

Direct Work Exchange Networks Synthesis of Isothermal Process Based on Superstructure Method

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Properly integrating mechanical energy between high-pressure (HP) and low-pressure (LP) streams via direct work exchangers has been a significantly promising strategy to improve energy efficiency of industrial process, thus achieving energy conservation and emission reduction. This paper presents a mixed-integer nonlinear programming (MINLP) model with the objective of minimized total annual cost (TAC) to synthesize direct work exchange networks (WEN) of isothermal process. Two upgraded stage-wise superstructures with and without stream splits are developed, which explicitly include entire feasible matches, and handle all the parameters and possible network structures. The proposed superstructures are relatively different from that of heat exchange networks (HEN) because it is essential to consider the optimized selection of utility compressors/expanders in each stage, and utility compressors for LP streams prior to entering the superstructure. Ultimately, a case study is conducted to demonstrate the synthesis of direct work exchange networks based on superstructure method can offer vitally considerable savings in TAC. The results indicate that our approach yields a network with 20.0 % lower TAC and 18.8 % lower TAC, compared with that of transshipment model and graphical method.

1. Introduction

The increment of energy efficiency is of vital significance in transformation process due to its dominating responsibility for a large portion of expenditures and decisive acts on environmental aspects (Huang and Karimi, 2013). The mature field of heat recovery systems can significantly improve plant energy efficiency, which has made notable advances in reducing utility consumption in chemical process industries (Fu and Gundersen, 2016). Currently, further development of methodologies for Heat Integration are still attracting interests, including novel graphical techniques for Pinch Analysis (Gadalla, 2015), exergy or entropy analysis (Cheng and Liang, 2012) and mathematical optimization (Nussbaumer and Thalmann, 2016). However, the notion of work integration to conserve relatively more costly mechanical energy has received limited attention so far, despite the fact that work is an equally important thermodynamic parameter.

In contrast to synthesis of HEN, work exchange networks synthesis mainly focuses on the matching between HP and LP streams via indirect or direct work exchangers in addition to stand-alone compressors and expanders (Razib et al., 2012). Energy in the indirect recovery devices is exchanged in two steps, thus it should be stressed that the energy recovery efficiency of these devices is relatively low (Chen and Wang, 2012). With respect to the direct work exchangers, mechanical energy can be directly transferred from work sources to work sinks, such that recovery efficiency of a direct work exchanger is much higher (Huang and Fan, 1996). It is essential to investigate the direct work exchange networks.

Liu et al. (2014) proposed a graphical integration methodology for work exchange networks of isothermal process by plotting Composite Curves of HP and LP streams in the logarithmic pressure versus work diagram, in which two linearly approximated auxiliary lines of LP streams and five matching rules were presented to assist identifying the feasible match between HP and LP streams. Nevertheless, the linearity hypothesis may be unreasonable to search the feasible match. To address this issue, Zhuang et al. (2015) introduced the condensed transshipment model to synthesize the direct WEN applied to isothermal process by constructing intermediate pressure of LP streams for achieving the optimal WEN structure according to the formulated mixed integer linear programming (MILP) model. Moreover, on account that the minimum utility consumption was

regarded as the objective function, overestimated capital investment had to be utilized, thus resulting in a complex WEN configuration. Therefore, we need efficient formulations and design approaches for direct WEN synthesis to simultaneously consider the operation and capital expenditure.

In this paper, an MINLP model aiming at minimized TAC for direct WEN synthesis applied to isothermal process is formulated. To achieve this goal, two upgraded stage-wise superstructures with and without stream splits are proposed to intuitively contain entire feasible matches and handle all the possible network structures. In addition, the proposed superstructures are comparatively different from that of HEN because it is essential to consider the optimized selection of utility compressors/expanders in each stage, and utility compressors for LP streams prior to entering the superstructure. A case study is conducted to verify the accuracy of the presented method.

2. Problem statement

Given a set of gaseous streams at high and low pressure with known volume flows, inlet and outlet pressure, inlet temperature, as well as utilities for work (mechanical energy), a network of work exchangers, stand-alone compressors and expanders is designed to attain the desired changes for stream conditions in such a way that the total annual cost is minimized. To simplify the synthesis procedure, we adopt the assumptions commonly utilized in the previous work (Zhuang et al., 2015), as the individual equipment operation is beyond the scope of our work.

3. Model formulation

3.1 Upgraded superstructures for WEN

Direct work exchange between HP and LP streams has more sophisticated pressure constraints than the temperature constraints in heat exchange, as it is required that the outlet pressure of HP streams should be lower than the inlet pressure of LP streams while the inlet pressure of HP streams must be higher than the outlet pressure of LP streams. Obviously, the traditional stage-wise superstructure for HEN cannot be utilized to synthesize WEN. Consequently, two upgraded stage-wise superstructures with and without stream splits are proposed for WEN synthesis on the basis of the stage-wise superstructure presented by Yee and Grossmann (1990), as shown in Figure 1.

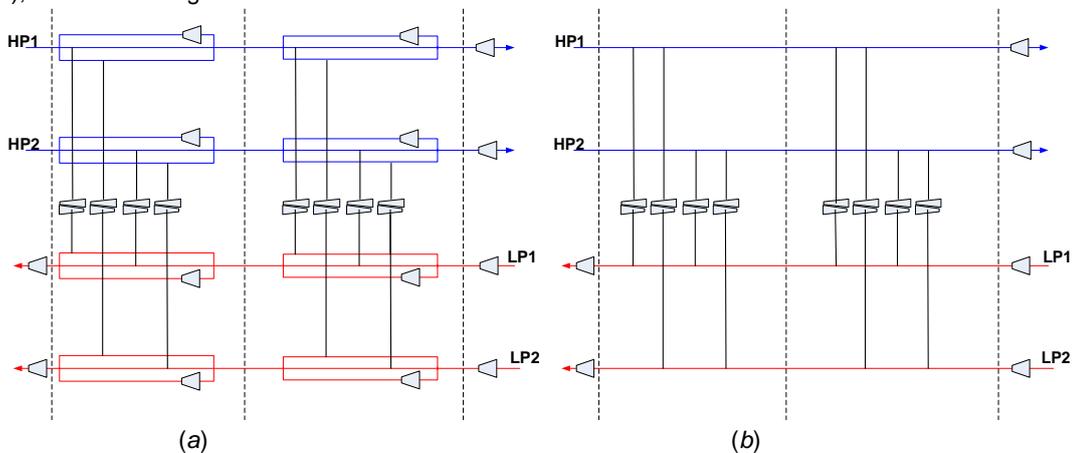


Figure 1: (a) WEN stage-wise superstructure with stream splits; (b) WEN stage-wise superstructure without stream splits. (The items in this figure represent the same meaning as illustrated in Liu et al. (2014).)

Regarding WEN stage-wise superstructure with stream splits, as illustrated in Figure 1 (a), the upgraded development is based on the following ideas:

- (1) In each stage ($k=1, 2, \dots, K$, $K = \max\{\text{the number of HP and LP streams}\}$), the parallel expanders or compressors should be allocated to achieve work balance at each stage for the purpose of meeting the requirement that the outlet pressure of LP streams is higher than the inlet pressure of HP streams.
- (2) It is necessary to allow for allocation of series compressors for LP streams prior to entering the final stage, so as to satisfy the end-pressure constraints of LP streams at the final stage.
- (3) Every single stream in each stage consists in the work exchange exists no more than once. Additionally, the same match between one HP and one LP stream should exist at most once in different stages.
- (4) The number of stream splits for HP streams is equal to the number of LP streams plus one while the number of LP streams splits is equivalent to the number of HP streams plus one. Subsequently, these sub-streams with different flow rates should be mixed at the end of each stage according to isobaric mixing.

Furthermore, relating to the superstructure without stream splits, as shown in Figure 1 (b), most ideas are the same as aforementioned, except that the downstream match depends on the work load of upstream match, which decreases the search space for subsequent matches due to the reduced pressure difference in the same stage. Besides, series utility compressors or expanders should be placed between adjacent direct work exchangers at each stage.

The detailed mathematical model is presented in the following sections, where WEN stage-wise superstructure can be generated based on the constraints and objective function below.

3.2 Constraints

In this context, the overall work balance for HP(*i*) and LP(*j*) streams is expressed by Eq(1) and Eq(2), while Eq(3) and Eq(4) denote work balance of HP and LP streams at each stage.

$$W_i = F_i \cdot R \cdot TIN_i \cdot \ln\left(\frac{PIN_i}{POUT_i}\right) = \sum_{k \in NK} \sum_{j \in LP} W_{i,j,k} + W_{HU,i} + \sum_{k \in NK} W_{HU,i,k} \quad (1)$$

$$W_j = F_j \cdot R \cdot TIN_j \cdot \ln\left(\frac{POUT_j}{PIN_j}\right) = \sum_{k \in NK} \sum_{i \in HP} W_{i,j,k} + W_{LU,j} + \sum_{k \in NK} W_{LU,j,k} \quad (2)$$

$$F_i \cdot R \cdot T_{i,k} \cdot \ln\left(\frac{P_{i,k}}{P_{i,k+1}}\right) = \sum_{j \in LP} W_{i,j,k} + W_{HU,i,k} \quad (3)$$

$$F_j \cdot R \cdot T_{j,k} \cdot \ln\left(\frac{P_{j,k}}{P_{j,k+1}}\right) = \sum_{i \in HP} W_{i,j,k} + W_{LU,j,k} \quad (4)$$

Where *HU* and *LU* represent the depressurized and pressurized utility.

For depressurized and pressurized utilities consumption, they can be calculated by the following equations.

$$W_{HU,i,t} = F_i \cdot R \cdot T_{i,K} \cdot \ln\left(\frac{P_{i,K}}{POUT_i}\right) \quad (5)$$

$$W_{LU,j,s} = F_j \cdot R \cdot TIN_j \cdot \ln\left(\frac{P_{j,K}}{PIN_j}\right) \quad (6)$$

$$W_{LU,j,t} = F_j \cdot R \cdot T_{j,1} \cdot \ln\left(\frac{POUT_j}{P_{j,1}}\right) \quad (7)$$

Where subscript '*t*' denotes target end and subscript '*s*' denotes initial end.

The assignment of superstructure inlet pressure and temperature is expressed by Eq(8) and Eq(9) according to the assumption of isothermal process.

$$TIN_i = T_{i,k} = TOUT_i, \quad TIN_j = T_{j,k} = TOUT_j \quad (8)$$

$$PIN_i = P_{i,1} \quad (9)$$

To ensure the work exchange between HP and LP streams with a more rapid work transmission rate, Eq(10) and Eq(11) show the minimum approach pressure constraints.

$$dP_{i,j,k} = P_{i,k} - P_{j,k} + \Gamma(1 - z_{i,j,k}) \geq \Delta P_{\min} \quad (10)$$

$$dP_{i,j,k+1} = P_{j,k+1} - P_{i,k+1} + \Gamma(1 - z_{i,j,k}) \geq \Delta P_{\min} \quad (11)$$

Afterwards, Eq(12) to Eq(14) express the decrease in pressures to guarantee that the pressure change along the *K* stages is monotonic.

$$P_{i,k+1} \leq P_{i,k}, P_{j,k+1} \leq P_{j,k} \quad (12)$$

$$P_{i,K} \geq POUT_i, P_{j,1} \leq POUT_j \quad (13)$$

$$P_{j,K} \geq PIN_j \quad (14)$$

The following logical constraints are necessary to promote the selection between the work-exchange equipment that will constitute the WEN,

$$W_{i,j,k} \leq \Omega_{i,j} \cdot Z_{i,j,k} \quad (15)$$

$$W_{HU,i,k} \leq \Omega_i \cdot Z_{HU,i,k}, W_{HU,i,t} \leq \Omega_i \cdot Z_{HU,i,t} \quad (16)$$

$$W_{LU,j,s} \leq \Omega_j \cdot Z_{LU,j,s}, W_{LU,j,k} \leq \Omega_j \cdot Z_{LU,j,k}, W_{LU,j,t} \leq \Omega_j \cdot Z_{LU,j,t} \quad (17)$$

where the upper bounds are as follows.

$$\Omega_{i,j} = \min\{W_i, W_j\}, \Omega_i = W_i, \Omega_j = W_j \quad (18)$$

The binary variables existing in these logical relationships ensure the feasible alternatives and constrain the search space for avoiding sub-optimal solutions or even solutions without physical meaning.

In regard to the stream-split superstructure, Eq(19) should be consistent with the third idea of the upgraded development so as to achieve the feasible match in WEN.

$$\sum_{k=1}^K z_{i,j,k} \leq 1 \quad (19)$$

In addition, as for the superstructure without stream splits, most constraints are the same as those of stream-split superstructure except that Eq(20) should be substituted for Eq(19), and Eq(21) should be added to implement the operation without stream splits.

$$\sum_{k=1}^K z_{i,j,k} \leq K \quad (20)$$

$$\sum_{j \in LP} z_{i,j,k} + z_{HU,i,k} \leq 1, \sum_{i \in HP} z_{i,j,k} + z_{LU,j,k} \leq 1 \quad (21)$$

All the variables above are non-negative.

3.3 Objective function

The objective is to minimize the total annual cost of the overall WEN configuration, composed of operation expenditures (OPEX) and capital expenditures (CAPEX), and expressed by the following Eq(22):

$$\min TAC = CAPEX + OPEX \quad (22)$$

In which,

$$CAPEX = f \cdot \left[\sum_{k \in NK} \sum_{i \in HP} \sum_{j \in LP} (C_{i,j,k}^{WE} \times z_{i,j,k}) + \sum_{i \in HP} (C_{i,t}^{Tur} \times z_{i,t}) + \sum_{i \in HP} (C_{i,k}^{Tur} \times z_{i,k}) \right] \\ + \sum_{j \in LP} (C_{j,s}^{Comp} \times z_{j,s}) + \sum_{j \in LP} (C_{j,t}^{Comp} \times z_{j,t}) + \sum_{j \in LP} (C_{j,k}^{Comp} \times z_{j,k}) \quad (23)$$

$$OPEX = \sum_{i \in HP} C_{HU} \times W_{HU,i,t} + \sum_{k \in NK} \sum_{i \in HP} C_{HU} \times W_{HU,i,k} + \sum_{j \in LP} C_{LU} \times W_{LU,j,s} \\ + \sum_{j \in LP} C_{LU} \times W_{LU,j,t} + \sum_{k \in NK} \sum_{j \in LP} C_{LU} \times W_{LU,j,k} \quad (24)$$

The cost parameters in the two equations are the same as those of Huang and Karimi (2016).

4. Case study

In this section, the case from Liu et al. (2014) is supplied with three HP and two LP streams where Table 1 lists their various properties. Afterwards, the corresponding comparison with solutions obtained by other authors is listed after the case study.

Table 1: Stream properties for the case study

Streams	Inlet pressure (P_{IN} , kPa)	Outlet pressure (P_{OUT} , kPa)	Volume flow-rate (F , $\text{Nm}^3 \cdot \text{s}^{-1}$)	Inlet temperature (T_{IN} , K)
HP1	2,000	150	1.23	525
HP2	780	180	0.57	480
HP3	780	220	0.85	420
LP1	200	700	1.85	330
LP2	200	1,600	0.83	360

According to the presented MINLP model, a three-stage superstructure with and without stream splits is introduced to deal with the case. By targeting the minimized TAC, $W_{i,j,k}$, $P_{i,k}$, $P_{j,k}$, and $dP_{i,j,k}$ are selected as decision variables to optimize the formulated model written in GAMS (version 24.0) and then solved by BARON solver. Further, the solutions gained for the case are shown in Table 2. Clearly, the corresponding optimal networks obtained can be illustrated as Figure 2.

Table 2: Solution comparison of our model with those of Liu et al. (2014) and Zhuang et al. (2015)

Methods	CAPEX (k\$/y)	OPEX (k\$/y)	TAC (k\$/y)
No integration	2,590	1,244	3,834
Graphical method	2,426	447	2,873
Transshipment Model	2,768	439	3,207
Stream-split superstructure	1,490	481	1,971
No stream-split superstructure	2,000	834	2,834

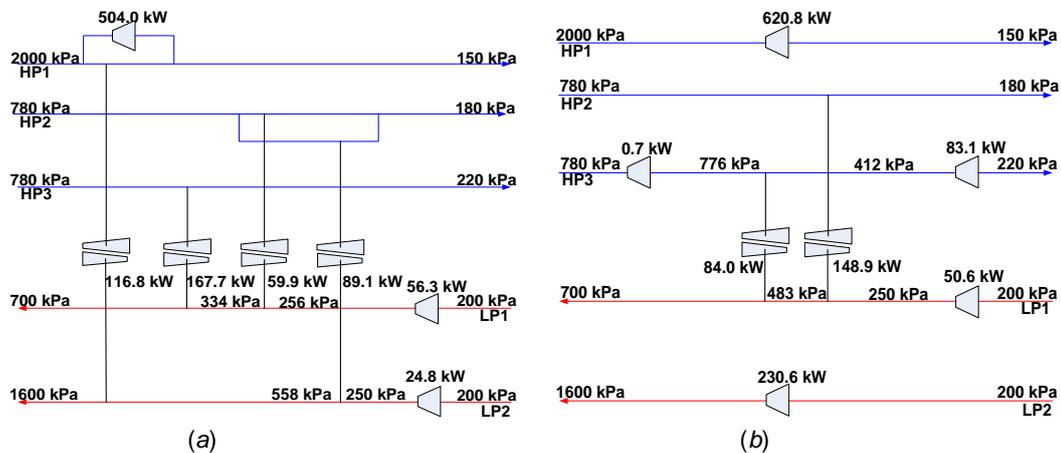


Figure 2: (a) Optimal WEN configuration with stream splits obtained for the case; (b) Optimal WEN configuration without stream splits obtained for the case.

From Table 2 and Figure 2, it can be found that the proposed model adopting stream-split superstructure yields an optimal network with a TAC of 1,971 k\$/y, a 20.0 % and 18.8 % decrease to the solutions obtained by the transshipment model and graphical method. Moreover, in respect to the superstructure without stream splits, the corresponding model acquires a relatively better solution for TAC of 2,834 k\$/y compared with TAC of previous works. Consequently, specific to this case, the optimal WEN configuration obtained by stream-split superstructure is better than that without stream splits.

5. Conclusions

The paper presents a mixed-integer nonlinear programming (MINLP) model targeting minimized total annual cost to synthesize direct work exchange networks of isothermal process. Two upgraded stage-wise superstructures with and without stream splits are developed to consider entire feasible matches, and handle all the parameters and possible network structures. The proposed superstructures are relatively different from that of heat exchange networks because it is essential to optimize selection of utility compressors/expanders in each stage, and utility compressors for LP streams prior to entering the superstructure. Ultimately, the case study demonstrates that the synthesis of direct work exchange networks based on superstructure with stream splits can offer vitally considerable savings in TAC, where the results indicate that our approach yields a network with 20.0 % lower TAC and 18.8 % lower TAC, compared with that of transshipment model and graphical method.

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