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A Linear Mathematical Formulation to Minimize Intermediate Fluid Flow in Batch Heat Exchanger Networks

Nitin Dutt Chaturvedi

Department of Chemical and Biochemical Engineering, Indian Institute of Technology Patna, Bihta, Patna, Bihar, India
nitind@iitp.ac.in

In heat integration of batch process, heat storage plays an important role. In batch processes heat can be transported from a prior time zone to later time zone via heat storage. Transfer of heat in batch process from one time zone to another requires intermediate fluid. The utilization of intermediate fluid involves operating (pumping) and capital cost (heat exchanger area, piping, capital investment and space). Minimizing intermediate fluid flow reduces power required for pumping along with storage requirement. Considering these factors, reducing intermediate fluid requirement becomes important for batch processes. On the other hand minimizing utility targets remains an essential concern. In this paper a linear mathematical model is proposed to minimize intermediate fluid requirement while satisfying minimum utility requirement. Developed model is linear hence solution can be guaranteed to be globally optimal. An example is solved via developed mathematical formulation in order to illustrate the proposed method. Proposed formulation can also be used to trade-off between intermediate fluid flow requirement and utility requirements.

1. Introduction

Rising energy needs and environmental issues of today's world necessitate efficient and cost optimal way of utilization of available resources. Noteworthy research efforts have been observed in past regarding heat integration in continuous (Klemeš et al., 2013) as well as batch processes (Fernández et al., 2012). Additional constraint which needs to be addressed in batch processes is time hence techniques developed for continuous process cannot be applied to batch processes directly. In a recent work, Varbanov et al.(2016) developed a methodology to optimize process scheduling and direct heat integration for batch Heat Exchanger Networks (HENs). Later, Varbanov et al. (2017) developed a procedure for the synthesis of batch water allocation networks for flexible scheduling framework.

One of the important issue which needs to be addressed for overall optimization of batch HEN is minimization of heat storage as heat storage requires space and capital investment. Sadr-Kazemi and Polley (1996) worked on heat storage design for batch HENs and inferred that a higher flexibility is provided by heat storage in comparison to direct heat recovery. Later, Krummenacher and Favrat (2000) developed a technique based to reduce heat storage units. Recently, Löffler (2017) demonstrated the application of batch processes to heat engines and showed potential improvements. In a important work, Shahane et al. (2017) formulated a model to design combined direct and indirect batch HEN. In another recent work, Anastasovski (2017) proposed an approach to design of units for heat storage in batch HEN. Minimizing volume of heat transfer medium can also be considered in order to minimize heat storage.

In indirect heat integration, heat from hot stream to cold stream is transported via intermediate fluid. For overall optimization of batch HEN minimization of intermediate fluid is an important aspect. Minimizing intermediate fluid flow reduces heat exchanger area, storage requirements, power required for pumping and size of piping. In this paper a linear mathematical model is proposed to minimize intermediate fluid requirement while satisfying minimum utility requirement.

2. Problem Statement

In a batch HEN, a general problem for minimizing intermediate fluid flow among intervals while satisfying minimum hot and cold utility requirement is pointed next:

- A set of hot streams (H_i) is given with their initial (T^s) and final (T^T) temperatures. Each stream exists for fixed time duration.
- A set of cold streams (C_j) is also given with their initial (T^s) and final (T^T) temperatures. Each stream exists for fixed time duration.
- The objective is to determine the minimum intermediate flow among intervals required to transport heat between different time intervals in order to achieve minimum external utility requirements. Note that heat losses along with time for transfer of heat are neglected.

3. Mathematical formulation

Each of the streams is sub-divided into sub intervals of time (say $E_1, E_2, E_3\ldots$) so as to accommodate end points of time durations of all cold and hot streams within the edges of these time intervals. The proposed formulation is based on Modified Grand Composite Curve (MGCC) of time intervals of batch process proposed by Chaturvedi and Bandyopadhyay (2014). The MGCC is generated by further shifting of hot and cold streams of Grand Composite Curve (GCC) (Cold streams up by ΔT_{min} /2 and hot streams down by ΔT_{min} /2). Initially the time horizon of the batch process is divided into interval (say $E_1, E_2, E_3\ldots$) so as to accommodate end points of time durations of all streams within the end at these points of these time intervals. Then, for a time interval E_k, its GCC is created via considering all streams in interval E_k, and next GCC is modified to create MGCC. In order to create MGCC from GCC positive slope segments of GCC are to be shifted upward by $1/2\Delta T_{\min}$ and negative slope segments of GCC are shifted downward by $1/2\Delta T_{min}$ and then intersecting portions produced by the shifted segments are removed.

Every segment of MGCC of a time interval may be assumed to be a pseudo-stream of that interval. Pseudo-streams of an interval can transfer its heat to subsequent interval using intermediate fluid. A thermodynamically equivalent heat exchanger with zero additional temperature driving force may be considered (Bade and Bandyopadhyay, 2014).

Let Z_k^S and Z_k^D stand for the number of sources and demands in an interval k. Heat capacity of the i^{th} source of the k^{th} interval at temperature T_{ik}^s is CP_{ik}^s , and CP_{jl}^d is the heat capacity of the j^{th} demand of the l^{th} interval at temperature T_{jl}^d . The heat capacity rate of the i^{th} source of the k^{th} interval transferred to the j^{th} demand of the l^{th} interval is represented as CP_{ikjl}. However, source in k^{th} interval can supply to a demand in l^{th} interval if and only if $k \le l$.

The mathematical model of the problem comprises the following sets, variables, parameters, and constraints:

Sets: - Batch process is segregated into time intervals, sources and demands of each individual time interval are sets for the formulation.

$E = \{e / e = \text{Intervals in total time horizon}\}$

$S_k = \{s_{ik} / s_{ik} = \text{Source of time interval } k\} \qquad \forall k \in E$

$D_l = \{d_{jl} / d_{jl} = \text{Demand of time interval } l\} \qquad \forall l \in E$

$ST_k = \{st_{ik} / st_{ik} = \text{set of streams in time interval } k\} \qquad \forall k \in E$

$H_k = \{h_{ik} / h_{ik} = \text{set of negative slope streams in time interval } k\} \qquad \forall k \in E$

$C_k = \{c_{ik} / c_{ik} = \text{set of positive slope streams in time interval } k\} \qquad \forall k \in E$

$D_l = \{d_{jl} / d_{jl} = \text{Demand of time interval } l\} \qquad \forall l \in E$

Parameters: - Following are parameters for the formulation

T_{ik}^s	Temperature of i^{th} source of time interval k
T_{jl}^d	Temperature of j^{th} demand of time interval l
CP_{ik}^s	Heat capacity value of i^{th} source of time interval k
CP_{jl}^d	Heat capacity value of j^{th} demand of time interval l

This is to be noted that the temperature of sources and demands are that of pseudo streams which are obtained from MGCC of each time interval.

Variables: -
Following variables are defined for the formulation:

CP_{ikjl}	Heat capacity transfer from i^{th} source of interval k to a j^{th} demand of interval l
HU_{jl}	Total external hot utility required for j^{th} demand of interval l
CU_{jl}	Total external cold utility required for j^{th} demand of interval l
F	Total heat capacity required for heat transfer
M	Total minimum utility requirement

Note that all the above variables need to be positive.
Constraints: Following constraints heat capacity balance, heat balance needs to be taken into account.
Heat Capacity Balance for demands: - For a positive slope stream in interval *l.* The heat can be supplied in same intervals or from the negative slope streams of previous intervals and similarly for a negative slope stream in interval *l* heat is stored and then supplied to streams of subsequent intervals. Eq(1) and Eq(2) balances total heat capacity transferred from each source in different intervals to a particular demand *j* in interval *l*.

$$\sum_{k<l}\sum_{i\in H_k} CP_{ikjl} + \sum_{k=l}\sum_{i\in ST_k} CP_{ikjl} = CP_{jl}^{d} \qquad \forall j \in C_k \tag{1}$$

$$\sum_{k>l}\sum_{i\in C_k} CP_{ikjl} + \sum_{k=l}\sum_{i\in ST_k} CP_{ikjl} = CP_{jl}^{d} \qquad \forall j \in H_k \tag{2}$$

Heat Capacity Balance for Sources: - Eq(3) and Eq(4) balances the availability of heat capacity from a particular source that can be transferred to demands in different intervals.

$$\sum_{k>l}\sum_{i\in C_k} CP_{ikjl} + \sum_{k=l}\sum_{i\in ST_k} CP_{ikjl} = CP_{jl}^{s} \qquad \forall j \in H_k \tag{3}$$

$$\sum_{k<l}\sum_{i\in H_k} CP_{ikjl} + \sum_{k=l}\sum_{i\in ST_k} CP_{ikjl} = CP_{jl}^{s} \qquad \forall j \in C_k \tag{4}$$

Energy Balance equation: -Eq(5) and Eq(6) provides heat balance for each demand of time interval. Note that, hot and cold utility need to include in this equation.

$$\sum_{k<l}\sum_{i\in H_k} CP_{ikjl}T_{ik}^{s} + \sum_{k=l}\sum_{i\in ST_k} CP_{ikjl}T_{ik}^{s} + HU_{jl} - CU_{jl} = CP_{jl}^{d}T_{jl}^{d} \qquad \forall j \in C_k \tag{5}$$

$$\sum_{k>l}\sum_{i\in C_k} CP_{ikjl}T_{ik}^{s} + \sum_{k=l}\sum_{i\in ST_k} CP_{ikjl}T_{ik}^{s} + HU_{jl} - CU_{jl} = CP_{jl}^{d}T_{jl}^{d} \qquad \forall j \in H_k \tag{6}$$

Minimum Utility Target: -
Eq(7) ensures the minimum utility requirement (*M*). The value of total minimum requirement of utility can be determined using method given by Chaturvedi and Bandyopadhyay (2014). Also, solving the same formulation (Eqs 1-6), with the objective of minimizing total utility requirement gives the minimum utility requirement (*M)*.

$$\sum_{l}\sum_{j} HU_{jl} + \sum_{l}\sum_{j} CU_{jl} = M \tag{7}$$

Objective: - Aim of this formulation is to minimize intermediate fluid flow among intervals which is given as following equation.

$$Objective = F = \sum_{i}\sum_{k}\sum_{j}\sum_{l} CP_{ikjl} \tag{8}$$

4. Illustrative Example

The stream data for this illustration are given in Table 1 (Chaturvedi and Bandyopadhyay, 2014). Minimum allowable temperature difference for heat exchange is given to be 10 °C. Five streams of this illustration are segregated into 3 time intervals (step 1). Table 2 shows the streams interval-wise with their shifted temperatures. Next, GCC of the three intervals are generated and their individual utility requirement are calculated, GCC of the three intervals are modified to generate MGCCs of these intervals.

Each segment of MGCC of a time interval may be assumed to be a pseudo-stream of that interval. Pseudo-streams of an interval can transfer its heat to subsequent interval using intermediate fluid. The total utility requirement is calculated to be 33 kWh (Eq(7)). The problem of minimizing intermediate fluid is then solved as linear programming model (Equation 1-6 as constraints) to minimize total intermediate flow (Eq(5) as objective). The model was solved by the GAMS/ MINOS solver on the computer (Intel(R) Core(TM) i7-4790, 3.6 GHz and 4 GB RAM). The solution time is within fraction of seconds. The total intermediate flow is calculated to be 0.565 kWh/°C. The size of this LP problem was 49 continuous variables. Figure 2 shows heat exchanger network for this example. In Figure 2 dotted line shows heat exchange via intermediate fluid, bold italic values are heat capacities when streams are split. The numbers in boxes shows heat duty in kWh.

Table 1: Stream data for illustrative example

Stream name	MCP (kW/ºC)	Target (ºC)	temp.Supply temp.(ºC)	Start time (h)	End time (h)
C1	9	135	20	0.1	0.3
H1	4	60	170	0.0	0.3
C2	8	140	80	0	0.1
H2	3	30	150	0.0	0.3
H3	1	35	155	0.3	0.4

Table 2: Interval wise data with shifted temperatures for illustrative example

E_1(0.0-0.1h)			E_2(0.1-0.3h)			E_3(0.3-0.4h)		
$MCp\Delta t$ (kWh/ºC)	T^T (ºC)	T^S (ºC)	$MCp\Delta t$ (kWh/ºC)	T^T (ºC)	T^S (ºC)	$MCp\Delta t$ (kWh/ºC)	T^T (ºC)	T^S (ºC)
0.8	145	85	1.8	140	25	0.1	30	150
0.4	55	165	0.8	55	165			
0.3	25	145	0.6	25	145			

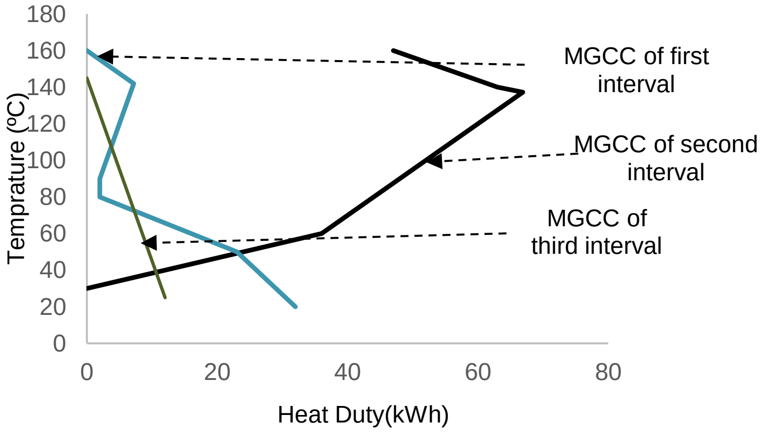

Figure 1: Modified Grand Composite Curves of first, second and third intervals

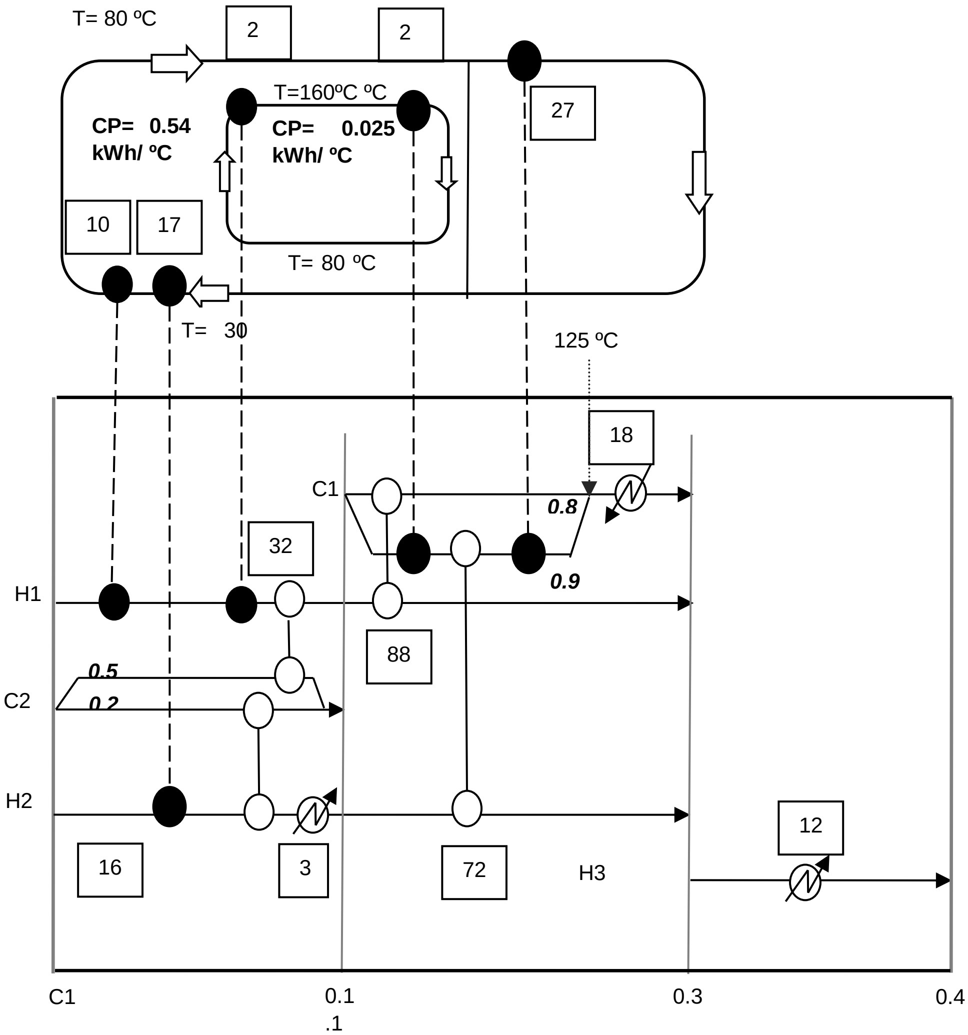

Figure 2: Heat exchanger network for illustrative example

5. Conclusions

Optimizing HEN in batch process is an important aspect of process design. Efficient utilization of space and capital investment is must for process plants. Minimizing intermediate fluid flow among intervals addresses heat transfer area optimization along with space limitation, capital investment, storage requirement and pumping costs. A mathematical formulation is proposed to calculate minimum intermediate fluid flow required among intervals while satisfying minimum utility requirements. The developed model is based on MGCC and is linear hence solution can be guaranteed to be globally optimal. Proposed formulation is based on source demand classifications of streams and also utilizes hot and cold stream classification via positive and negative slope in MGCC and incorporates both time and temperature constraints of batch processes. In addition proposed formulation also incorporates the minimum temperature difference constraints needs to be satisfied while heat transfer. An example is solved via proposed mathematical formulation in order to illustrate the developed method. In the example, the total intermediate flow is calculated to be 0.565 kWh/°C. The formulation presented in this paper can be extended to trade-off between intermediate fluid flow and hot/cold utility requirements. Current research is directed towards this issue.

Nomenclature

CP_{ikjl}	Heat capacity transfer from i^{th} source of interval k to a j^{th} demand of interval l (kWh/ºC)
CP^{s}_{ik}	Heat capacity value of i^{th} source of time interval k (kWh/ºC)
CP^{d}_{jl}	Heat capacity value of j^{th} demand of time interval l (kWh/ºC)
T^{s}_{ik}	Temperature of i^{th} source of time interval k (ºC)
T^{d}_{jl}	Temperature of j^{th} demand of time interval l (ºC)
CU_{jl}	Total external cold utility required for j^{th} demand of interval l (kWh)
HU_{jl}	Total external hot utility required for j^{th} demand of interval l (kWh)
F	Total heat capacity required for heat transfer (kWh/ºC)
M	Total minimum utility requirement
ΔT_{min}	Minimum temperature difference for feasible heat transfer (ºC)
c_{ik}	i^{th} positive slope stream in time interval k
C_k	Set of positive slope streams in time interval k
d_{jl}	j^{th} demand in in time interval l
D_l	Set of demands in time interval l
E	Set of time interval
h_{ik}	i^{th} negative slope stream in time interval k
H_k	Set of negative slope streams in time interval k
s_{ik}	i^{th} source in in time interval k
S_k	Set of sources in time interval k
st_{ik}	i^{th} stream in time interval k
ST_k	Set of streams in time interval k
Z_k^D	Number of demands in an interval k.
Z_k^S	Number of sources in an interval k.

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