times of simulation;
- (2) To determine the distribution model of the fluctuation ratio in fuel prices $k$ and determine the parameters regarding the distribution model of the decrease rate of fuel consumption saving in weight;
• (3) To generate a random number by a random number generator in the computer in line with the
determined distribution function k and l, and is a sampled value of fluctuation ratio in fuel prices and
decrease rate of fuel consumption savings in weight;
• (4) Record t; the cumulative time of fluctuation in fuel price, calculate CF, cumulative fuel cost savings
during this period;
• (5) Determine whether t<T is true or not, if not, steps (2), (3) and (4);
• (6) Simulate the specific number of times, collect the data obtained and calculate expected value and
variance.

3. Analysis of Simulation Results

3.1 Case analysis of predicted fuel cost saving simulation

Suppose the service time of the refitted system is 10 years, 360 days per year, the weight of fuel saved in
flight is 30kg/h, the average daily utilization rate of the aircraft is 10 hours/day, totally 4.5 tons of fuel is saved
per cycle Tt =15 days, and the annual decrease rate of fuel saving in weight is subject to uniform distribution
on U(-0.1, 0); the initial fuel price is Pf0 =468.75 (USD/ton) and the fuel price fluctuates once every Tt =15 days,
then the fluctuation ratio of fuel price is subject to distribution as follows:

\[
k \sim \begin{cases} 
N_1(0.02, 0.02^2), & P = 0.2 \\
N_2(0.05, 0.02^2), & P = 0.2 \\
U_1(0.1, 1.5), & P = 0.1 \\
U_2(-0.15, -0.1), & P = 0.1 
\end{cases}
\]

If the aircraft runs for 1,000 times as simulated, the results are shown in Table 1 and Figure 1; Pf is the
expected value of fuel price, D(Pf) is the variance of fuel price, CF is the expected value of fuel cost saving,
and D(CF) is the variance of fuel cost saving, unit (USD 10,000).

According to the results of simulation for 1,000 times as shown in Figure 1 (a), the overall trend of fuel price
fluctuation is decreasing, which is related to the determined distribution of the assumed fluctuation ratio of fuel
price. If more sufficient and more accurate information is acquired, a more realistic fuel price forecast is more
likely to be obtained. According to Table 1, the actual fuel saving rate of the system will decline with its
increasing service life, so the annual fuel cost saving will also decrease.

### Table 1: Expected value and variance of fuel price and fuel cost saving

<table>
<thead>
<tr>
<th>Time</th>
<th>Pf</th>
<th>D(Pf)</th>
<th>CF</th>
<th>D(CF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st year</td>
<td>0.04688</td>
<td>0.0001500</td>
<td>5.04958</td>
<td>0.6521</td>
</tr>
<tr>
<td>2nd year</td>
<td>0.04623</td>
<td>0.0003571</td>
<td>9.82094</td>
<td>4.9637</td>
</tr>
<tr>
<td>3rd year</td>
<td>0.04596</td>
<td>0.0004727</td>
<td>14.3313</td>
<td>15.7735</td>
</tr>
<tr>
<td>4th year</td>
<td>0.04557</td>
<td>0.0006601</td>
<td>18.5557</td>
<td>34.6829</td>
</tr>
<tr>
<td>5th year</td>
<td>0.04531</td>
<td>0.0008874</td>
<td>22.5538</td>
<td>63.4786</td>
</tr>
<tr>
<td>6th year</td>
<td>0.04525</td>
<td>0.0011573</td>
<td>26.3497</td>
<td>103.9908</td>
</tr>
<tr>
<td>7th year</td>
<td>0.04452</td>
<td>0.0014876</td>
<td>29.9331</td>
<td>155.8535</td>
</tr>
<tr>
<td>8th year</td>
<td>0.04501</td>
<td>0.0019552</td>
<td>33.3277</td>
<td>216.6565</td>
</tr>
<tr>
<td>9th year</td>
<td>0.04470</td>
<td>0.0022037</td>
<td>36.5613</td>
<td>290.8323</td>
</tr>
<tr>
<td>10th year</td>
<td>0.04473</td>
<td>0.0024227</td>
<td>39.5927</td>
<td>373.5397</td>
</tr>
</tbody>
</table>

Under the influence of fuel price Pf and fuel saving quantity Qj of the system per cycle, the annual fuel cost
saving varies with the increasing service life. When the increase of cost saving resulted from the increased
fuel price is larger than the decrease of cost saving resulted from the decreased annual fuel saving quantity of
the system, the fuel cost saving of the system will increase; otherwise, it will decrease.

According to the fluctuation ratio of fuel price and the fuel cost saving variance curve shown in Figure 1 (b)
and (d), as simulation time increases, the variance fluctuates dramatically and the added value of variance
also presents an increasing trend, which indicates that the accuracy of forecast will decrease as the time for
simulation extends. When the expected fuel cost saving in simulation is taken as a reference for assessment
and decision-making, the risk must be considered.
3.2 Influence of fluctuation ratio of fuel price on fuel cost saving

Suppose the service time of the refitted system is 10 years, 360 days per year, the weight of fuel saved in flight is 30kg/h, the average daily utilization rate of the aircraft is 10 hours/day, totally 4.5 tons of fuel is saved per cycle $T_t = 15$ days, and the annual decrease rate of fuel saving in weight is subject to uniform distribution on $U(-0.1, 0)$; the initial fuel price is $P_0 = 468.75$ (USD/ton) and the fuel price fails to fluctuate every $T_t = 15$ days, then the fluctuation ratio of fuel price $k$ is subject to distribution as follows: unit (USD 10,000), $CF$ is the fuel cost saving, and $D(CF)$ is the variance of fuel cost saving:

$$
\begin{align*}
&k_1 \sim \begin{cases} 
N_1(0.02, 0.02^2), & P = 0.2 \\
N_2(0.05, 0.02^2), & P = 0.2 \\
N_3(-0.03, 0.02^2), & P = 0.4 \\
U_1(0.1, 0.15), & P = 0.1 \\
U_2(-0.15, -0.1), & P = 0.1 
\end{cases} \\
&k_2 \sim \begin{cases} 
N_1(0.02, 0.02^2), & P = 0.2 \\
N_2(0.05, 0.02^2), & P = 0.2 \\
N_3(-0.03, 0.02^2), & P = 0.2 \\
U_1(0.1, 0.15), & P = 0.1 \\
U_2(-0.15, -0.1), & P = 0.1 
\end{cases} \\
&k_3 \sim \begin{cases} 
N_1(0.02, 0.02^2), & P = 0.2 \\
N_2(0.05, 0.02^2), & P = 0.2 \\
N_3(-0.03, 0.02^2), & P = 0.3 \\
U_1(0.1, 0.15), & P = 0.15 \\
U_2(-0.15, -0.1), & P = 0.15 
\end{cases} \\
&k_4 \sim \begin{cases} 
N_1(0.02, 0.02^2), & P = 0.2 \\
N_2(0.05, 0.02^2), & P = 0.2 \\
N_3(-0.03, 0.02^2), & P = 0.4 \\
U_1(0.1, 0.15), & P = 0.15 \\
U_2(-0.15, -0.1), & P = 0.15 
\end{cases}
\end{align*}
$$

Table 2: Fuel cost saving under different distribution functions of fluctuation ratio of fuel price

<table>
<thead>
<tr>
<th>$k$</th>
<th>$CF$</th>
<th>$D(CF)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>k1</td>
<td>39.5927</td>
<td>373.5397</td>
</tr>
<tr>
<td>k2</td>
<td>40.8998</td>
<td>438.6750</td>
</tr>
<tr>
<td>k3</td>
<td>41.3498</td>
<td>689.2063</td>
</tr>
<tr>
<td>k4</td>
<td>41.4683</td>
<td>907.2918</td>
</tr>
</tbody>
</table>

The distribution of different fluctuation ratios of fuel price has significant influence on fuel simulation results. As shown in Figure 2 (a) and (b), k1, k2 and k3, k4 are two sets of fluctuation ratio distribution functions with the same piecewise function and different weighted values. The more dramatic the change of price fluctuation ratios is, the more significant the change in the expected fuel price and variance will be. It can be seen from Figure 2 (c) and (d) that the contact ratio of fuel cost saving variance curve which includes the same piecewise function and different weighted values of fluctuation ratios of fuel price is higher than that of the fuel cost saving curve which includes different piecewise functions of fluctuation ratios of fuel price, which means that
correctly estimating the piecewise function of fluctuation ratios of fuel price is an important part to accurately forecast the value and determines the probabilistic risk of fuel cost saving.

Figure 2: Fuel price and fuel cost saving at different fluctuation ratios of fuel price \( k \)

### 3.3 Influence of fluctuation cycle of fuel price on fuel cost saving

Suppose the above conditions remain unchanged, the fluctuation ratio of fuel price is \( k_2 \) and fluctuation cycle of fuel price \( T_v \) is respectively 5, 10, 15 and 20 days, the simulation result will be as shown in Table 3 and Figure 3; unit (USD 10,000)

Table 3: Fuel cost saving and variance at different fluctuation cycles of fuel price

<table>
<thead>
<tr>
<th>( T_v )</th>
<th>( CF )</th>
<th>( D(CF) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>41.7379</td>
<td>2441.2560</td>
</tr>
<tr>
<td>10</td>
<td>40.8998</td>
<td>853.8993</td>
</tr>
<tr>
<td>15</td>
<td>41.0318</td>
<td>438.6750</td>
</tr>
<tr>
<td>20</td>
<td>40.0120</td>
<td>278.8930</td>
</tr>
</tbody>
</table>

Figure 3: Fuel cost saving and variance at different fluctuation cycles of fuel price
It can be seen from Table 3 and Figure 3 that under the condition that the distribution function of fluctuation ratio of fuel price is determinate, the fluctuation cycle of fuel price will influence the expected value of weight fuel cost saving in the future. In the same time period, the shorter the fluctuation cycle of fuel price is, the greater the change in the expected value and variance of fuel price will be. The reason why the results exist as shown in Figure 3 (a) and (b) is that the expected fluctuation ratio of fuel price is not zero. It can also be seen that the expected value and variance curve of fuel price at $T_v=5$ fluctuates dramatically after it fluctuates for more than 500 times, which means that the longer the time the simulation lasts, the greater the risk of prediction result will be. This problem can be resolved by increasing the number of simulations.

Under the condition that the fuel consumption is determinate, the fuel price fluctuation determines the variation trend of fuel cost saving. As shown in Figure 3 (d), by comparing the two variance risks at $T_v=5$ and $T_v=20$, it can be seen that the variance risk of fuel cost saving at $T_v=5$ is much greater than that at $T_v=20$, which indicates that the fluctuation cycle of fuel price is the biggest factor influencing the probability risk of fuel cost saving.

4. Conclusion

From the MATLAB simulation results, we can tell that when predicting the future weight fuel cost saving, it is necessary to focus on the main impact of the volatility distribution of fuel price and the fluctuation cycle of fuel price on fuel economy. The distribution of volatility mainly affects the expected value of fuel cost savings. The variation of both of them determines the probability risk of simulation results. The model presented a sound evaluation over the expectation and variance risks of fuel cost saving under future environmental fluctuations to cover the shortage of current studies.

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