

Modelling of Multi-Scale Uncertainties in Biofuel Supply Chain Optimization

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Although strategic and operational uncertainties differ in significance of impact, a “one-size-fits-all” approach has been typically used to tackle all types of uncertainty in the optimal design and operations of supply chains. In this work, we propose a stochastic robust optimization model that handles multi-scale uncertainties in a holistic framework, aiming to optimize the expected economic performance while ensuring the robustness of operations. Stochastic programming and robust optimization approaches are integrated in a nested manner to reflect the decision maker’s different levels of conservatism towards strategic and operational uncertainties. The resulting multi-level mixed-integer linear programming model is solved by a decomposition-based column-and-constraint generation algorithm. To illustrate the application, a county-level case study on optimal design and operations of a spatially-explicit biofuel supply chain in Illinois is presented, which demonstrates the advantages and flexibility of the proposed modeling framework and efficiency of the solution algorithm.

1. Introduction

When hedging against uncertainties in the optimal design and operations of supply chains, only one uniform approach has been typically used to tackle all types of uncertainty (Shah, 2004). However, uncertainties at strategic and operational scales may differ in their significance of impact (Yue et al., 2014). Strategic uncertainties have impacts over a significant duration of the project’s lifetime. Once realized, they would remain unchanged for a considerable period of time. Examples of strategic uncertainties include climate and weather, technology evolution, incentives and policies, and network stability (Gao et al., 2015). In contrast, operational uncertainties change more frequently and often lead to immediate adjustment in operational decisions. Examples of operational uncertainties include variations in raw material quality and composition, supply and demand, cost and price, as well as in lead times of production, transportation, and material handling activities (Gebreslassie et al, 2012). Moreover, the realizations of operational uncertainties may be associated with that of strategic uncertainties. For example, the yields of agricultural products are expected to be dependent on the climate and weather. In addition, a decision maker may hold different levels of conservatism towards strategic and operational uncertainties. For instance, one might be less conservative towards strategic uncertainties and willing to explore all possibilities, but be more conservative towards operational uncertainties considering factors such as demand fulfill rate (Garcia and You, 2015a).

Due to all the concerns above, a “one-size-fits-all” approach may not be sufficient for efficiently handling multi-scale uncertainties in optimal design and operations of supply chains. Therefore, a heterogeneous modeling framework is required. Consequently, the goal of this work is to propose a holistic modeling framework and solution algorithm to handle both strategic and operational uncertainties for supply chain optimization problems. Major novelties of this work are summarized below:

- A novel framework that handles multi-scale uncertainties in optimal design and operations of supply chains;
- A nested stochastic robust optimization framework that models the different characteristics of and the decision maker’s different levels of conservatism towards strategic and operational uncertainties via integration of stochastic programming and robust optimization approaches;
- A decomposition-based C&CG algorithm that efficiently solves the resulting multi-level MILP problem.

2. General modeling framework

2.1 Deterministic model

Before introducing the proposed stochastic robust optimization model for supply chain design and operations under multi-scale uncertainties, we first present the corresponding deterministic model. A general form of the deterministic model is given below, based on which an uncertainty counterpart will be developed.

$$\min \left\{ C^{str}(x, y) + C^{opr}(x, y, z) : Ax + By \geq d, Ex + Fy + Gz \geq h, x \in \{0, 1\}^n, y \in \square^m, z \in \square^r \right\} \quad (\text{PD})$$

where x denotes the binary variables for strategic decisions, which model the selection of facility locations; choice of manufacturing technologies, storage types, and transportation modes, etc.; y denotes the continuous variables for strategic decisions, which include the capacities of manufacturing facilities and warehouses; z represents the continuous operational decisions, which involve the quantities of raw materials to acquire from suppliers and products to sell to customers, target of production to reach, as well as amounts of materials to transfer through the transportation links and to hold in the inventory.

The deterministic model (PD) can be divided into two parts. The first part corresponds to supply chain design. $C^{str}(x, y)$ stands for the amortized capital cost associated only with strategic decisions. The inequality $Ax + By \geq d$ represents the strategic-level constraints, involving network design, capacity limits, as well as economic models for capital investment and fixed operating cost. The second part corresponds to supply chain operations. $C^{opr}(x, y, z)$ stands for the operational cost that is influenced by both strategic and operational decisions. The inequality $Ex + Fy + Gz \geq h$ represents the operational level constraints, covering material balances, bounding constraints, and cost calculations of various supply chain activities. Problem (PD) is often formulated as a mixed-integer linear programming (MILP) model.

2.2 Stochastic robust optimization model

When supply chain design decisions must be made before the realization of uncertain parameters, two-stage optimization models are often employed (Birge and Louveaux, 2011). In such models, the first-stage (strategic) decisions are made “here-and-now” before realization of any uncertainty, and the second-stage (operational) decisions are made in a “wait-and-see” manner after all uncertainties are revealed. Instead of handling all types of uncertainty with a uniform approach, we handle strategic and operational uncertainties differently and reflect the decision maker’s different levels of conservatism with the following stochastic robust optimization model.

$$\min \left\{ C^{str}(x, y) + E_{\omega \in \Omega} \left[\bar{C}_{\omega}^{opr}(x, y) \right] : Ax + By \geq d, x \in \{0, 1\}^n, y \in \square^m \right\} \quad (\text{P0})$$

where $\bar{C}_{\omega}^{opr}(x, y) = \max_{u_{\omega} \in U_{\omega}} \min_z \left\{ C_{\omega}^{opr}(x, y, z, u_{\omega}) : Gz_{\omega} \geq h_{\omega} - E_{\omega}x - F_{\omega}y - K_{\omega}u_{\omega}, z_{\omega} \in \square^r \right\}, \forall \omega \in \Omega$. ω denotes a particular realization of strategic uncertainties, and u_{ω} denotes a particular realization of operational uncertainties. As can be seen, the proposed model (P0) explicitly reflects the decision maker’s different levels of conservatism. An expectation over all strategic uncertainty realizations is considered in the objective function, showing that the decision maker is considering all possibilities. On the other hand, the value of $\bar{C}_{\omega}^{opr}(x, y)$ represents the worst-case operating cost under strategic uncertainty realization ω at given first-stage decisions according to the max-min problem (Shi and You, 2016). Therefore, the decision maker hedges against the worst-case realization of operational uncertainties defined by the uncertainty set U_{ω} in problem (P0) (Gong et al., 2016). Please also note that the uncertainty set U_{ω} for operational uncertainties is indexed by realization ω of the strategic uncertainty. In this way, we allow the range of variation in operational uncertainties to be dependent on the realization of strategic uncertainties. This is critical as strategic uncertainties may have significant impacts on operational uncertainties. As there can be infinite realizations of strategic uncertainties, the expectation function in (P0) is usually transformed approximately into a tractable optimization problem by introducing a finite number of scenarios. We name the resulting model a nested stochastic robust optimization model, because a robust optimization problem is nested in a two-stage stochastic program.

$$\min \left\{ C^{str}(x, y) + \sum_{s \in S} p_s \bar{C}_s^{opr}(x, y) : Ax + By \geq d, x \in \{0, 1\}^n, y \in \square^m \right\} \quad (\text{P1})$$

where $\bar{C}_s^{opr}(x, y) = \max_{z \in U_s} \min_{u_s \in U_s} \{C_s^{opr}(x, y, z_s, u_s) : Gz_s \geq h_s - E_s x - F_s y - K_s u_s, z_s \in \square^r\}$, $\forall s \in S$. The index ω in (P0) is replaced by scenario s . The expectation function is replaced by the summation over all scenarios, where p_s denotes the probability of scenario s of strategic uncertainty realization. This is relevant when we have a number of long-term projections at hand (e.g., good, normal, and bad scenarios for the future climate) or we have a good estimate of the distribution of the strategic uncertainties (e.g., process conversion efficiency from historical data). The proposed stochastic robust optimization model (P1) is a multi-level MILP, which cannot be solved directly by any off-the-shelf solver.

3. Solution strategy

To solve problem (P1), we take advantage of the C&CG algorithm (Zeng and Zhao, 2013). It can be shown that the value of uncertain parameters at the optimal solution will be at one of the extreme points of the uncertainty set. Therefore, if we enumerate all the extreme points, then problem (P1) can be reduced to an equivalent (probably large-scale) MILP by removing the max-min function. Specifically, we can replace the content in the curly bracket in problem (P1) with the following.

$$\left\{ \bar{C}_s^{opr}(x, y) \geq C_{s,l}^{opr}(x, y, z_{s,l}), Gz_{s,l} \geq h_s - E_s x - F_s y - K_s u_{s,l}, z_{s,l} \in \square^r, \forall l \in L \right. \quad (\text{EP})$$

where $u_{s,l}$ denotes an extreme point of uncertainty set U_s .

Next, we decompose the problem using partial enumeration, and obtain a master problem that has the same structure as the equivalent MILP but contains only a subset of the extreme points. Thus, the master problem constitutes a valid relaxation and provides a lower bound. In order to obtain upper bounds and feasible solutions, we solve a series of max-min operational-level subproblems at given first-stage decisions and strategic uncertainty realization s . It can be proved that a set of new extreme points would be generated in each iteration if the termination criterion is not met. Based on these extreme points, we can add an optimality cut to the master problem, which is in the form of (EP) but corresponds only to one new extreme point. Explicit form of the master and subproblems are omitted here. Implementation of the improved C&CG algorithm is shown in Figure 1.

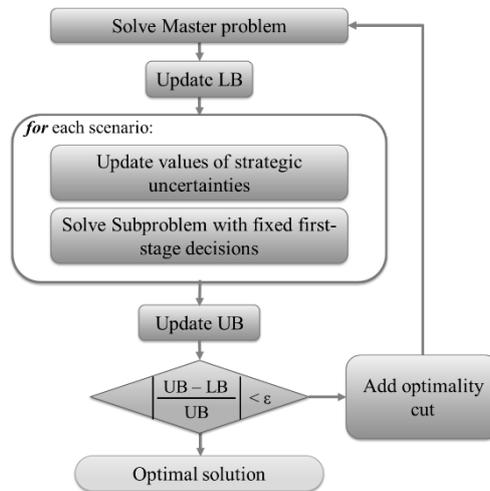


Figure 1: Flowchart of the improved C&CG algorithm

4. Case study

4.1 Case description

To demonstrate the applicability of the proposed modeling framework and solution algorithm, we consider an application on the optimal design and operations of a potential biofuel supply chain, which shows great promise for sustainable energy supply (Gong et al., 2015). We modify the deterministic spatially explicit and multi-period MILP model in (You et al., 2012) to handle strategic and operational uncertainties in a biofuel supply chain. The underlying supply chain superstructure is shown in Figure 2a. In this case study, the biomass feedstock is corn stover, and the biofuel product is fuel ethanol (Yue et al., 2015). The biomass-to-biofuel conversion is via a bio-chemical pathway. The supply chain optimization is performed at the county level in Illinois (Yue et al., 2013). We consider 25 biomass suppliers, 40 biofuel customers, and 10 candidate

sites for building biorefineries. Three capacity levels of ethanol production at biorefineries are considered, namely 10 – 50, 50 – 100, and 100 – 150 MM gallons/year. We assume that corn stover can only be harvested in October and November (Yue and You, 2015). The first-stage decisions include the selection of location and determination of capacity for biorefineries, which must be made before the realization of both strategic and operational uncertainties. The second-stage decisions involve the various continuous variables on biomass acquisition, biofuel sales, inter-site transportation, and inventory management. These second-stage decisions are made after the realization of both strategic and operational uncertainties. Complete recourse is assumed by allowing the use of imported biofuel from an external source to satisfy the unmet demand with a penalty cost.

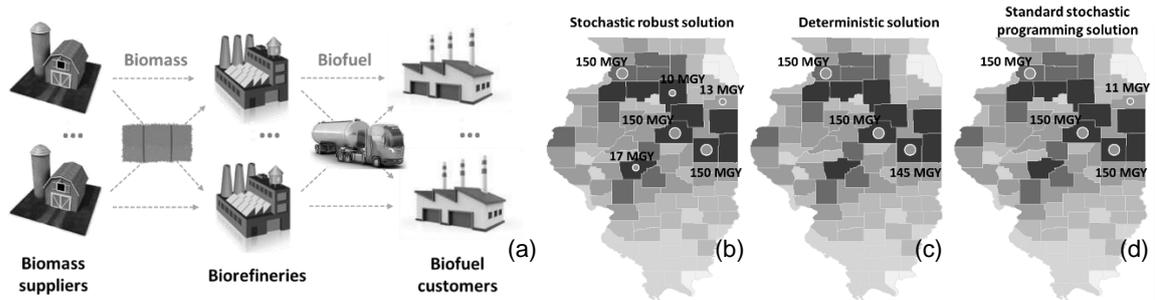


Figure 2: (a) Superstructure of the three-echelon biofuel supply chain; and optimal supply chain layout of (b) stochastic robust solution, (c) deterministic solution, and (d) standard stochastic programming solution. Backgrounds represent the nominal biomass availability in Illinois

The objective is to minimize the expected total cost over different scenarios of strategic uncertainty realization while ensuring the robustness against operational uncertainties. As shown in Table 1, the strategic uncertainties considered include technology evolution and climate and weather. Uncertainty in technology evolution is reflected as the deviation of actual process conversion efficiency from the designated value, which may originate from design or implementation errors. Uncertainty of climate and weather is reflected in the variations of precipitation and insolation, which greatly impact the yield of biomass feedstock. The operational uncertainties include variations in biomass supply and biofuel demand.

Table 1: Scenarios and uncertainty sets

Scenario	Probability	Strategic uncertainties		Operational uncertainties	
		Conversion	Climate	Biomass availability	Biofuel demand
Nominal		α	Normal	a	d
S1	1/12	0.9α	Bad	[0.7a, 0.9a]	
S2	1/6	α			
S3	1/12	1.1α			
S4	1/12	0.9α	Normal	[0.9a, 1.1a]	[0.9d, 1.1d]
S5	1/6	α			
S6	1/12	1.1α			
S7	1/12	0.9α	Good	[1.1a, 1.3a]	
S8	1/6	α			
S9	1/12	1.1α			

4.2 Results and discussions

The stochastic robust optimal solution under both strategic and operational uncertainties are shown in Figure 2(b). The optimal first-stage decisions indicate that a total of six biorefineries should be built. For comparison, we also solve a deterministic MILP by setting the process conversion efficiency, biomass availability, and biofuel demand at the nominal values. The resulting supply chain layout is given in Figure 2(c). To compare with the solutions from a uniform modeling approach, we also solve the problem using a standard two-stage stochastic programming approach with 90 scenarios. The resulting supply chain layout is shown in Figure

2(d). As can be observed, the stochastic programming solution is like a compromise solution between the stochastic robust solution and the deterministic solution.

To compare all three supply chain designs in an uncertain environment, we perform a simulation in all the aforementioned 90 scenarios for each supply chain design. The results are plotted in Figure 3. The triangles denote the perfect information solutions, which are obtained by solving a deterministic model for each scenario that optimizes both strategic and operational decisions. The circles denote the stochastic robust solutions corresponding to Figure 2(b). The squares denote the deterministic solutions corresponding to Figure 2(c), among which the crossed squares indicate that ethanol import from the external source are needed in 55 out of the 90 scenarios. The diamonds denote the standard stochastic programming solutions corresponding to Figure 2(d), among which the crossed diamonds indicate that ethanol import from the external source are needed in 14 out of the 90 scenarios. From the horizontal lines, we can see that the perfect information solutions set the baseline for expected cost. The expected cost of the stochastic robust solution is higher than that of the stochastic programming solution. The deterministic solution has an expected cost that is even slightly higher than the stochastic robust solution. This indicates that deterministic optimization at nominal values without considering uncertainties can lead to lower demand fulfillment rate and higher expected total cost.

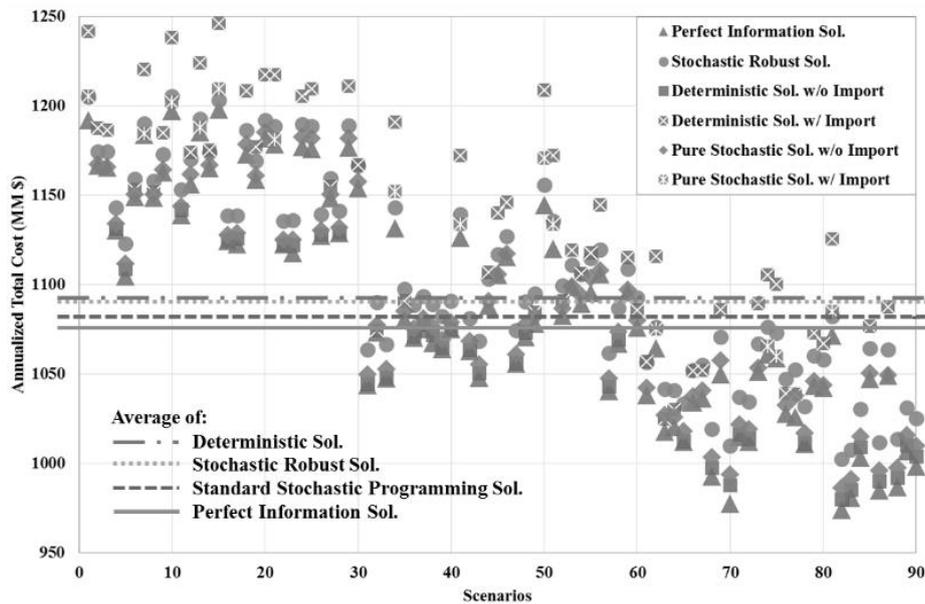


Figure 3: Simulation results for different supply chain designs

To take a close look at the stochastic robust solution, we present its cost breakdown and inventory profile in the scenario where all uncertain parameters are set to their nominal values, as shown in Figure 4.

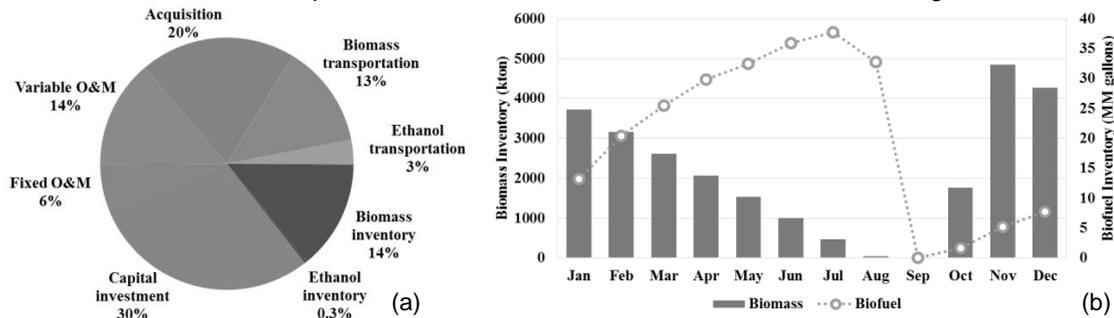


Figure 4: (a) Cost breakdown and (b) inventory profile of the stochastic robust solution under nominal scenario

4.3 Computational performances

We solve the MILP master problem and subproblem with CPLEX 12. All models and solution procedures are coded in GAMS 24.5. The optimality tolerance for CPLEX 12 is set to 0. As aforementioned, the C&CG

algorithm involves iterative solution of one master problem and multiple subproblems. The subproblem in all iterations has 65 binary variables, 2,161 continuous variables, and 11,103 constraints. The master problem in the first iteration has 30 binary variables, 63,421 continuous variables, and 19,551 constraints. As the algorithm iterates, the size of the master problem increases because an optimality cut is added at the end of each iteration. Among the 36 instances shown in Figure 5, the minimum number of iterations is 1, and the maximum number of iterations is 8. The master problem at the 8th iteration has 30 binary variables, 506,941 continuous variables, and 155,694 constraints. The shortest solution time is 117 CPU s, while the longest solution time is about 14.5 CPU h. Overall, the C&CG is shown to be capable of achieving a small optimality gap in its early iterations and converge within a reasonable number of iterations and solution time.

5. Conclusions

A nested stochastic robust optimization model was proposed to simultaneously handle strategic and operational supply chain uncertainties. Considering the cases that a decision maker is less conservative towards strategic uncertainties and more conservative towards operational uncertainties, the authors modeled the multi-scale uncertainties via integration of stochastic programming and robust optimization approaches. The resulting formulation was a multi-level MILP involving expectation over multiple scenarios as well as max-min problems that guarantee the robustness of operations. The authors employed a decomposition-based C&CG algorithm for the solution, which was based on partial enumeration. The application was demonstrated by a county-level case study on optimal design of a spatially-explicit biofuel supply chain in Illinois.

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