CHEMICAL ENGINEERING TRANSACTIONS

VOL. 70, 2018

A publication of AIDIC

The Italian Association
of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Timothy G. Walmsley, Petar S. Varbanov, Rongxin Su, Jiří J. Klemeš

ISBN 978-88-95608-67-9; **ISSN** 2283-9216

DOI: 10.3303/CET1870164

Synthesis of a Cost-Optimal Heat Exchanger Network using Genetic Algorithm and Differential Evolution

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The heat exchanger network (HEN) synthesis problem can be characterized as non-convex, non-linear and highly combinatory. Mainly for large-scale HEN, the complexity increases even more and mathematical formulation may require very sophisticated methods. In this work, the simultaneous synthesis problem is solved by a bi-level hybrid method: in the upper level a Genetic Algorithm (GA) is used to optimized discrete variables and in the lower level, the continuous variables are optimized by Differential Evolution (DE). Two examples from the literature were used to test the applicability of the proposed solution and results found in terms of the total annual cost (TAC) were lower than those published in the literature.

1. Introduction

The greatest challenge in the synthesis of chemical processes is to confront excessive energy consumption in processing plants. The objective of HEN synthesis is to find the best network of heat exchangers that facilitates the exchange of heat between a set of hot and cold streams with a minimum cost of investment and operation costs. The HEN synthesis problem can be tackled sequentially or simultaneously. When using sequential methods, the problem is decomposed into smaller subproblems that are solved in sequence. An example of using such approach were presented by Linnhoff and coworkers and is known as Pinch Technology. Basic aspects of Pinch Technology are presented in the work of Linnhoff and Hindmarsh (1983). The reader is also referred to the Klemeš (2013) for more recent development on Pinch Technology as well as other topics of process integration. When simultaneous techniques are used the problem is solved in such a way that there is no decomposition. Simultaneous models are, generally, based on superstructures. The most used superstructure in the literature was the one developed by Yee and Grossmann (1990). This stage-wise superstructure gives rise to a mathematical programming model to simultaneously optimize the energy cost, the number of heat exchangers and the cost of capital considering the isothermal mixing at the end of each stage of the superstructure. For a comprehensive study on stage/interval-based superstructures for HEN synthesis, the reader is referred to the work of Fraser et al. (2016), who compared different superstructure-based mathematical models and the influence of approximations for logarithmic mean temperature differences (LMTD) on total annual costs.

Due to the great importance and complexity to perform HEN synthesis, solution approaches based on mathematical programming, heuristics and stochastic methods are being developed and, with computational advances, better solutions are found to complex case studies.

Stochastic methods use only information from the objective function evaluation. It is not required studies about differentiability and discontinuities are not complex to manipulate. Important examples of stochastic methods are those motivated by analogies with nature, such as Genetic Algorithms (GA), by Holland (1975), Simulated

Please cite this article as: Aguitoni M.C., Pavao L.V., Siqueira P.H., Jimenez L., Ravagnani M.A.S.S., 2018, Synthesis of a cost-optimal heat exchanger network using genetic algorithm and differential evolution , Chemical Engineering Transactions, 70, 979-984 DOI:10.3303/CET1870164

Annealing (SA), by Kirkpatrick et al. (1983), Particle Swarm Optimization (PSO), by Kenned and Eberhart (1995) and Differential Evolution (DE), by Storn and Price (1997).
Stochastic methods have found important results in HEN synthesis problem. Athier et al. (1997) presented a hybrid model which used SA to optimize binary variables and Sequential Quadratic Programming (SQP) to optimize continuous variables. Lewin (1998) used GA to optimize the continuous and discrete variables of the problem but worked with each set of variables on a different level. Yerramsetty and Murty (2008) proposed a model that includes streams splits considering non-isothermal mixing using DE algorithm to optimize the continuous and discrete variables simultaneously. Zhaoyi et al. (2013) used a GA-PSO combination, with a new version of the stage-wise superstructure of Yee and Grossmann (1990) containing stream splits only in some of the stages. Pavão et al. (2017a) worked problems using a SA in the upper level and rocket fireworks optimization (RFO) in the lower level. The effectiveness of this method was also demonstrated in the studies of Pavão et al. (2017b) in which HEN multi-objective optimization is carried out considering cost and environmental impacts.
GA and DE are uncomplicated to implement, robust, versatile and are able to handle non-linear, non-differential, non-convex, multimodal functions and work with discrete or continuous parameters. GA has been much explored in HEN synthesis problems and has presented good results. DE is still little explored for that end. The combination of these two algorithms was approached in the present work. They work together to perform the simultaneous synthesis of heat exchangers networks based on the superstructure proposed by Yee and Grossmann (1990) without considering isothermal mixing. GA works in the upper-level to find optimal HEN topologies, while DE is employed to perform continuous optimization in the lower-level, finding optimal heat loads and stream fractions to the topologies proposed by GA.

2. Mathematical model

The formulation of the problem involves the use of the stage-wise superstructure (SWS) model proposed by Yee and Grossmann (1990), which considers the non-isothermal mixing at the end of each stage. The objective is to minimize the sum of costs related to heat exchanger units and the costs of utilities used in the heat exchanger network:

$$\begin{aligned} TAC = \sum_i \sum_j \sum_k Z_{ijk}\left(CF + C.A_{ijk}^{B}\right) + \sum_i Zcu_i\left(CFcu + Ccu.Acu_i^{Bcu}\right) \\ + \sum_i Zhu_j\left(CFhu + Chu.Ahu_j^{Bhu}\right) + \sum_i CCU.Qcu_i + \sum_j CHU.Qhu_j \end{aligned} \tag{1}$$

Where for the heat exchangers there are *Z*, *A*, *CF*, *C* and B which are the binary variables representing the existence of the heat exchanger, area, fixed deployment cost, area cost coefficient and area costs exponent, respectively. Similarly, for the coolers these variables are *Zcu*, *Acu*, *CFcu*, *Ccu* and *Bcu* and for the heaters these variables are *Zhu*, *Ahu*, *CFhu*, *Chu* and *Bhu*. *CCU* and *CHU* are cold and hot utility costs, respectively. To calculate the heat transfer area, Chen's approximation (Chen, 1987) for the logarithmic mean temperature difference (*LMDT*) is used.
Some constraints are essential for the model. Here we highlight the maximum heat constraints in the heat exchangers and utilities, thermodynamic process constraints and constraints for the stream split. If these constraints are violated a variable penalty function is applied and summed to the value of the objective function.

3. Optimization approach

In the present paper a two-level optimization method for solving the HEN synthesis problem was developed. GA is used to propose new topologies at the upper-level and for each topology random populations of heat load and split-streams are generated and optimized by DE algorithm. The algorithm was implemented in Matlab (2014b), on a 3.5 GHz, intel i5 and 16 GB of RAM computer. All parameters used in GA and DE are in Table 1.

Table 1: GA and GE parameters

GA parameters		DE parameters	
Top	40	*Pop*	50
Couple	20	*F*	0.5
P_C	0.85	P_c	0.8
P_M	0.01	*iterMax*	100 or 200
iterMaxD	40		

The *Top* and *Pop* parameters describe the number of individuals (topologies for GA / heat load and split-stream for DE) in the population, the *couple* parameter is the number of crossover per generation, the P_C parameter is the crossover probability in GA and DE, the P_M parameter is the mutation probability, *F* parameter is the perturbation factor, which is a real number in the interval [0, 2] and the *iterMaxD and iterMax* parameters are the maximum number of generations in the set of discrete and continuous variables.

3.1 Upper-level (Genetic Algorithm)

Topologies are suitable for the chromosome coding of genetic algorithms. So, they are a very attractive option for working with the binary variables in HEN synthesis problems. In these problems, each chromosome represents a network structure (topology) and the set of all configurations that the chromosome generates is its search space. The topology is represented by the binary matrix $Z_{i,j,k}$ that encodes the presence/absence of the matches.

All GA parameters are initialized together with streams data and costs of the problem. The fitness for each individual in *Top* is calculated by:

$$FITNESS_d = \frac{1}{Ctotal_d} \tag{2}$$

Where $Ctotal_d$ is the total annual cost of the individual $d \in Top$. Those costs are minimized at the lower-upper by DE and are returned to GA. Such procedure is carried out until a certain number of iterations is reached. During each iteration, the principles of selection and reproduction are applied to a population of topologies. The selection determines which individuals will be able to reproduce, generating a given number of descendants that will suffer mutation and generate a population of offspring, with a probability determined by their fitness index

$$P_d = \frac{FITNESS_d}{\sum FITNESS_d} \tag{3}$$

That is, *d* individuals with greater fitness occupy a larger portion in a roulette wheel. Parents are randomly selected and will generate their children according to a probability of P_C. The mutation in each of the genes of each chromosome (children) is determined by a fixed probability P_M. The population of children has their fitness calculated. The new population that survives to the next generation will be composed by the fittest individuals (least cost) among all (parents and children). In the next generation, this whole procedure is repeated until a maximum number of iterations is reached (*iterMaxD = 40)*.

3.2 Lower-level (Differential Evolution)

The strategy of differential evolution applied is *DE/rand/1/bin.* This means that the vector to be disturbed is of the “rand” type (randomly generated), the number of weighted differences for the disturbance is 1 and the type of crossing is binomial. For each chromosome (topology), initial populations with random values of heat loads (*F*) and stream split fractions (*FC* and *FH*) are generated. *DE* will be applied in these populations. Initially, the fitness of each individual of the population is calculated by Eq(2) and thus there is a topology with several different solutions. In order to improve each solution, populations of heat loads and stream split fractions will undergo genetic operations. The first genetic operation is a mutation. A new chromosome *X* (heat loads and stream split fractions) is generated and each one of its *ijk* positions is determined by:

$$Xmut_{ijk}^{(p)} = X_{ijk}^{(p)} + F\left(X_{ijk}^{(w)} - X_{ijk}^{(y)}\right) \tag{4}$$

where *p, w* and *y* are distinct, randomly selected and denote the position of the individual in the populations of heat load and stream split fractions and *F* controls the amplitude of the difference between the variables. Then, the crossover is performed in each individual through Eq(5):

$$Xcross_{ijk}^{(p)} = \begin{cases} Xmut_{ijk}^{(p)}; r < P_c \vee l = p \\ X_{ijk}^{(p)}; otherwise \end{cases} \tag{5}$$

Where *r* has a uniform distribution in the interval [0,1], P_C is the probability of crossover and *l* is a random index chosen in the interval [1, *NP*], which ensures that population *Xcross* receives at least one component of population *Xmut*. The selection is made by comparing the costs coming from populations *X* with the cost coming from populations *Xcross*. The populations of heat loads and stream split fractions that give rise to the lowest costs will constitute the new population in the next generation. This will be repeated until it reaches a maximum number of generations (*iterMax = 100 or 200).*

4. Cases studies

4.1 Case 1

This case was originally proposed by Adjiman (2000) and has four process streams, being two hot and two cold ones. Problem data are presented in Table 2, where *A* is the heat exchanger area in m^2, *CHU* is the hot utility cost*, CCU* is the cold utility cost and *U* is the global heat transfer coefficient.

Table 2: Stream and cost data for case 1

Stream	Tin(K)	Tout(K)	CP(kW/K)
H1	650	370	10
H2	590	370	20
C1	410	650	15
C2	350	500	13
Hot utility	680	680	-
Cold utility	300	320	-

U = 0.5 kW/m^2 for all matches except for those involving hot utility. *U* = 0.833 kW/m^2 for matches involving hot utility. *Capital cost* = 5500+150*A*\$/y (A in m^2). *CHU* = 110 \$/(kW·y), *CCU* = 15 \$/(kW·y).

Adjiman (2000) worked with deterministic algorithms and found a minimum *TAC* of 154,997 \$/y after 315 s. Li and Miller (2004) utilized a Tabu Search (TS) algorithm and found a slightly lower cost of 154,910 \$/y, after 9,102 s. Escobar *et al*. (2014), employed a heuristic Lagrangean approach for the synthesis of multiperiod heat exchanger networks, but also tackled this case considering a single-period. Their method took 0.63 s to identify a solution with TAC of 154,892.97 \$/y. Faria et. al (2015), used the bound contraction methodology to solve a model derived from the stage-wise superstructure and found a TAC 154,995 \$/y after 250 s.
In the present paper, the number of stages in the superstructure used is *ST* = 2. The number of independent continuous variables is 16 and the number of binary variables is 8. With a total of 24 independent variables, there are 255 possible structures. The best solution was found after 39 s and has a *TAC* of 154,853 \$/y.
The best network is shown in Figure 1. Heat exchanger areas (A_{ijk}) are presented above each unit, heat loads Q_{ijk} are shown below the corresponding device and outlet temperatures (*Thoutijk* and *Tcoutijk*) are shown on the corresponding side.

Table 3: Comparison of results for case 1

	TAC(\$/y)	Time (s)
Adjiman (2000)	154,997	315
Li and Miller (2004)	154,910	9,010
Escobar *et al.* (2014)	154,893	0.63
Faria *et al.* (2015)	154,995	250
This work	154,853	39

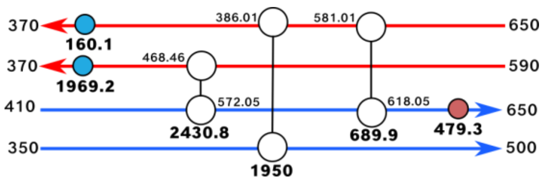

Figure 1: Optimum solution for case 1.

4.2 Case 2

This is the popular 10 SP1 problem introduced by Pho and Lapidus (1973). The problem data is presented in Table 4. The results are presented in Table 5. Two values for the annual cost of the heat exchangers were used in the literature. Therefore, the analysis of the minimum *TAC* was made considering these values.
Lin and Miller (2004) utilized a Tabu Search (TS) approach to the HEN synthesis and presented an optimal network with *TAC* = 43,329 \$/y, after 28,207 s. The authors used, for the annual cost for the heat exchangers 140 $A^{0.6}$ \$/y. Pariyani *et al.* (2006) worked with a randomized algorithm capable of sequentially generating more

flexible topologies. Using the capital cost as 145.63 $A^{0.6}$ \$/y, the TAC found was 43,661 \$/y after 10,012 s. Peng and Cui (2015) used a two-level SA algorithm for solving this example, finding TAC of 43,414 \$/y after a processing time of 3,142 s.

In the current work the number of stages in the superstructure used is ST = 2. With a total of 150 continuous variables, 50 binary variables and a total number of combinations equal to 1.13×10^{15}, the hybrid method proposed was able to find a network with TAC of 43,227 \$/y (Exchanger cost$_1$ = 140 $A^{0.6}$ \$/y) and 43,596 \$/y (Exchanger cost$_1$ = 145.63$A^{0.6}$ \$/y). The best solution is showed in Figure 2 and was found after 4,029 s.

Table 4: Stream and cost data for case 2

Stream	Tin(°C)	Tout(°C)	CP(kW/K)
H1	160	93	8.79
H2	249	138	10.55
H3	227	66	14.77
H4	271	149	12.56
H5	199	66	17.73
C1	60	160	7.62
C2	116	222	6.08
C3	38	221	8.44
C4	82	177	17.28
C5	93	205	13.9
Steam	236	236	-
Water	38	82	-

U=0.852 kW/m^2 for all matches except for those involving steam. U=1.136 kW/m^2 for matches involving steam.
Exchanger cost$_1$ = 140$A^{0.6}$ \$/y (A in m^2). Exchanger cost$_2$ = 145.63$A^{0.6}$ \$/y (A in m^2)
CHU = 37.64 \$/(kW·y), CCU = 18.12 \$/(kW·y).

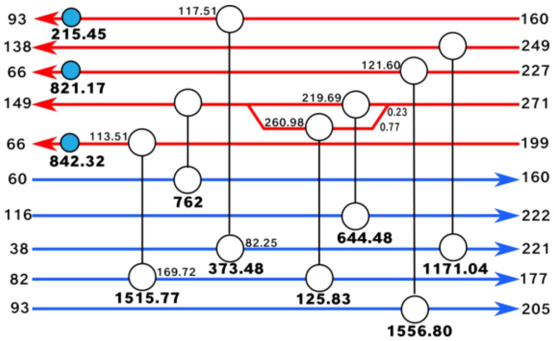

Figure 2: Optimum solution for case 2.

Table 5: Comparison of results for case 2

	TAC(\$/y)	Time (s)
Lin and Miller (2004) [1]	43,329	28,207
Pariyani et al. (2006) [2]	43,661	10,012
Peng and Cui (2015) [2,3]	43,411	3,142
This work [1]	43,227	4,029
This work [2]	43,596	4,029

[1]Exchanger cost$_1$; [2]Exchanger cost$_2$, [3] Solution with stream splits

5. Conclusions

A new hybrid method combining the GA and DE algorithms was developed to perform HEN synthesis. The stage-wise superstructure proposed by Yee and Grossmann (1990) is the model adopted considering non-isothermal mixing. The GA algorithm works at upper-level by finding optimal topologies and supplying them to the lower-level where DE acts. DE finds optimal heat loads and stream fractions for these topologies. The proposed method is applied to two case studies from the literature, a small-scale and a large-scale one. The obtained results are better than other approaches and still have a good computational performance. Case 2 was a little more explored than case 1. In case 2 the study was directed for situations with split stream. In the literature, there are two approaches that work with split stream and the difference between them is the cost of heat exchangers.

Acknowledgments

The authors would like to acknowledge the Brazilian Coordination for the Improvement of Higher Education Personnel (CAPES, Brazil) and the Brazilian National Council for Scientific and Technological Development (CNPq, Brazil) for the financial support.

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