

Effects of Nanoparticles Shape and Concentration on Natural Convection of Al₂O₃-Water Nanofluid in a Cubic Enclosure

Gang Wang*, Yao Zhang, Jin Zhang, Bingshan Ma

School of Civil Engineering, Lanzhou University of Technology, Lanzhou, 730050, China
wang_g@lut.cn

Three-dimensional numerical simulations of natural convection heat transfer of Al₂O₃-water nanofluid in a differentially-heated, cubic enclosure is performed considering nanoparticles shape. The effects of nanoparticles shape, nanoparticles volume fraction and Rayleigh number on natural convection heat transfer of Al₂O₃-water nanofluid are analysed. The results show that the thermal conductivity of nanofluid increases with the addition of Al₂O₃ rod-shaped nanoparticles, and the increasing degree of viscosity of nanofluid is higher than that of thermal conductivity. When Al₂O₃ spherical nanoparticles are dispersed in water, the strength of flow and heat transfer increases with increasing nanoparticles volume fraction. The average Nusselt number increases with the increase of nanoparticles volume fraction at $Ra=10^3$ for Al₂O₃-water nanofluid with different nanoparticles shape. When $Ra \geq 10^4$, the average Nusselt number decreases with the increasing volume fraction of Al₂O₃ rod-shaped nanoparticles, and the average Nusselt number increases with the increase of spherical nanoparticles volume fraction. The heat transfer rate increases with increasing Rayleigh number for nanofluid with different nanoparticles shape.

1. Introduction

The development of nanotechnology leads to the emergence of nanofluid which is a new kind of heat transfer fluid with higher thermal conductivity (Choi, 1995). Since then, a large number of studies on nanofluids in various aspects have been performed. These studies on nanofluids include thermophysical properties (Bubbico et al., 2015), forced convection heat transfer (Pendyala et al., 2015), natural convection heat transfer, and boiling heat transfer, etc.

For natural convection of nanofluids, Khanafer et al. (2003) appeared to be the first who investigated numerically the heat transfer enhancement of nanofluid in a differentially heated square enclosure. They showed that the suspended nanoparticles substantially increase the heat transfer rate at any given Grashof number. Recently, there are many studies on natural convection of nanofluids in enclosures with different geometries and with various boundary conditions as summarized by Haddad et al. (2012). Most of these studies also indicated that the heat transfer rate increases with the increase of nanoparticle volume fraction. However, it should be noted that the contradictory results have been reported in the literature which considered that the dispersion of nanoparticles in the base fluid may cause the deterioration of heat transfer. Putra et al. (2003) performed an experimental study of natural convection of Cu-water or Al₂O₃-water nanofluids inside a cylindrical enclosure heated and cooled from the two ends respectively. They found a systematic and definite deterioration in the heat transfer for the high Rayleigh numbers and the degree of deterioration depended on the density and concentration of nanoparticles. Ho et al. (2010) conducted an experimental investigation on natural convection heat transfer of Al₂O₃-water nanofluid in vertical square enclosures. The experimental results indicated that systematic heat transfer degradation for the nanofluid containing nanoparticles of $\phi \geq 2\%$ over the entire range of the Rayleigh number considered. Meanwhile, Li and Peterson (2010) also found the dispersion of nanoparticles in the base fluid causes the decrease in the heat transfer rate.

As seen from the above literature, most of the published papers are concerned with the analysis of two-dimensional natural convection heat transfer of nanofluids in 2D enclosures. However, studies of three-dimensional natural convection in nanofluid-filled enclosures are scarce. This paper presents a numerical

study on natural convection of Al_2O_3 -water nanofluid in a differentially-heated, cubic enclosure. The main purpose of this paper is to examine the effects of nanoparticles shape, the Rayleigh number and nanoparticles volume fraction on the natural convection characteristics.

2. Mathematical formulation and numerical method

2.1 Mathematical formulation

As shown in Figure 1, a three-dimensional cubic enclosure of side length L filled with Al_2O_3 -water nanofluid is considered ($Pr=6.2$). The left sidewall of the enclosure is maintained at a constant hot temperature T_H , while the opposite wall has a constant cold temperature T_C . Four other walls of the enclosure are adiabatic. The nanofluid is assumed to be Newtonian, incompressible and the flow is laminar. It is assumed that the base fluid (water) and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermophysical properties of the base fluid (water) and Al_2O_3 nanoparticles are given in Table 1. The thermophysical properties of the nanofluid are taken to be constant except for the density variation in the buoyancy force, which is estimated by using the Boussinesq approximation.

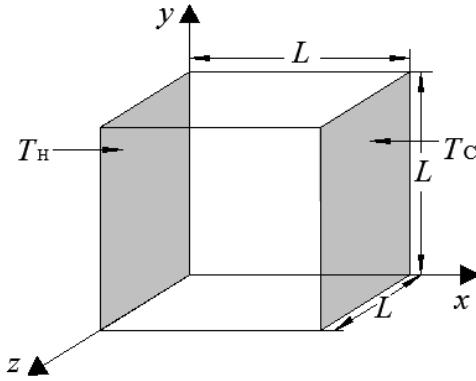


Figure 1: A schematic diagram of the physical model

Table 1: Thermophysical properties of water and Al_2O_3 nanoparticles

Thermophysical properties	ρ [kg/m ³]	c_p [J/(kg·K)]	k [W/(m·K)]	$\beta \times 10^6$ [1/K]
Water	997.1	4,179	0.613	210
Al_2O_3	3,970	765	40	8.5

Based on the foregoing assumptions, the non-dimensional governing equations for the laminar, steady-state natural convection in the cubic enclosure filled with the nanofluid can be written as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0 \quad (1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + W \frac{\partial U}{\partial Z} = -\frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2} \right) \quad (2)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + W \frac{\partial V}{\partial Z} = -\frac{\partial P}{\partial Y} + \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial Z^2} \right) + \frac{(\rho\beta)_{nf}}{\rho_{nf} \beta_f} Ra Pr \theta \quad (3)$$

$$U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} + W \frac{\partial W}{\partial Z} = -\frac{\partial P}{\partial Z} + \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left(\frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} + \frac{\partial^2 W}{\partial Z^2} \right) \quad (4)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} + W \frac{\partial \theta}{\partial Z} = \frac{\alpha_{nf}}{\alpha_f} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2} \right) \quad (5)$$

where the dimensionless variables in Eqs. (1)-(5) are defined as follows:

$$X = \frac{x}{L}, Y = \frac{y}{L}, Z = \frac{z}{L}, U = \frac{uL}{\alpha_f}, V = \frac{vL}{\alpha_f}, W = \frac{wL}{\alpha_f}, P = \frac{\bar{p}L^2}{\rho_{nf} \alpha_f^2}, \theta = \frac{T - T_0}{T_H - T_C}, Pr = \frac{\nu_f}{\alpha_f}, Ra = \frac{g \beta_f (T_H - T_C) L^3}{\nu_f \alpha_f} \quad (6)$$

where T_0 is the reference temperature and $T_0=(T_H+T_C)/2$. In Eqs. (2)-(6), the subscripts f and nf represent the base fluid and nanofluid, respectively. The effective density of the nanofluid is given by

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s \quad (7)$$

where the subscript s stands for nanoparticles and φ is the nanoparticles volume fraction. Thermal diffusivity of the nanofluid is expressed as

$$\alpha_{nf} = k_{nf}/(\rho c_p)_{nf} \quad (8)$$

where the thermal capacity of the nanofluid is given by

$$(\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_s \quad (9)$$

The thermal expansion coefficient of the nanofluid can be determined by

$$(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_f + \varphi(\rho\beta)_s \quad (10)$$

The nanofluid thermal conductivity is calculated from the Hamilton-Crosser equation (Hamilton and Crosser, 1962) as follows:

$$\frac{k_{nf}}{k_f} = \frac{k_s + (n-1)k_f - (n-1)(k_f - k_s)\varphi}{k_s + (n-1)k_f + (k_f - k_s)\varphi} \quad (11)$$

where the symbol n stands for the empirical shape factor of nanoparticles. For spherical and rod-shaped nanoparticles, $n=3$ and 6. The proposed expression of Xuan and Li (2010) is used for calculating the effective dynamic viscosity of the nanofluid. The expression is:

$$\mu_{nf} = 1 + K\varphi \quad (12)$$

where the symbol K denotes the asymmetrical degree of nanoparticles. $K=2.5$ for spherical nanoparticles and $K=80$ for rod-shaped nanoparticles.

The non-dimensional boundary conditions are as follows.

$$\begin{aligned} X=0: & \quad U=V=W=0, \theta=0.5 \\ X=1: & \quad U=V=W=0, \theta=-0.5 \\ Y=0,1: & \quad U=V=W=0, \partial\theta/\partial Y=0 \\ Z=0,1: & \quad U=V=W=0, \partial\theta/\partial Z=0 \end{aligned} \quad (13)$$

The overall heat transfer characteristics are described by the average Nusselt number at the isothermal walls ($X=0, 1$), which is defined as follow:

$$Nu = \frac{k_{nf}}{k_f} \int_0^1 \int_0^1 \frac{\partial\theta}{\partial X} \Big|_{X=0,1} dYdZ \quad (14)$$

2.2 Numerical method

The dimensionless governing Eqs.(1) - (5) along with the dimensionless boundary conditions are solved using Fluent software. The numerical calculation code is developed using SIMPLEC algorithm. The convective terms are discretized by using the QUICK scheme, and a second-order central difference scheme is used for the diffusion terms. Grid independence tests are performed using three sets of staggered, non-uniform grids: $40 \times 40 \times 40$, $60 \times 60 \times 60$, and $80 \times 80 \times 80$. The results show insignificant differences between the $60 \times 60 \times 60$ and $80 \times 80 \times 80$ grids. The $60 \times 60 \times 60$ grids is used for computations in this study. Meanwhile, in order to validate the numerical code, we solve the problem of natural convection in a cubic enclosure filled with nanofluid.

Table 2: Code validation for natural convection of water and Al_2O_3 -water nanofluid in a cubic enclosure against Ravnik et al. (2010) ($Pr=6.2$)

φ	Ra	Nu	
		Present	Ravnik et al. (2010)
0	10^3	1.081	1.071
	10^6	9.219	9.032
0.2	10^3	1.724	1.718
	10^6	10.550	10.390

As shown in Table 2, it is found that the present average Nusselt numbers are in good agreement with those reported by Ravnik et al. (2010). The above verification efforts demonstrate the robustness and accuracy of the present numerical method.

3. Results and discussion

In this part of the study, the fluid flow and heat transfer characteristics of the cubic enclosure filled with Al_2O_3 -water nanofluid is studied for a range of the Rayleigh number ($10^3 \leq Ra \leq 10^6$), nanoparticles volume fraction ($0 \leq \phi \leq 0.1$) and a choice of spherical and rod-shaped Al_2O_3 nanoparticles.

3.1 Representative velocity and temperature fields

The computed results show that the flow field in the enclosure is very complex. Both dimensionless velocities V and U are employed to describe the flow field because they represent the primary fluid motion in the enclosure. For pure water and Al_2O_3 -water nanofluids, Figures 2 and 3 present the distributions of V velocity (at $Y=0.5$) and U velocity (at $X=0.5$) in the $Z=0.5$ plane at $Ra=10^5$. When $Ra=10^5$, we can see from Figure 2 that pure water has the highest V velocity, while the flow of nanofluid slows down with increase in nanoparticles volume fraction from $\phi=0$ to 0.1. As nanoparticles volume fraction increases, it is clear from Figure 2a and Figure 2b that the V velocity of nanofluid with rod-shaped Al_2O_3 nanoparticles decreases faster than that of nanofluid with spherical Al_2O_3 nanoparticles. This is due to the fact that the viscosity of the nanofluid with rod-shaped nanoparticles is larger than that of the nanofluid with spherical Al_2O_3 nanoparticles, as shown in Equation (12). That is to say, the shape of nanoparticles has a significant effect on the flow of nanofluid and the increasing viscous resistance of nanofluid would suppress the flow of nanofluid. Furthermore, when $Ra=10^5$, we can see from Figure 3a that the U velocity of nanofluid with rod-shaped Al_2O_3 nanoparticles at $X=0.5$ and $Z=0.5$ also decreases with increasing nanoparticles volume fraction. However, as shown in Figure 3b, the U velocity of nanofluid with spherical Al_2O_3 nanoparticles changes slightly as nanoparticles volume fraction increases. Based on the above analysis, the circulation strength of the nanofluid with spherical Al_2O_3 nanoparticles is stronger than that of the nanofluid with rod-shaped Al_2O_3 nanoparticles.

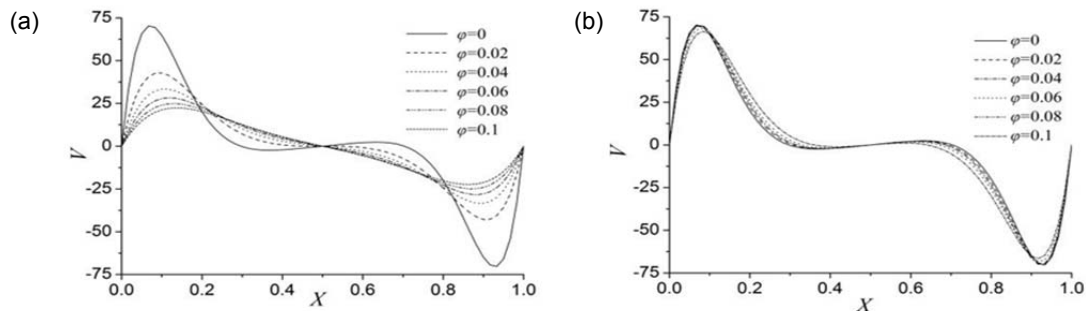


Figure 2: Velocity profiles of V at $Y=0.5$ in the $Z=0.5$ plane at $Ra=10^5$ for water and Al_2O_3 -water nanofluids with (a) rod-shaped nanoparticles and (b) spherical nanoparticles

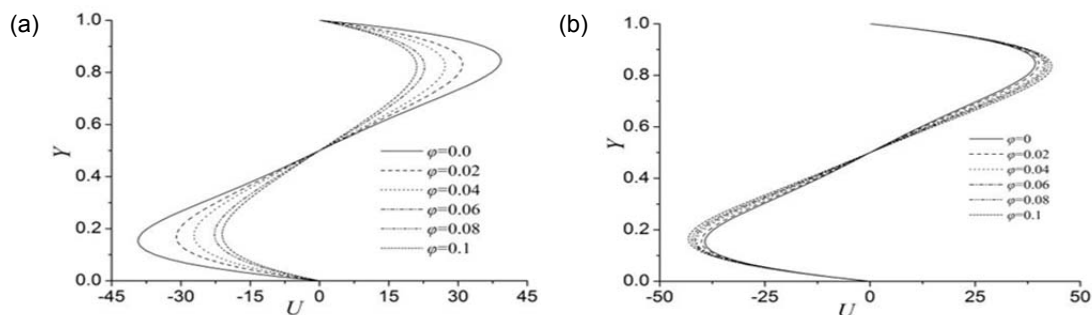


Figure 3: Velocity profiles of U at $X=0.5$ in the $Z=0.5$ plane at $Ra=10^5$ for water and Al_2O_3 -water nanofluids with (a) rod-shaped nanoparticles and (b) spherical nanoparticles

Figure 4 shows the temperature profiles at $Y=0.5$ in the $Z=0.5$ plane of the enclosure at $Ra=10^5$ for water and nanofluid. As shown in Figure 4a, the temperature gradient decreases with the increase of nanoparticles volume fraction for the nanofluid with rod-shaped Al_2O_3 nanoparticles. Consequently, it is obvious that the heat transfer rate would decrease with increasing nanoparticles volume fraction for the nanofluid with spherical Al_2O_3 nanoparticles. This is due to the increasing viscous resistance of nanofluid with rod-shaped Al_2O_3 nanoparticles suppresses the flow of nanofluid. However, for the nanofluid with spherical Al_2O_3 nanoparticles, the temperature gradient decreases changes slightly with the increase of nanoparticles volume fraction as indicated in Figure 4b. These results show that the shape of nanoparticles can affect the heat transfer characteristics of nanofluid.

3.2 Average heat transfer rate

Figure 5 displays the variation of average Nusselt number with nanoparticles volume fraction at different Rayleigh numbers for Al_2O_3 -water nanofluids with spherical nanoparticles and rod-shaped nanoparticles. As shown in Figure 5a, for Al_2O_3 -water nanofluids with rod-shaped nanoparticles, the average Nusselt number Nu increases with an increase in nanoparticles volume fraction at $Ra=10^3$. This is due to the fact that conduction dominates the heat transfer process at $Ra=10^3$ and the thermal conductivity of nanofluid increases with increasing nanoparticles volume fraction. When $Ra=10^4$, the average Nusselt number decreases first and then increases at about $\phi=0.04$ with the increase of nanoparticles volume fraction. When $Ra=10^4$, the flow of fluid is weak convection and dispersing the rod-shaped Al_2O_3 nanoparticles into the water makes the viscosity of nanofluid higher. Consequently, the average Nusselt number decreases with ϕ increasing from $\phi=0$ to 0.04. Then the average Nusselt number increases with ϕ increasing from $\phi=0.04$ to 0.1. This is because the thermal conductivity of nanofluid increases with increasing ϕ . When $Ra=10^5$ and 10^6 , the average Nusselt number decreases with the increasing nanoparticles volume fraction. This indicates that, with increasing ϕ , the increasing viscosity of nanofluid with rod-shaped Al_2O_3 nanoparticles has a more pronounced effect on the heat transfer rate. However, as shown in Figure 5b, the average Nusselt number increases with increasing ϕ for Al_2O_3 -water nanofluids with spherical nanoparticles when $Ra=10^3$, 10^4 , 10^5 , and 10^6 . This is due to the fact that, with increasing ϕ , the increasing thermal conductivity of nanofluid with spherical Al_2O_3 nanoparticles has a more significant effect on the heat transfer rate than the viscosity of nanofluid.

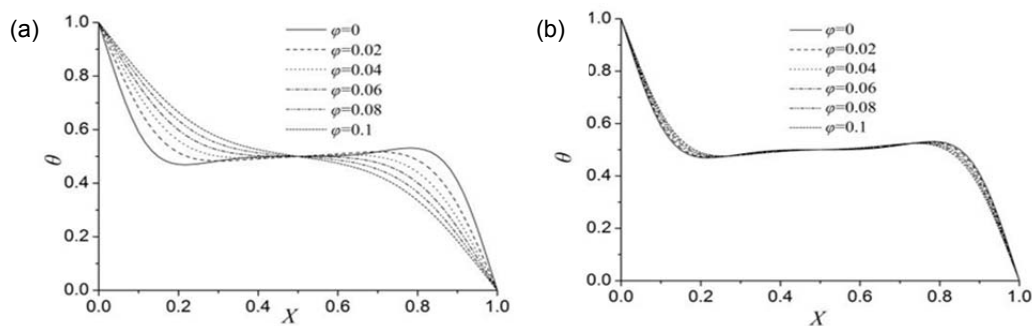


Figure 4: Temperature profiles of at $Y=0.5$ in the $Z=0.5$ plane at $Ra=10^5$ for water and Al_2O_3 -water nanofluids with (a) rod-shaped nanoparticles and (b) spherical nanoparticles

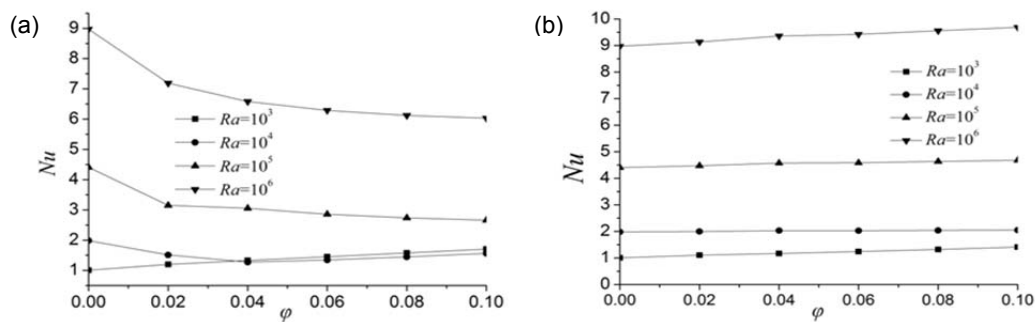


Figure 5: Variation of average Nusselt number with solid volume fraction at different Rayleigh numbers for Al_2O_3 -water nanofluids with (a) rod-shaped nanoparticles and (b) spherical nanoparticles

4. Conclusions

This paper presents a numerical study on natural convection heat transfer of Al_2O_3 -water nanofluid in a differentially-heated, cubic enclosure using the finite volume method. The study focuses on the effects of the shape of nanoparticles, nanoparticles volume fraction, and Rayleigh number on the natural convection of Cu-water nanofluid. The numerical results lead to the following conclusions. The shape of nanoparticles has a significant effect on natural convection heat transfer of Al_2O_3 -water nanofluid in the cubic enclosure. Dispersion of rod-shaped Al_2O_3 nanoparticles into water can weaken the flow strength of nanofluid and cause a decrease in the average heat transfer coefficient at higher Rayleigh numbers. However, the average heat transfer rate of the nanofluid with rod-shaped Al_2O_3 nanoparticles increases with increasing nanoparticles volume fraction at $Ra=10^3$. For the nanofluid with spherical Al_2O_3 nanoparticles, the average heat transfer rate increases with the increase of nanoparticles volume fraction for all the Rayleigh numbers considered. This indicates that using the nanofluid with spherical Al_2O_3 nanoparticles is more effective for the enhancement of natural convection heat transfer.

Acknowledgments

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