

# Study on the Position Control System Framework of 3R Underactuated Robot based on Fuzzy Control

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Fuzzy control theory is adopted to study the position control system framework of 3R underactuated robot. Lagrange equation is used to establish the block matrix to solve the trajectory tracking control issue. Later, fuzzy control strategy is used to further plot the control flow chart and establish the simulation model. Based on the comparison between expected value and experiment value, the maximum of related error is 2.22% and the maximum absolute error is 1.43°. The system framework designed in this article may achieve better control over the position of 3D underactuated robot.

## 1. Introduction

A mechanical system not only has passive joints, but also is provided with control input less than the degree of freedom of the system. Such system is called 3R underactuated robot. On the one hand, the system has less actuation systems, thus lowering the weight of the robot significantly and saving the cost. On the other, less actuation devices result in reduced energy consumption of the robot, which is beneficial to drive the reform of the industry. Moreover, if the actuation device of any joint fails, we may convert it into the passive joint to continue the operation. It can be seen that 3R underactuated robot has better theoretical study value and application prospective, and is a new trend for future robot research. However, the underactuated system is subject to two-order non-holonomic constraint, resulting in a great difference between a non-linear structure and general system. Since the underactuated system meets Brockett condition, i.e. no smooth status feedback control law, it is impossible to control the passive joint directly with a kinetic method.

Based on domestic and international literature, this article uses the fuzzy control theory to study the position control system framework of 3R underactuated robot. This article establishes the block matrix with Lagrange equation to solve the trajectory tracking control issue. Later, the article utilizes the fuzzy control strategy to further plot the control flow chart, and establishes the simulation model. The model as established in this article is featured by simple rule, less calculation, low operation difficulty and simple process.

## 2. Literature review

Before the mid-1990s, all the robots studied were all driven, that is, the joints of these robots were controlled by their motors respectively. The characteristic of such a robot is that the dimension of the input space (i.e. control space) is equal to the structure of the space dimension and is easy to realize control; the disadvantage is that because of the many driving devices, it causes much energy, high cost, and heavy robot, especially when the robot is a series structure, the quality of the latter drive is the former drive. Compared with the robot's terminal drive, the load of the actuator will be very large. However, the drive motor cannot be done very lightly at present. To meet the requirements of light and low consumption, some of the driving devices in the robot have been omitted in recent years. These joints degenerate into passive joints with free motion, and an underactuated robot with passive joints has emerged. The robot is a kind of mechanical system which contains passive joints and control input number is less than system freedom. It has the advantages of low cost, tight structure, good flexibility and low energy consumption. Therefore, it has important theoretical significance and broad application prospects. As the underactuated robot belongs to the two order nonholonomic constraint system, the control difficulty is greatly increased compared with the full drive robot. At present, this kind of robot has attracted more and more attention and become a new hot spot in robot research.

The underactuated robot system is a kind of special nonlinear system, which receives non-complete constraints and has special nonlinear structure. Because of the existence of non-driving joints, the system is not completely controllable. With the development of science and technology, while studying the nonholonomic system mechanics gradually, people began to apply the theory of flying integrity mechanics to many aspects. For the researchers in the field of control theory and engineering applications, the most concerned problem is how to design and apply control based on fully understanding of the characteristics of the nonholonomic system, so that the system has achieved the desired control effect according to the practicable trajectory under the inherent constraints.

Research on underactuated robots has been done home and abroad, and these research results have played a positive role in practice. Ding and others analysed the model of a kind of underactuated system, established coordinate transformation, represented it as a special chain form, and then used recursive method to stabilize the system. The proposed form is simpler than the standard recursive technique, because it avoids the use of analytical differentiation, and finally applies the results to the Acrobot system (Ding et al., 2017). Xiong uses a stable hybrid scheme to control underactuated mechanical systems. The hybrid controller includes state feedback controller and discrete event monitor. When the continuous state collides the switch boundary, a new controller is applied to the equipment. The system uses the Lyapunov theory to determine the switch boundary and ensures the stability of the closed loop system (Xiong et al., 2017). The position control of the two rotating joint plane underactuated robot is considered in the work, and the dynamic model of the system and the fuzzy control strategy are proposed. The stability of the proposed method is verified by the simulation and the experimental results. But this result cannot be widely applied to other systems, because it is only derived from practical considerations, and there is no analytical proof of stability. But it also shows that, in a case, the fuzzy logic can solve the control problem in the case that the demand for the work solution is more important than the verification of the theorem (Liu et al., 2017). Urakubo studies the stability of a class of standard chain two order nonholonomic systems. First, two typical two order nonholonomic systems, the third joint underactuated three bar planar operating arm and all joint free motion redundant manipulators, are proposed by the force / torque driven model applied on the terminal operator and converted from coordinate and input to the two ordered chain forms. Then the law of discontinuous control is proposed. When the initial state is or is driven into a specific region of the state space, all the states of the system will be exponentially stabilized to the predetermined equilibrium point. The effectiveness of the proposed control scheme is verified by simulation of a planar 3R manipulator with underactuated joints (Urakubo, 2017). Aiming at the problems in Acrobot control, a functional integrated control method based on fuzzy control, variable structure control and LQR control is proposed by Hayashi. Finally, the control strategy of Acrobot is extended to the control of the multi degree of freedom underactuated arm robot, and a control strategy of the multi degree of freedom underactuated arm robot is proposed, which provides a method for the control of the multi degree of freedom underactuated mechanical system (Hayashi et al., 2017). Based on the stability of the non-driving arm at the highest point and the lowest point, a neural network based open loop control method for a non-driven joint robot is proposed by Cerkala and Jadlovska (Cerkala and Jadlovska, 2017). Li has studied the stabilization and tracking control of uncertain nonholonomic dynamic systems. A variety of adaptive and robust feedback positive definite and trajectory tracking control strategies are proposed for uncertain nonholonomic dynamic systems and non-holonomic wheeled mobile robots. The stability of the closed loop system is proved and simulated. The control problem of this kind of system is solved well (Li et al., 2017). The underactuated two connecting rod robot is studied by Massou and Boumhidi. The hardware improvement of the robot system, the derivation and analysis of the traditional dynamic model and the simulation demonstration under the different operating system are studied (Massou and Boumhidi, 2017).

To sum up, the main work of the above research is to study the model of the underactuated robot system and the different methods applied, but there are still some problems in the field of under actuated robot control. Therefore, based on the above research status, this paper takes the underactuated 3R robot as the research object, establishes the dynamic model and analyses it, and uses the intelligent control theory to study the new method of using the robot position control in the operating space, and carries out the simulation analysis. To further the feasibility and effectiveness of the inflammatory control strategy, the underactuated experimental platform was designed and built, and the experimental research was carried out, and the control of the underactuated 3R robot was carried out in the experiment.

### 3. Method

#### 3.1 Underactuated robot kinetic model

The schematic diagram of a horizontal 3R underactuated robot is shown in Figure.1, in which the 1st joint and the 2nd joint are of active joint with an actuation device and the 3rd joint is of passive joint (or free joint) without an actuation device. Parameters are specified as follows: for joint angle, angular acceleration and joint

control torque (or control voltage), any of them in the counterclockwise direction is positive while that in the clockwise direction is negative;  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  represents three rotation angles;  $\alpha_1$  is the included angle between the connection line of joint 1 and joint 3 and the rod 3;  $\alpha_2$  is the included angle between the connection line of joint 2 and joint 3 and the rod 3; and  $\alpha_1$  and  $\alpha_2$  represents the configuration parameter of rod 3 relative to active rod 1 and active rod 2. The curve S refers to the any curve in the motion plane of the underactuated system.  $d$  is the normal distance from the robot end point to the curve S. It is specified that the normal distance from a point on the side where the origin O is to the curve S is negative while the normal distance from a point on the other side to the curve S is positive.

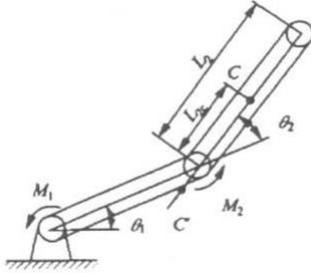


Figure 1: Schematic diagram of a horizontal 3R underactuated robot

Since the robot moves in the horizontal plane, gravity potential energy item is not taken into account. The kinetic equation established based on Lagrange equation of the second kind is:

$$M(\theta)\ddot{\theta} + F(\theta, \dot{\theta}) = \tau \quad (1)$$

Where,  $M$  is mass inertia matrix,  $F$  is an item relating to angular velocity including Coriolis, centrifugal force and friction damping,  $\theta$  is joint angle matrix, and  $\tau$  is joint actuation torque matrix. The equation (1) is expanded into a block matrix:

$$\begin{Bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{Bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{Bmatrix} + \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{Bmatrix} \tau_1 \\ \tau_2 \\ 0 \end{Bmatrix} \quad (2)$$

It can be seen from the equation (2) that the coupling relationship exists between the actuation joint acceleration and the passive joint acceleration and the motion of the passive joint may be coupled-controlled through control over various active joint. So, the coordinates of the end point of the passive joint is controlled indirectly by the input torques of two active joints, which means the trajectory tracking control issue of the underactuated robot may be solved kinetically through an appropriate control strategy.

### 3.2 Fuzzy control strategy

The motion of the underactuated robot is broken down into the rotation motion of the active joint and the extension or contraction motion of the passive joint. The extension or contraction of the passive joint is the response of the rotation motion of two active joints to the kinetic coupling of the passive joint. To realize the trajectory tracking control of end point of 3R underactuated robot, it is necessary to control the rotation of the active joint and the expansion or contraction of the passive joint to ensure that the normal distance  $d$  from the end point to the target trajectory S and the velocity at the end point is greater than zero. Here, we only consider the formation of the end point trajectory while have no quantitative requirement for the velocity and acceleration of the end point.

In order not to lose the generality, it is necessary to install the brake at the free joint. The trajectory tracking control process is divided into two stages. In the first stage, the brake is locked and the underactuated robot is degenerated into fully actuated to control the end point of active joint robot to approach the target trajectory from any initial position; in the second stage, the brake is unlocked when the end point reaches the target trajectory for the first time, the passive joint is completely free, and the underactuated control stage begins.

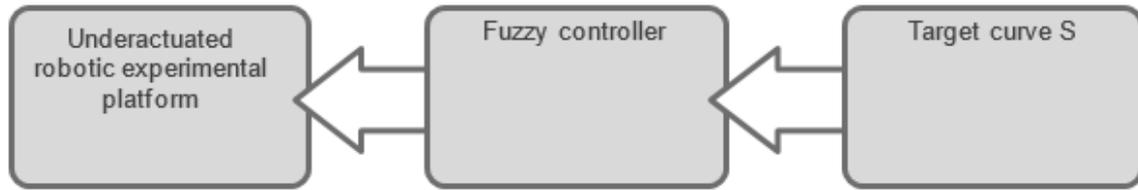


Figure 2: trajectory tracking control block diagram

The control flow chart of the underactuated control stage is shown in Figure 2. Two active joints adjust and control the deviation of both ends, respectively, and two control layers are independent from each other. Below is the specific process: when  $\tau_1$  and  $\tau_2$  are considered separately and  $\tau_1$  is calculated separately, the rod 1 and the rod 2 are relatively still and considered a whole,  $d$  and  $\dot{\alpha}_1$  are fed back as the input variable to the fuzzy controller 1 to get the input torque  $\tau_1$  of the joint 1; when  $\tau_2$  is calculated separately,  $d$  and  $\dot{\alpha}_2$  are fed back as the input variable to the fuzzy controller 2 to get the input torque  $\tau_2$  of the joint 2; finally, the motion of the end rod is controlled jointly through the kinetic coupling to enable the coordinates of the end point to track the target trajectory curve.

For Mamdani 2D fuzzy controller, the input variables are the normal distance  $d$  from the robot end to the target curve and its change rate  $\dot{\eta}$ , while the output variables are the control torques of two active joints (or control voltage), i.e.  $\tau_1$  and  $\tau_2$ .

Based on the kinetic coupling law and control experience, the fuzzy control rule may be summarized as follows: when the end error and the error change rate are positive, impose a large reverse torque on the actuated joint and allow the passive joint to generate a reverse acceleration to compensate the positive error; when the end error and the error change rate are negative, impose a large torque on the actuated joint to compensate the negative error. Other intermediary circumstances may be based on the following principles: if the error is great, mainly focus on eliminating the error by adjusting and controlling the variables; if the error is small, mainly focus on controlling the system stability by adjusting the control to avoid over-adjustment. The experiment system of the underactuated robot is shown in Figure.3. 3R underactuated robot mainly consists of two parts, i.e. mechanical structure and electric part. Since only the motion of three joints is taken into account in this experiment, the rod that is connected to the pedestal during the motion is kept still. Active joint 1 and active joint 2 adopt Maxon brushless DC servo motor and planetary reducer, while passive joint 3 is equipped with an electromagnetic clutch. Each joint is attached with a high-accuracy incremental coder (initially 10000P/R, 40000P/R following segmentation) which is used to detect the joint angle in a real-time manner. The limit switch on each rod is used to limit the rod position and play a role of limit protection. When the rod contacts the limit switch, the motor will stop. Structural parameters of the robot (rod length and mass) are same as the simulation setting values.



Figure 3: Experiment system of underactuated robot

#### 4. Results and Discussions

Figure.4 shows the angular velocity curve of joint 1. During 3 s~4 s, there is a great difference between the actual angular velocity of the joint and the theoretical angular velocity of the joint mainly because: on one hand, the issue of zero drift exists in the control card itself, i.e. the motion status generated by the same control voltage when the motor rotates clockwise is inconsistent with that when the motor rotates counterclockwise; on the other one, the motion of the rod 2 and the rod 3 has certain influence on the rod 1,

and the delay of the control system may cause the experiment curve to be lagged behind the theoretical curve generally.

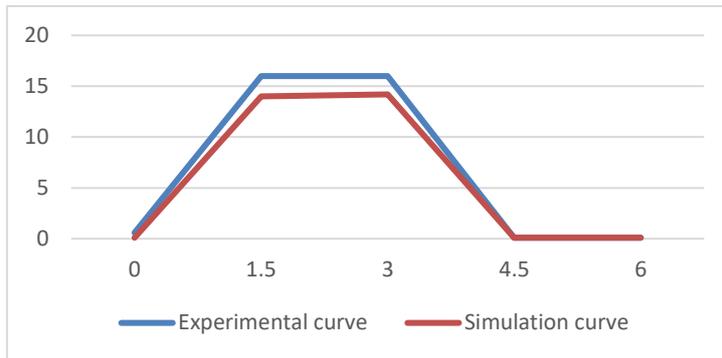


Figure 4: The angular velocity of joint 1

When the motor control voltage of the joint 2 is higher than 0.72V, the end rod has higher velocity and acceleration and moves rapidly to contact the limit switch to cause the motor to stop and result in an uncontrollable situation. Therefore, the maximum voltage is limited in the experiment, the voltage curve initially is a straight line in Figure.5. It can be seen from Figure.4 that the value of each joint becomes stable, and the curve is similar to that obtained from simulation. Compared with the simulation curve, the experiment curve is relatively lagged behind and also caused by the delay of the control system. The difference at about 3s is greatest mainly due to the influence of velocity and acceleration of the motor at 3s~4s in the active joint I. The end position of the joint converges to the expected value, and similar to the shape of the simulation curve. Parameters and errors at the final position of the experiment are shown in Table 1.

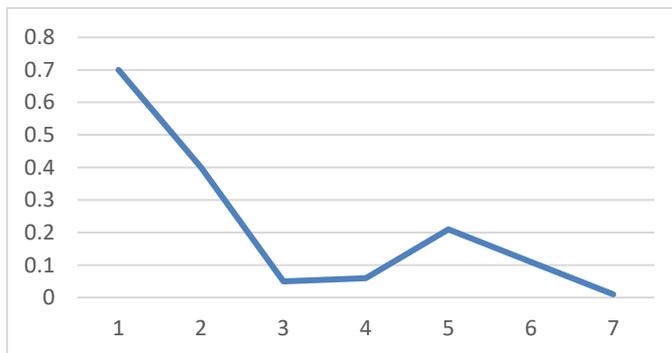


Figure 5: The control voltage of joint 2

Table 1: The experiment parameters and error analysis

	Expected value	Experimental value	Relative error
$\Theta 1$	$45^\circ$	$45.09^\circ$	0.22%
$\Theta 2$	$56^\circ$	$56.03^\circ$	0.05%
$\Theta 3$	$-52^\circ$	$-50.57^\circ$	2.74%
$\alpha$	$30^\circ$	$30.73^\circ$	2.60%
r	360	361.64	0.46%
x	270	276.00	2.22%
y	520	523.60	0.69%

It can be seen from Table 1 that compared with the expected value, the maximum relative error at the end position of the robot in the operation space is 2.22%, the maximum relative error of the joint angle in the joint space is 2.74%, the maximum relative error of the position control parameter ( $\Theta 1$ , r) during the motion is 2.60%, and the relative error on the whole is within 3%. All these values indicate that the experiment accomplishes the operation task of the robot very well and verify the validity and reliability of the theoretical

method. Compared with the simulation values, the maximum relative error at the end position of the robot in the operation space is 2.31%, the maximum relative error of the joint angle in the joint space is 3.71%, the maximum relative error of the position control parameter (01, a, r) during the motion is 1.31%, and the relative error on the whole is within 4%.

It can be also seen from the value simulation and the experiment results that, based on the same control principle, an experiment curve similar to the simulation curve is achieved by adjusting structural parameters and control parameters of the robot in value simulation and controlling necessary parameters of the robot. It is meaningful in two aspects: on one hand, the validity of the theoretical method is verified through the experiment; on the other, the results from theoretical simulation and related parameters of the robot are fundamental to future value analysis and offer guidance to future experimental research.

## 5. Conclusion

Based on a lot of references, this article adopts the fuzzy control theory to study the position control system framework of 3R underactuated robot. In this article, Lagrange equation is used to establish the block matrix to solve the trajectory tracking control issue. Later, fuzzy control strategy is used to further plot the control flow chart and establish the simulation model. The model as established in this article is featured by simple rule, less calculation, low operation difficulty and simple process. Results show that, based on the comparison between the expected value and the experiment value, the maximum of related error is 2.22% and the maximum absolute error is 1.43°. Results also show that the system framework designed in this article may achieve better control over the position of 3D underactuated robot.

Such factors as gravity needs to be taken into account for activities of 3R underactuated robot in this article since under the gravity condition, the position of 3R underactuated robot can be better controlled with lower difficulty in operation. Therefore, the control principle and simulation model in this article is more suitable for a 3D space environment.

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