CHEMICAL ENGINEERING TRANSACTIONS
VOL. 53, 2016

A publication of AIDIC
The Italian Association of Chemical Engineering
Online at www.aidic.it/cet

Guest Editors: Valerio Cozzani, Eddy De Rademaeker, Davide Manca

ISBN 978-88-95608-44-0; **ISSN** 2283-9216

DOI: 10.3303/CET1653039

Catastrophic Transitions in a Forest-Grassland Ecosystem

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The paper analyses catastrophic transitions in a forest-grassland ecosystem from a nonlinear dynamical point of view. Recently, forest-grassland ecosystems have been experimentally proved to be bistable systems and thus bistable dynamical vegetation models has been proposed in the literature. We consider in this paper a recently proposed mathematical model which includes in a dynamic way the human influence based on the rarity perception of the grass and the forest value. Bifurcation analysis, conducted with continuation technique, has been conducted to trace and analyse the steady state solutions of the ecosystem considering the parameter related to the human influence as bifurcation parameter. Multiplicility and bistability have been observed in a wide parameter range, which are mainly organized by multiple hysteresis and transcritical bifurcations. Catastrophic transitions are then observed both as consequence of parameter perturbations in correspondence of limit point bifurcations or as consequence of the state vector perturbations in the multistability parameter region.

1. Introduction

Ecosystems may respond gradually (and thus predictably) to external human and/or environmental disturbance or they may change their equilibrium state in a sudden and sharp manner. It is now well accepted that such sharp change (catastrophic transitions) may be explained with the existence in the ecosystems of two or more stable states (May, 1977; Groffmann et al. 2006). Indeed, alternative states have been found in many ecosystems and many studies have explained mathematically catastrophic transitions in terms of possible regime shifts and/ or bifurcations. Examples of these studies range from coral reefs (Knowlton et al.1992, Hoegh-Guldberg et al. 2007) and deserts (Brovkin et al.1998) to lakes (Scheffer et al. 2001) and tidal flats (Rietkerk et al. 2004; vanNes and Scheffer 2007).

In such systems sudden shifts to one state or to another, may manifest as consequence of external disturbances and thus mathematical models which possess two or more stable states have been adopted to explain such behaviour. The most simple cases are bistable systems where two stable states coexist and depending on the initial conditions or on the external perturbations the system may approach one of the two stable states. Human interactions are frequent external disturbances in many ecosystems and to understand the way disturbances influence the dynamics of ecosystems is nowadays particular challenging as a sudden shift to an undesired stable state may represent an ecological disaster.

In this context, dynamic systems where vegetation-environment feedbacks are present are of particular challenge. Forest-grassland mosaic ecosystems, such as savanna, are typical examples of such ecosystems where two species (forest and grass) compete for the same food (soil, sunlight and space) (Walker and Noy-Meir, 1982; Sarmiento 1984; Sankaran et al 2005).

Recent experimental observations suggest that forest–grassland ecosystems may have bistable behaviour where fire is the main responsible of the feedback mechanism which lead to the bistability (Staver et al. 2011; Alexandridis et al. 2008, 2011a, 2011b). Thus, it is of primary importance for the management and the control of the forests, to understand how the stability of these two states is affected by external perturbations. Indeed forest management policies such as deforestation, which is often used to control fire (Russo et al. 2013, 2014, 2015; Evaggelidis et al. 2015), are able to maintain savanna in stable grassland state. Because of this

Please cite this article as: Russo L., Spiliotis C., Siettos C., 2016, Catastrophic transitions in a forest-grassland ecosystem, Chemical Engineering Transactions, 53, 229-234 DOI: 10.3303/CET1653039

feedback mechanism, mathematical models have been constructed which consider the human-environment interaction Horan et al. 2011;Innes et al. 2013).
In this paper we demonstrate through the bifurcation analysis (Russo and Spliliotis 2016) that a forest-grassland ecosystem model, proposed by (Innes et al. 2013) which also includes human inference, may have multiplicity and multistability. Catastrophic transitions are then analysed and explained from a nonlinear point of view.

1. Mathematical model and steady states

We consider a minimal model of a forest-grassland mosaic which includes the human influence in a dynamic way. The system equations are the following (Innes et al. 2013):

$$\begin{cases} \dfrac{df}{dt} = w(f)(1-f)\,f - \nu f - J(x) \\ \dfrac{dx}{dt} = sx(1-x)U(f) \end{cases} \qquad (1)$$

f is the forest population expressed in terms of fraction of land occupied by the forest and x is the fraction of people who prefer forest. In Eqs. (1), the forest growth rate is regulated by three terms: the first one is a generation term which is proportional to the forest fraction *f*, the grass fraction *(1-f)* and to a nonlinear term *w(f)* which takes into account the fire incidence; the second one, νf , is the rate at which forest reverts to grassland through natural processes; in contrast, $J(x) = \frac{h}{2}\left(\frac{1}{2} - x\right)$, represents only human-driven transitions and it is proportional to the relative value of the forest. A more detailed explanation of the model can be found in (Innes et al. 2013).
Setting the derivatives equal to zeros, the Eqs. (1) gives rise to an algebraic system of nonlinear equations, the solutions of which are the steady states of the model. Three kinds of steady states may be found solving the following subsystems:

$$\begin{cases} x = 0 \\ w(f)(1-f)\,f - J(0) = \nu f \end{cases} \qquad (2)$$

$$\begin{cases} x = 1 \\ w(f)(1-f)\,f - J(1) = \nu f \end{cases} \qquad (3)$$

$$\begin{cases} f = \dfrac{1}{2} \\ w\left(\dfrac{1}{2}\right)\dfrac{1}{4} - \nu\dfrac{1}{2} = J(x) = \dfrac{h}{2}\left(\dfrac{1}{2} - x\right) \end{cases} \qquad (4)$$

A Newton-Raphson iteration method has been used to find the solutions of the nonlinear systems equations (2) and (3). Graphically, for x=0, these solutions correspond to the intersections of the straight line νf with the nonlinear function $w(f)(1-f)\,f - J(0)$, whereas for x=1, they correspond to the intersections of the straight line νf with the nonlinear function $w(f)(1-f)\,f - J(1)$. Setting the parameters $s = 10$, $k = 6.5$, $\nu = 0.18$, these intersections are reported in Figure 1 as the *h* parameter is changed. In particular, in Figure1(a) we show the steady state solutions corresponding to x=0 and in Figure1(b) the ones corresponding to x=1. As it clearly appears, for x=0, as h is higher then 0, the number of solutions passes from three to two, indicating that h=0 is a bifurcation point. When h is higher then ~0.33, there are not solutions corresponding to x=0. For the case of x=1, when h is higher then 0, there are three solutions, two of them disappearing at ~0.36. Again for very high values of h, there are not solutions corresponding to x=1. Finally, it should be considered the steady state solution which is solution of the system (4), which exists for all values of h. Clearly, in both cases, decreasing the parameter related to the rate of transformation of forest in grass, the number of the steady states increases, leaving just the solutions corresponding to *f=0.5*. While this graphical analysis can give a quick idea of the number of solutions for different values of *h*, usually it cannot give a precise information about their stability. Moreover, dynamic regimes like periodic solutions cannot be detected. In order to obtain a more complete picture of the system dynamics, phase portraits have been constructed tracing the trajectories for different initial conditions. Figure 2 shows two phase portraits, for h=0.2 (Figure2(a)) and for h=0.4 (Figure2(b)). It is apparent that, while a weak human influence (h=0.2, Figure2(a)) leads to a multiplicity of steady states (two of which stable, i.e.there is bistability), as h passes a critical value, there is one steady states which attracts all the trajectories.

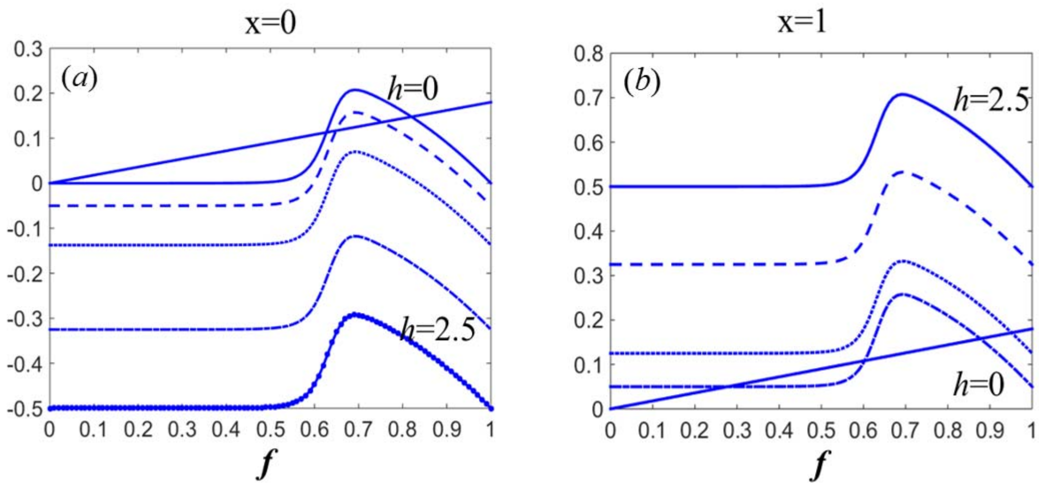

Figure 1. Steady states solutions for $s=10$, $k=6.5$, $v=0.18$. The parameter h varies from zero to 3. Depending on h the number of fixed points varies from zero to three. (a) Solutions correspond to $x=0$. (b) Solutions corresponds to $x=1$

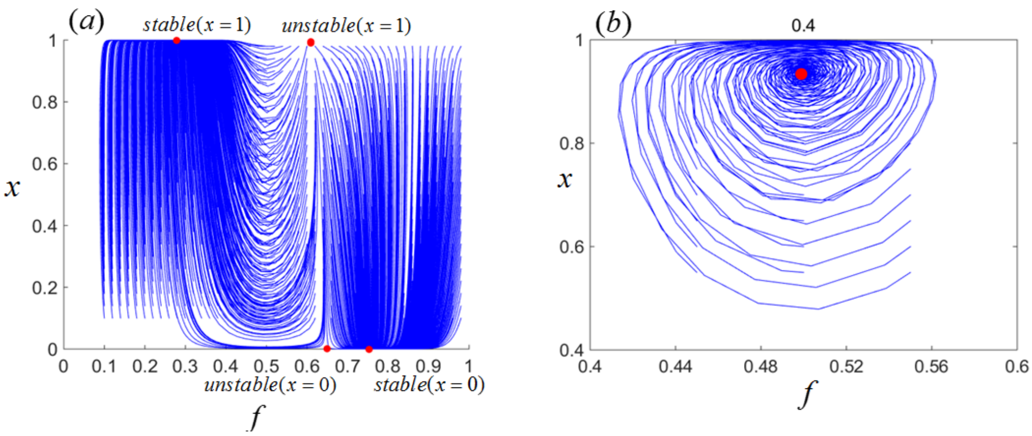

Figure 2. Phase portraits for $s=10$, $k=6.5$, $v=0.18$, and trajectories for different initial conditions. (a) $h=0.2$ multi-stability of steady states (b) $h=0.4$ one stable steady state.

2. Bifurcation analysis and hysteresis

Multiplicity of regime solutions as well as stability of steady and, eventually, dynamic regimes may systematically studied through the bifurcation analysis of the system as the main bifurcation parameter *h* is changed. Continuation based methodologies are the tool of choice for this analysis. Indeed, starting from a steady state solution, it is possible to trace the branch of solutions, to compute their stability and to detect the bifurcations involved in the stability change and/or in the number of solutions as the bifurcation parameter is varied. Numerical bifurcation analysis conducted through parameter continuation is then applied to study the nonlinear dynamics of the system and in particular the multistability and the multiplicity of regimes.

Fixing the values of the parameters at s=10, k=6.5, v=0.18, we construct the bifurcation diagram in Figure 3 with respect to the parameter *h*. Solid lines depict stable solutions, while dashed lines the unstable ones. It is apparent from Figure 3 (a), that there are three branches of steady states: one corresponding to x=0; the S

branch which corresponds to the locus of steady states with x=1, and the straight constant line corresponding to the steady states with f=0.5.
In order to understand from a nonlinear point of view the origin of x=0 branch, we performed the bifurcation analysis also for negative values of h (Figure3b). In particular, in Figure3(b) the stability of each steady state is reported: (+,+) are stable nodes with two positive eigenvalues; (+,–) are saddles with one negative and one positive eigenvalue and (–,–) unstable nodes with two negative eigenvalues. It is apparent that the branch of the steady states x=0 appears with a S-shape in specular manner respect to the x=1 S branch. Both the S-shaped branches (x=0 and x=1) cross the horizontal line f=0.5, corresponding of two transcritical bifurcations (BP2 for the x=0 branch, BP1 for the x=1 branch) where the steady states exchange their stability.
Thus, looking just at the positive values of h (Figure3(a)), up to the h value corresponding to limit point bifurcation LP4, there are 6 steady states: two stable nodes (one for x=0 and one for x=1); three saddles (one for x=0,one for x=1 and one for f=0.5); and one unstable node (for x=0). Multistability and multiplicity is observed in a wide range of parameter, whereas as the parameter h passes the one corresponding to the limit point LP1 only one stable steady state exists.

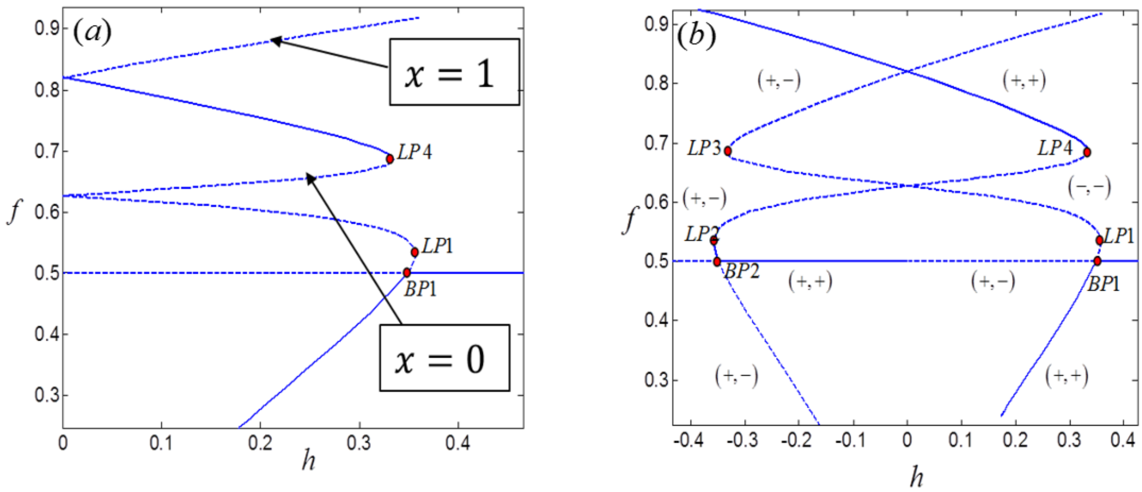

Figure 3. Bifurcation diagram with respect the parameter h . $k = 6.5$ $\nu = 0.18$. *Solid lines correspond to stable steady states and dashed lines to unstable steady states. LP are limit point bifurcation and BP transcritical bifurcations. (a) The diagram only for positive value of h. (b) the complete diagram where it is shown the specular dynamical behavior. The diagram consists of two branches one of S type (* $x = 1$ *) and one S inverted (x=0). The S branch bifurcates at* $BP1$ *via a transcritical bifurcation at the point* $(h, f) = (0.349, 0.5)$ *whereas the S inverted branch bifurcate at BP2.*

From a practical point of view, when the human influence is strong enough the system reaches an equilibrium with 50% of grass and 50% of forest, that is neither the grass neither the forest will be rare. On the other hand when the human influence is weak, then multiple stable steady states may occur and thus catastrophic transitions are possible between steady states with a very different percent of grass and forest.

3. Catastrophic shifts

In an ecosystem, catastrophic shifts occur when the system passes from a stable regime to another completely different in an abrupt way as a consequence of environmental or external disturbances. When this happens, the recovery of the previous state is extremely difficult due, in the majority of cases, to hysteresis effects. From a dynamical point of view, a catastrophic shift may occur in two ways. In the first case, these shifts are a consequence of perturbations of the vector state in a parameter region where the system has multistability. Indeed, if the system has more the one stable regime for a specific value of the parameter (h), then as a consequence of a perturbation in the vector state (x or f), the system may jump from one equilibrium to another.
As each stable regime is characterized by its own basin of attraction (the set of all the initial conditions leading to the same stable regime), a perturbation to the vector state may bring it to jump to another basin of attraction which end the system to a different stable state.

This phenomenon is discussed in Figure 4, where simulation results are reported for a h parameter value belonging to the region of multistability [0;h_{LP4}]. In this region there are two stable steady states: one corresponding to x=0 and one to x=1. Starting from an initial condition belonging to the basin of attraction of x=1, the system reaches first the state x=1 and then, when a perturbation is given (at time around t=100), the system finally reaches the steady state x=0 (Figure 4 (a) and (b)). Indeed, as a consequence of the state vector perturbations, the system trajectory has crossed the separatrix of the attraction basins of the two stable states (Fig.4 (c)).

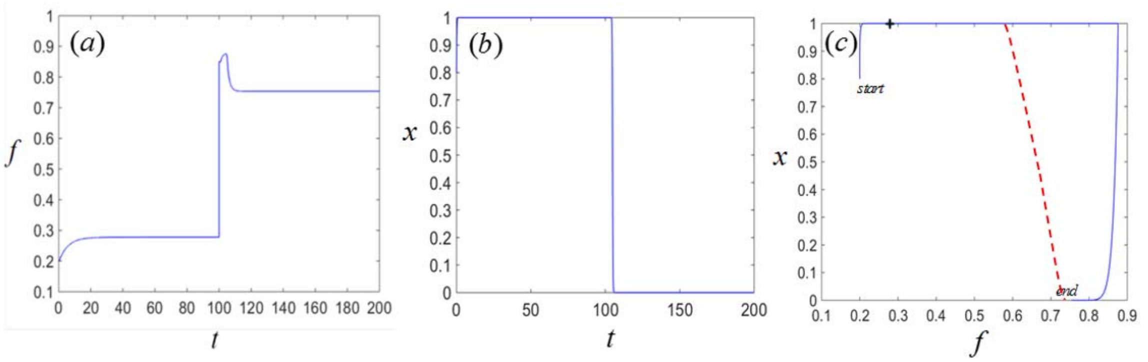

Figure 4. Catastrophic shifts after perturbation of the steady vector. (a) Perturbation of f at time $t=100$. (b) Perturbation of x at time $t=100$ (c) The corresponding phase diagram (f,x). The dashed line is the separatrix between attraction basins of the two stable states.

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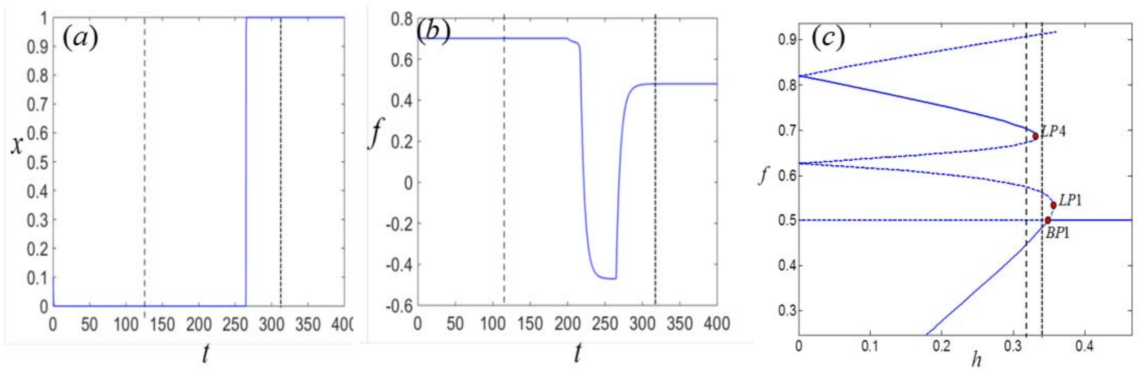

Figure 5. Catastrophic shifts after perturbation of the parameter h (a) and (b) simulations of the jump from the x=0 steady state to the x=1 steady state. (c) The bifurcation diagram with the corresponding parameter perturbation. The coarse dashed line is the value of parameter h before the perturbation and the dense dashed is the value of h after the perturbation.

The second way with which a catastrophic transition may occur, is when one equilibrium disappears as a consequence of a perturbation of a system parameter. In this case, the system reaches a new stable equilibrium in an abrupt way because it cannot reach anymore the pre-existing regime. Such transitions are observed for example when a parameter crosses a critical value corresponding to a catastrophic bifurcation such as a limit point. This case is discussed in Figure 5, where it is clearly shown that if the system is in the stable steady state x=0 just before the critical value of LP4, then a small perturbation of the parameter leads the system to another equilibrium (x=1) with a sharp and sudden transition.

4. Conclusions

We analysed steady states solutions and multistability in a model of a forest grassland model which considers dynamically the human interactions and their preference based on the rarity perception value. We analysed the change of the number of steady states and their stability as the parameter related to strength of human influence is varied. The analysis is conducted graphically and numerically by adopting continuation techniques. Bifurcation analysis is then performed systematically and bifurcation diagrams are constructed and thoroughly examined. The main results are that multiplicity and multistability are observed in a wide range of the parameter when human influence is weak. There are two stable steady states x=0 (where all human beings prefer grass) and x=1 (where all human beings prefer forest) characterized by a very different percent of grass and forest. Catastrophic transitions may then occur from one state to another as the vector state or parameter related to the human influence are perturbed. Strong human influence (high value of the parameter h) brings to a unique stable steady state characterized by 50% of forest and 50% of grass.

Acknowledgments

Lucia Russo would like to thank Marco Imparato for his technical support.

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