Robust Waste Transfer Station Planning by Stochastic Programming

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Regarding the infrastructure planning in waste management, the future situation in the Czech Republic or in some other waste developing countries is unknown due to the undecided support from the government or the EU. Furthermore, the production of waste depends on many factors (e.g. separation rate) that are uncertain. The level of recycling will be influenced by technical progress while taking into account the compromise between maximum waste material recovery and the economic impact of the process. Nevertheless, it is important to plan the transportation infrastructure that will be able to support the realization of future projects/development. Such a feature might be the transfer station grid, which can decrease the total transportation costs/impact on the environment. The grid design should be robust with respect to all the possible technological solutions (establishment of waste treatment facilities). In this paper, the mathematical model for grid design of transfer stations is proposed. The model will be presented as a two-stage mixed-integer stochastic programming problem. The model will be tested through a case study on the current situation and possible legislation changes regarding waste management in the Czech Republic. The model scale will be on the micro-regional level proposing a robust transfer station grid design. The realization of these projects takes into consideration possible investments and decides about the capacity of the facility with regards to future government support and donations. The output in the form of recommendation will serve possible investors, municipalities and/or stakeholders from the field of waste management to plan more sustainable projects.

1. Introduction

Since the situation in waste management is unknown due to the undecided support to the particular technology system and treatment from the government or the EU, the planning of future infrastructure is not secured from the investment point of view. The state-of-the-art in the field of location and network flow problem is extensive. The paper by Klemes et al. (2017) is worth mentioning, because they summarised the progress in the sustainability applications from the recent year. Another important result was published by Walmsley et al. (2017), where the network was utilised for the organic and dry fractions of municipal waste through the p-graph approach. Šomplák et al. (2017) analysed the current state of the waste handling, which is an important input for the simulations of future development. However, the all the previous planning is performed globally and for all subsystems at the same time. Some papers deal with sequential development and construction as in Eiselt and Marianov (2015). The individual decisions are not robust enough to comprise the unknown future development (the problems were not handled as multi-stage as in Hrabec et al. (2015)). This paper proposes a novel approach in the planning of transport infrastructure for efficient treatment of residual waste which is in line with all the possible cases of future development of waste management system. The future uncertainty (legislative development and support for different systems) in the treatment grid design is projected through the processing cost for different facilities at various locations. The computational approach...
was designed to handle real-life tasks in reasonable time. In section 3, the case study is presented with the use of data from the Czech Republic.

2. Problem Description

2.1 Decisions, Layout and Inputs

The problem consists of deciding where to construct the transfer stations, what should be their respective capacities and from which producer of waste to which waste-processing plant should the cargo be send, provided that some of the data are uncertain. This problem can be categorized as a two-stage stochastic facility location problem (Birge and Louveaux, 1997), where the so-called first-stage decision must be made prior to the realization of the uncertainty (this corresponds to the future construction of the transfer stations and their capacities). The second-stage decision then depends on the realization of the uncertainty (in this case, all the other decisions about transport and processing are second-stage). The uncertainty is modelled using a large number of possible realizations called scenarios. The more scenarios are considered, the better the model is, but the more difficult it is to solve.

Possibly the most important data regarding this problem is the road network partly depicted in Figure 1 (and described by an incidence matrix in the mathematical model). This network had 24,770 arcs (roads) connecting the 6,258 nodes (waste producers and waste-processing plants).

The second important piece of data are the locations of the waste producers, the waste-processing plants and the possible locations for transfer stations – some of these are depicted in Figure 1. There are 6,258 places producing waste, 44 waste-processing plants (where 15 correspond to foreign facilities – potential export of waste abroad) and 116 possible places for the transfer stations (these sets were not mutually exclusive).

Figure 1: A map showing the producers of waste (blue dots) and the places processing waste (red dots). The road network (black lines) and the possible transfer stations (black rings) are shown on two separate parts.

To be able to differentiate between the transportation of waste that does or does not use the transfer stations, a separate road network was computed – for each possible transfer station was found the shortest path to each
waste-processing plant. In this pre-processing step, 5,075 shortest path optimisation problems were solved, resulting in the additional network with 5,075 arcs (omitting the ones that started and ended at the same place). The transfer of waste when using the transfer stations is assumed 3 times cheaper than the regular one. Each of the possible transfer stations can be constructed with 6 different capacities (higher capacities have higher construction costs, but the unit cost decreases). Combining this with the 116 locations results in 696 binary first-stage decisions. The second-stage decisions are the flows on the arcs of the two networks and the amounts of waste processes at the plants, in total 29,889.

The uncertain parameter that is considered in the model is the cost for processing the waste at the 44 different plants, which correspond with the legislation development and local conditions (such as the demand for heat, etc.). The number of scenarios for this model was set to 1,000 and so the model has almost 30 M variables.

2.2 Mathematical Model

The notation that is used to develop the mathematical model is described in Table 1.

<table>
<thead>
<tr>
<th>Type</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sets</td>
<td>s ∈ \mathcal{S}</td>
<td>Set of scenarios</td>
</tr>
<tr>
<td></td>
<td>j ∈ \mathcal{J}</td>
<td>Set of nodes (cities)</td>
</tr>
<tr>
<td></td>
<td>i ∈ \mathcal{I} \subseteq \mathcal{J}</td>
<td>Set of possible transfer stations</td>
</tr>
<tr>
<td></td>
<td>t ∈ \mathcal{T}</td>
<td>Set of possible transfer station capacities</td>
</tr>
<tr>
<td>Parameters</td>
<td>A_1</td>
<td>The first incidence matrix (Figure 1)</td>
</tr>
<tr>
<td></td>
<td>A_2</td>
<td>The second incidence matrix (from pre-processing)</td>
</tr>
<tr>
<td></td>
<td>c_1</td>
<td>Transfer costs, without the transfer stations (on A_1)</td>
</tr>
<tr>
<td></td>
<td>c_2</td>
<td>Transfer costs, using transfer stations (on A_2)</td>
</tr>
<tr>
<td></td>
<td>p_s</td>
<td>Probability of a scenario s</td>
</tr>
<tr>
<td></td>
<td>\theta_{i,t}</td>
<td>Cost of a construction of a transfer station at location i, with capacity t</td>
</tr>
<tr>
<td></td>
<td>k_{i,t}</td>
<td>Capacity of a transfer station at location i, with capacity t</td>
</tr>
<tr>
<td></td>
<td>f_{j,s}</td>
<td>Cost of processing waste at node j, scenario s</td>
</tr>
<tr>
<td></td>
<td>r_j</td>
<td>Production of waste at node j</td>
</tr>
<tr>
<td></td>
<td>q_j</td>
<td>Waste processing capacity of node j</td>
</tr>
<tr>
<td>Variables</td>
<td>d_{i,t}</td>
<td>Decision on building the transfer station at location i, with capacity t; binary, first-stage</td>
</tr>
<tr>
<td></td>
<td>x_{1,s}</td>
<td>Flows on A_1 in scenario s; continuous, second-stage</td>
</tr>
<tr>
<td></td>
<td>x_{2,s}</td>
<td>Flows on A_2 in scenario s; continuous, second-stage</td>
</tr>
<tr>
<td></td>
<td>y_{j,s}</td>
<td>Amount of processed waste in node j, scenario s; continuous, second-stage</td>
</tr>
</tbody>
</table>

To simplify the notation, some subscripts were hidden, meaning that the appropriate parameters/variables were stacked to form a vector of a fitting size (and the associated equalities/inequalities are meant for each element in the vector). The objective function given by Eq(1) minimizes the expected waste transportation and processing costs and the building cost for building the transfer plants:

\[
\text{minimize } \sum_{s \in \mathcal{S}} \theta_s \left( c_1^T x_{1,s} + c_2^T x_{2,s} + f_s y_s \right)
\]

(1)

The constraints then take the following form:

\[
A_1 x_{1,s} + A_2 x_{2,s} + y_s = r, \quad \forall s \in \mathcal{S},
\]

(2)

\[
y_s \leq q, \quad \forall s \in \mathcal{S},
\]

(3)

\[
\sum_{\text{flows from } i} x_{2,s} \leq \sum_{t \in \mathcal{T}} k_{i,t} d_{i,t}, \quad \forall s \in \mathcal{S}, \forall i \in \mathcal{I},
\]

(4)

\[
\sum_{t \in \mathcal{T}} d_{i,t} \leq 1, \quad \forall i \in \mathcal{I},
\]

(5)
\[ x_{1,s}, x_{2,s}, y_s \geq 0, \quad \forall s \in S, \]  
\[ d_{i,t} \in \{0,1\}, \quad \forall i \in I, \forall t \in T. \]  

The constraint Eq(2) is the conservation of waste – at each node and for each scenario, the amount produced must be equal to the amount transported (by one of the two possibilities) plus the amount processed. The constraint Eq(3) is an upper bound on the amount of waste that can be processed at a given node. The constraint Eq(4) guarantees that the amount transferred using the transfer station \( i \) is less than the installed capacity of that transfer station. The constraint Eq(5) ensures that at most one of the possible capacities is installed at location \( i \). The last two constraints Eq(6) and Eq(7) are the nonnegativity and integrality constraint, respectively. The only constraints that do not depend on the scenarios are Eq(5) and Eq(7). The total number of constraints that depend on scenarios is 36,307, meaning that the model has over 36M constraints.

3. Implementation and Results

3.1 Algorithms and Software

The model was solved using the Benders decomposition scheme described in (Kůdela et al., 2017a) enhanced by the warm-start cuts developed in (Kůdela et al., 2017b). It was programmed in the high-performance dynamic language JULIA (Bezanson et al., 2017) with the JuMP package for mathematical optimization (Dunning et al., 2017). In this scheme, the first stage problem was solved using the branch-and-cut method for mixed-integer problems, calling the CPLEX 12.6.3 solver. The MIP gap parameter was set at 1.5%. The individual subproblems in the second stage were solved by the primal-dual simplex method, calling the GUROBI 7.5 solver. This combination of solvers and algorithms achieved the best overall performance – this scheme reached the 1.5% optimality gap for the problem formulation with 1,000 scenarios within 24 h. These computations were carried out on an ordinary computer (3.2 GHz i5-4460 CPU, 16 GB RAM).

Figure 2: A map showing the results for one of the scenarios – thick black rings correspond to the selected places for transfer stations, red lines are flows from these transfer stations, black lines are regular flows. Another suitable solution strategy could be a heuristic based on genetic algorithms as in (Kůdela and Popela, 2015) or differential evolution as in (Viktorin et al., 2016). All of these strategies can utilize parallel computing to accelerate the execution.
3.2 A Summary of the Results

The results of the computation are best summarized in Figure 2 and Figure 3. Of the 116 possible locations, 71 were chosen as optimal places for the transfer stations. One scenario of optimal flows and the optimal places for the transfer stations is depicted in Figure 2 (the optimal places are the same for all scenarios, the flows are different).

The optimal expected cost was 260.14 M EUR and the expected total distance travelled by all vehicles was 8.23 M km, assuming that the regular flows are serviced by vehicles with capacity 10 t and the flows from transfer stations are serviced by vehicles with capacity 24 t (all fully loaded).

The histograms in Figure 3 represent the results for the 1,000 generated scenarios and show in detail the impact of building the transfer stations. The expected costs are 8 % lower on average when building the transfer stations, the costs for transportation alone are 21 % lower. The expected total distance travelled by all vehicles is reduced by 9 % on average when building the transfer stations. However, this quantity has a much higher variance and, in some scenarios, is worse than the situation with no transfer stations. This inconvenience stems from the objective focusing only on costs – if some form of trade-off between costs and total distance was added to the objective function, the results would be more favourable towards lower total distance (at the price of increased costs). This might represent the situation when taking into account the environmental aspects is more important than the overall cost.

Figure 3: Histograms of optimal cost and total distance travelled – the red one is using the transfer stations, the blue one is not.

4. Conclusions

In this paper, the mathematical model for grid design of transfer stations is proposed. The planning was modelled by a two-stage mixed-integer stochastic programming problem. The uncertainty is included in the cost of treatment, which corresponds to the possible future development of legislation and government support. An approach was tested through a case study on the current situation and possible legislation changes regarding waste management in the Czech Republic. It was scaled on the micro-regional level where the network had 24,770 roads connecting the 6,258 waste producers and treatment plants. With these features, the robust transfer station grid design was proposed. The realization of these projects takes into consideration possible investments and decides also about the capacity of the facility.

The output is in the form of recommendation for possible investors, municipalities and/or stakeholders from the field of waste management. The optimal solution with the 1.5 % gap was to design 71 sustainable projects, while the total expected cost was 260.14 M EUR and the expected total distance travelled by all vehicles was 8.23 M km. The possible extension for the proposed model would lead to consider the environmental aspect as the additional criterion or to calculate with the uncertain future waste production of the municipalities.
Acknowledgments

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