

Dynamic Connectivity between Wells Based on a New Grey Correlation Degree Method

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To overcome the under injection in water-flooding treatment of low permeability reservoir, this paper introduces the grey correlation degree method to the dynamic connectivity between injection and production wells. First, it is proved that Deng's model, focusing on the proximity between sequences, does not apply to the dynamic connectivity between injection and production wells. Then, the author built a new correlation degree model which reflects the similarity between sequences, and analysed the properties of the model. The new model can reflect the negative correlation between sequences, and outperforms Deng's model in the simulation. Then, the results of Deng's model and the new model were compared based on the parameter sequences of the real injection and production wells. The results show that the results of the new model are consistent with the actual production situation. Compared with Deng's model, the new model reflect the exact difference and connectivity variation between wells.

1. Introduction

According to the Self-Assessment Guide for Oil and Gas Reserves Based on SEC Rules (Q/SY182) issued by China National Petroleum Corporation, an oil-gas reservoir (CNPC, 2007) in clastic rocks has ultra-low permeability if its permeability is less than $1 \times 10^{-3} \mu\text{m}^2$. Featuring low porosity, poor permeability, fine throat and high bound water saturation, the ultra-low permeability oil-gas reservoir obviously needs artificial modification. In foreign countries, such an oil-gas reservoir is generally classified as low permeability reservoir, and developed with gas injection. In China, however, water injection remains the main way of development for any type of oil-gas reservoirs. The basic task of water injection is to inject enough water of good quality. The success of the task depends on the following factors: water quality, water source, water treatment, injection well performance, technical measures of water injection well, etc.

Inspired by grey correlation theory, this paper introduces grey correlation degree to determine the dynamic connectivity between injection and production wells without affecting the normal production. The proposed method was proved to be effective through application.

2. Status Quo of XX Block

XX block is an ultra-low permeability block with an average porosity of about 10.5%, and an average permeability of less than $1 \times 10^{-3} \mu\text{m}^2$. The water injection in this block can be traced back to 2009. After all these years, the situation in many injection wells falls short of expectations and even gets worse. The oil production plants have taken various measures to intensify the injection, such as increasing injection pressure, acidification, fracturing, Pad-acid-hydraulic fracturing, and so on. The injection effects of these measures vary greatly from one to another. Owing to the difference in validity period, some wells failed to achieve the expected results and some even underwent decrease in the injection rate.

The most intuitive reason for the poor effect lies in the connectivity difference between injection and production wells. The injection rate will increase if the inter-well connectivity grows after the intensification treatment, and the inverse is true.

3. Grey Correlation Degree Method

The grey system theory was proposed by the Chinese scholar Deng Julong in 1982. The theory holds that "grey" imperfect information is the actual state of most real-world systems. It is mainly targeted at small samples with both known and unknown information and uncertain systems with poor information. The grey correlation analysis is a model developed by the grey system theory (Deng, 1990; Liu, 2010). The model has been extensively applied in grey system analysis, modelling, prediction and decision making (Chen and Wei, 2010; Cheng et al., 2015; Cheng et al., 2015; Cheng et al., 2016).

The grey correlation analysis weighs the correlation between different factors from the perspective of development. In other words, the development consistency of different factors is investigated after comparing the data sequences on the change trend of these factors from the geometric angle (Cao et al., 2010; Liao et al., 2011; Liu et al., 2012; Bao, 2012; Dong et al., 2015). It should be noted that the analysis results and application effect of all grey correlation analyses ultimately depend on the grey correlation degree model. There are about 20 kinds of grey correlation degree models in existing literature (Liu et al., 2013). These models are mainly created on the following bases: (1) the geometrical proximity between sequences (Zhang et al., 1996; Dang et al., 2004); (2) the change trend similarity between sequences (Liu et al., 2010; Wei, 2006); (3) both the geometrical proximity and the change trend similarity (Zhao et al., 2007; Zhou et al., 2005; Peng, 2008).

In fact, the correlation between sequences should not be discussed out of context. For instance, when a doctor diagnoses a patient, the purpose is to determine the type of disease. The way to achieve this is to compare the pattern sequence of diagnostic results with the standard pattern sequence of diagnostic criteria. If the pattern sequence is close enough to the standard pattern sequence, the doctor can determine the type of disease. In this example, the doctor focuses on the geometrical proximity between sequences, without considering the change trend similarity.

Reference (Liu, 2010) gives a good example for the identification of main influencing factors. The research is targeted at the productivity of township enterprises in China. The purpose is to find out the main influencing factors on the productivity, and support the decision-making of such enterprises. To this end, the change trend of productivity was contrasted with that of the following factors: fixed assets, current assets, labour force and retained profit. The factors sharing similar change trend with productivity were regarded as the main influencing factors. Geometrically, the similarity is the consistency between sequence curves. It is worth mentioning that the productivity sequence and factor sequence have different dimensions.

To sum up, the geometrical distance reflects the proximity of two sequences, but does not mean the two sequences share the same change trend. The shorter the geometrical distance, the higher the proximity between the two sequences, and the closer the patterns corresponding to the two sequences. By contrast, the change trend similarity helps determine the correlation degree between two sequences of different dimensions. The similarity is positively correlated with the correlation degree.

The following example is given in this research.

Let us construct the following three sequences which is shown in Figure 1:

- (1) $x_1=(1,3,2,4,6,5,7,6,8)$
- (2) $x_2=(21,23,22,24,26,25,27,26,28)$
- (3) $x_3=(3,2,3,4,3,4,6,7,6)$

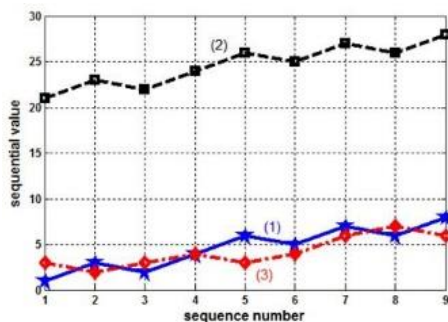


Figure 1: The graphical representation example data sequences

As shown in Figure 1, sequence (1) shares the same change trend with sequence (2), which signifies a significant positive correlation between the two sequences; sequence (1) differs greatly from sequence (3) in change trend, and the two sequences have a low correlation degree. However, sequences (1) and (3) are quite close to each other in terms of proximity. If the problem is pattern recognition, then the patterns corresponding to sequences (1) and (3) are similar, while those corresponding to sequences (1) and (2) are not.

Let us consider Deng's correlation degree model in References (Deng, 1990; Liu, 2010).

Let $X_0 = (x_0(1), x_0(2), \dots, x_0(n))$ be the reference sequence and $X_i = (x_i(1), x_i(2), \dots, x_i(n))$ be the comparison sequence. For $\xi \in (0, 1)$, it is assumed that:

$$\gamma(x_0(k), x_i(k)) = \frac{\min_i \min_k |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \xi \max_i \max_k |x_0(k) - x_i(k)|} + \frac{\xi \max_i \max_k |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \xi \max_i \max_k |x_0(k) - x_i(k)|} \quad (1)$$

Then, Deng's correlation degree model is:

$$\gamma_{0,i} = \gamma(X_0, X_i) = \frac{1}{n} \sum_{k=1}^n \gamma(x_0(k), x_i(k)) \quad (2)$$

For the above sequence, let $X_0 = x_1$, $X_1 = x_2$, $X_2 = x_3$ and $\xi = 0.5$, and obtain $\gamma_{0,1} = 0.3333$, $\gamma_{0,2} = 0.8868$.

As above, Deng's correlation degree is 0.3333 between sequences x_1 and x_2 and 0.8868 between the sequence x_1 and x_3 . It is clear that Deng's correlation degree focuses more on geometric proximity, and does not apply to similarity between sequences.

As stated in Reference (Wei, 2006), there is no correlation degree model that simultaneously demonstrate proximity and similarity. Thus, the following aspects must be considered before applying the grey correlation degree method to problem solving (Kimura et al., 2016; Casano et al., 2016): what is the exact nature of the problem, whether to focus on the proximity or the similarity, and which correlation degree model best fits in with the computational analysis.

Based on the daily production data of injection and production wells, this research aims to disclose the dynamic connectivity between injection and production wells. The investigation is based on the following basic knowledge. If the injection well is connected to the production well, then the parameters of the two wells will change in a similar way with the progress of production. The stronger the connectivity, the greater the similarity. For example, if the injection and production wells are well connected, the parameters of the production well will change in a similar way with those in the injection well; if the connectivity is bad, the production parameters will not be affected by the variation in injection parameters.

Therefore, the change trend similarity of parameter sequences between injection and production wells was taken to reflect the connectivity between injection and production wells (SHI G. et al., 2006; DENG Y. et al., 2003). Concerning the similarity parameter sequences, it is unwise to directly apply Deng's correlation degree model. Hence, the author decided to build a new correlation degree model that can reflect the similarity between sequences.

4. New Correlation Degree Model

4.1 Definition of new correlation degree model

Let us denote the parameter sequence as $X_0 = (x_0(1), x_0(2), \dots, x_0(n))$, and the comparison sequence as $X_i = (x_i(1), x_i(2), \dots, x_i(n))$, ($i = 1, 2, \dots, m$). The correlation degree is defined as follow.

If x_0 or x_i is constant sequence, then $r_{0,i} = 0$. Otherwise:

$$r_{0,i} = \frac{\sum_{k=2}^n ((x_0(k) - x_0(k-1)) \cdot (x_i(k) - x_i(k-1)))}{\sqrt{\sum_{k=2}^n (x_0(k) - x_0(k-1))^2} \sqrt{\sum_{k=2}^n (x_i(k) - x_i(k-1))^2}}$$

4.2 Properties of new correlation degree model

The new model should reflect both the change trend similarity and the cosine value of the intersection angle between vectors.

The new model should reveal the positive and negative consistencies of change trend. For sequences x_1 and x_2 (Figure 1), the correlation value is calculated as 1 by the new model. This means the new model has considered the positive consistency of change trend. In fact, the two sequences can coincide with each other through shifting. For sequences x_1 and x_3 (Figure 1), the correlation value is calculated as -0.1886 by the new model. Despite the closeness in the figure, sequences x_1 and x_3 have weak and negative consistency according to the new model. For sequences $x_4 = (1, 2, 3, 4, 5, 6, 7, 8, 9)$ and $x_5 = (9, 8, 7, 6, 5, 4, 3, 2, 1)$, the correlation value is calculated as -1 by the new model.

For each vector, the component is constituted by the difference between the adjacent components of the sequence. The intersection angle between the zero vector and any random vector of the same dimension is set to $\frac{\pi}{2}$.

Suppose $\Delta_i(j) = x_i(j+1) - x_i(j)$, $i = 0, 1, 2, \dots, m$, $j = 1, 2, \dots, n-1$ and $\Delta_i = (\Delta_i(1), \Delta_i(2), \dots, \Delta_i(n-1))$.

Then, the new correlation can be expressed as $r_{0,i} = \begin{cases} 0 & \Delta_0 \text{ or } \Delta_i \text{ is zero vector} \\ \frac{\Delta_0 \cdot \Delta_i}{\|\Delta_0\| \cdot \|\Delta_i\|} & \text{otherwise} \end{cases}$, where

$\Delta_0 \cdot \Delta_i$ is the inner product of two vectors; $\|\Delta_0\|$ and $\|\Delta_i\|$ are the vector norms.

The geometric definition of the inner product of vectors is $\Delta_0 \cdot \Delta_i = \|\Delta_0\| \cdot \|\Delta_i\| \cdot \cos(\varphi)$, where φ is the intersection angle between vectors.

Hence, if the intersection angle between the zero vector and any random vector of the same dimension is $\frac{\pi}{2}$, then there is $r_{0,i} = \cos(\varphi)$ according to the new model.

5. Application of the New Correlation Degree Model

One injection-production well group was selected from the XX Block. The well group consists of a water injection well WJW, and two production wells OW-1 and OW-2. The relative position of each well is shown in Figure 2. The record shows that the WJW is under-injected. The grey correlation degree analysis was performed based on the injection-production data collected between June, 2015 and June, 2017. The data cover the monthly injection rates of the WJW (m³/d) and the monthly production rates of OW-1 and OW-2 (m³/d).



Figure 2: Distribution of injection and production wells

Based on the above data, Deng’s model and the new model were adopted to calculate the grey correlation degree between injection and production wells, respectively. The results are listed in Table 1.

Table 1: Results of two different correlation degree models

Injection well WJW		
Wells	Deng’s correlation degree	Correlation degree of the new model in this paper
OW-1	0.620	0.252
OW-2	0.591	0.073

As shown in Table 1, Deng’s correlation degree values are greater than 0.5, which implies a very small difference. The result is contrary to the reality. The correlation degree values calculated by the new model are all relatively small, signifying a poor connectivity between injection and production wells. The values agree with the reality. The new model also reveals that the connectivity varies from well to well. The connectivity between the OW-1 and the WJW is superior to that between the OW-2 and the WJW.

To improve the water absorption capacity of the under-injected well, the CNPC Petrochemical Research Institute transformed the injection well with acidification, fracturing and other techniques. The injection well W-2 was acidized on October 9, 2014, but the validity period lasted only 6 days. Deng's model and the new model were used to evaluate the grey correlation degree between injection and production wells. The results are shown in Table 2.

Table 2: Comparison of the correlation degree between W-2 before and after acidification and the parameter sequence of O-1/2/3/4

Well Name	Well W-2			
	Before acidification		After acidification	
	Deng's model	The model in this paper	Deng's model	The model in this paper
O-1	0.4972	0.4063	0.3792	0.1420
O-2	0.5215	-0.6605	0.3953	0.0375
O-3	0.5005	0.0962	0.3931	-0.1928
O-4	0.4982	0.1303	0.4004	-0.0967

As mentioned above, the under-injected well W-2 was acidified to improve the water injection rate. However, the injection effect only lasted 6 days. At beginning of the acidification, the near-injection well area became more permeable under the corrosive effect, which in turn augmented the injection effect. The duration of augmentation was short lived because of the poor match between the acid formula and the reservoir's physical property and the strong heterogeneity of the reservoir.

According to Table 2, the results of Deng's model show that the acidification failed to enhance the injection-production well connectivity and even induced the decline of the correlation degree. The results of the new model demonstrate the plunge of the correlation degree between well W-2 and well O-1, the shift from strong negative correlation to weak positive correlation between well W-2 and well O-2, and the transition from weak positive correlation to weak negative correlation between well W-2 and well O-3 (O-4).

Through comparison, it is clear that Deng's model has a relatively weak distinction ability. The model cannot reflect the negative correlation between sequences, and its results deviate greatly from the actual connectivity between injection and production wells. On the contrary, the new correlation degree model can accurately reflect the connectivity between injection and production wells.

6. Conclusions

Through the analysis of Deng's model, this paper constructs a new grey correlation degree model for the dynamic connectivity between injection and production wells in low permeability oil reservoir. The proposed model is consistent with the cosine formula of the vector angle. It is capable of reflecting both the positive and negative correlations between sequences.

Based on the sequences of injection-production parameters, both Deng's model and the proposed model were adopted to analyse the connectivity between injection and production wells. The results of the two models were compared to verify the correctness and feasibility of our method. Both theoretical analysis and practical application show that the new grey correlation model outperforms Deng's model in discovering the dynamic connectivity between injection and production wells in low permeability oil reservoir.

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