

Development of the Frequency PID Tuning Method with Filter

Kozin A. Kozin^{*,a}, Anna M. Egorova^a, Sergei S. Mikhalevich^a, Flavio Manenti^b,
 Francesco Rossi^b

^a National Research Tomsk Polytechnic University, Department of Electronics and Automation of Nuclear Plants, Lenin Street 50, 634050 Tomsk, Russian Federation

^b Politecnico di Milano, Dipartimento Chimica Materiali e Ingegneria Chimica "Giulio Natta", Piazza Leonardo da Vinci 32 20133 Milano, Italy
 kozin@tpu.ru

In the previous research authors developed tunable method for PID controllers to achieve the desired phase margin. Crossover frequency is used as a parameter that allows correcting time response of closed-loop system. The aim of the research is to find proportional, integral, derivative and filter coefficients of PID controller that will provide desired characteristics (phase margin and crossover frequency) in the system. The analytical approach of proposed PID tuning method is presented. The comparison proposed method with conventional was done. Practical approach contains developing the control system of extraction process. This system is a part of Russian Federation large project – closed-loop nuclear fuel cycle. The plant was linearized and digital control system was developed.

1. Introduction

One of the actual research trends of control theory is a development of PID tuning methods. It is due to wide spread PID controllers in engineering practice. As is well known, PID controller is appropriate for the most linear or quasi-linear processes (Moradi and Vossoughi, 2015). At the present time there are a lot of different methods that are based on a desired time-domain or frequency characteristics. But most of them are developed for ideal PID controller, despite of all controllers contain filter for derivative part of PID controller in modern engineering practice. Ideal PID controller contains ideal differentiation that could not be hardware realized. Therefore the first order filter is used for practical reasons.

Nowadays, there are a lot of different PID tuning rules and methods which ignore filter constant in tuning procedure and then recommend choosing it additionally (Kristiansson and Lennartson, 2006). Any deviation of filter constant changes desired stability or time-response characteristics.

There are several PID tuning methods which allow to determinate four unknown (proportional gain, integral, derivative and filter time constants). For instance, method (Sanchis et al., 2010) used as desired data: phase margin, gain margin and additional parameter, input data is frequency plant characteristics. The method (Sanchis et al., 2010) also uses an optimization algorithm to provide desired characteristics of the system. Obvious disadvantage of this method is necessary to provide a lot of initial information about process and desired control system. In the article (Vadutov, 2014) the author suggests to use constrained optimization to provide minimum of IAE or ISE. In addition, the author imposes restrictions to the poles of closed-loop transfer function. This method can't be used for plant transfer function without delay. There is also research (Aström et al., 1998), where authors place filter in structural scheme before PID controller. This way decreases transient rate of control system.

In the previous research (Mikhalevich et al., 2015), authors developed tuneable method for PID controllers to achieve the desired phase margin. Crossover frequency is used as a parameter that is allowed to correct time response of closed-loop system. The aim of research is to find proportional, integral, derivative and filter coefficients of PID controller that will provide desired characteristics (phase margin and crossover frequency) in the system.

The paper consists of introduction, theoretical research, comparison study of proposed method with known, description of line of centrifugal extractors and development of control system.

The criterions of desired control system that are based on previous research and Aström recommendations (Aström and Häggglund, 2006) are shown in the theoretical part. Then, the analytical approach of proposed PID tuning method is presented. After, comparison proposed method with conventional was done.

Practical approach contains developing the control system of extraction process (Goryunov and Mikhaylov, 2012). Block of centrifugal extractors, as a plant, is used. This block contains 12 centrifugal extractors which are connected in serial line and used for extraction uranium and plutonium. This equipment is a part of Russian Federation large project – closed-loop nuclear fuel cycle. The plant was linearized and digital control system was developed.

2. Theoretical research

Let us consider the control system that is presented on Figure 1. The system consists of PID controller and plant. Initial information for PID tuning is transfer function of plant, phase margin φ_m , crossover frequency ω_c .

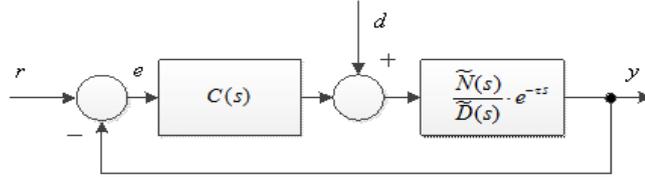


Figure 1: Block diagram of control system, where r is a set-point, y is output, d is a disturbance

Transfer function of plant could be presented as

$$P = \frac{\tilde{N}(s)}{\tilde{D}(s)} e^{-zs} = \frac{\tilde{N}(s)}{\tilde{D}(s)} \cdot \frac{L(s)}{M(s)} = \frac{N(s)}{D(s)}, \quad (1)$$

where $L(s)$ and $M(s)$ are numerator and denominator of lagging, that is expanded into the Padé series. The controller assumes the following form:

$$C(s) = \left(k_p + \frac{k_i}{s} + \frac{k_d s}{T_i s + 1} \right). \quad (2)$$

By inserting $s=j\omega$ into Eq.(1), plant transfer function is stated, as follows:

$$P(j\omega) = \left. \frac{N(s)}{D(s)} \right|_{s=j\omega} = \text{Re}(\omega) + j\text{Im}(\omega), \quad (3)$$

where $\text{Re}(\omega)$, $\text{Im}(\omega)$ are real and imaginary parts of a plant. The same substitution to controller transfer function is

$$C(j\omega) = \left(k_p + \frac{k_i}{j\omega} + \frac{k_d \cdot j\omega}{T_i \cdot j\omega + 1} \right), \quad (4)$$

According to criterion described in the article (Mikhalevich et al., 2015) and phase margin definition we could formulate the system of equations:

$$\begin{aligned} \text{Re} G(j\omega_c) &= -\cos(\varphi_m), \\ \text{Im} G(j\omega_c) &= -\sin(\varphi_m), \\ \frac{d\text{Re}[G(j\omega_c)]}{d\omega_c} &= 0. \end{aligned} \quad (5)$$

where $G(j\omega_c) = P(j\omega_c) C(j\omega_c)$ is a transfer function of open-loop system.

The system Eq.(5) allows to find proportional, integral and derivative parts of PID controller. Solutions of the system Eq.(5) are functions of phase margin, crossover frequency and filter constant T_i .

In common case, transfer function of open-loop system is

$$G = \frac{(\text{Re} T_i \omega^2 + \text{Im} \omega) k_p + (\text{Im} T_i \omega - \text{Re}) k_i + \text{Re} \omega^2 k_d - j \cdot (\text{Re} \omega - \text{Im} T_i \omega^2) k_p + (\text{Re} T_i \omega + \text{Im}) k_i - \text{Im} \omega^2 k_d}{(T_i \omega^2 - j\omega)}. \quad (6)$$

So, analytical solutions of the system of equations Eq(5) are

$$k_p = \frac{-(T_i^2 \omega^2 + 1)(4 \cos(\varphi)(\operatorname{Re} T_i \omega - \operatorname{Im})(\operatorname{Im}^2 + \operatorname{Re}^2) + (2a + 2b)(\operatorname{Im}^2 + \operatorname{Re}^2) + 2(\operatorname{Re} T_i \omega - \operatorname{Im})a}{4(\operatorname{Re} T_i^2 \omega^2 + \operatorname{Re})(\operatorname{Re} T_i \omega - \operatorname{Im})(\operatorname{Im}^2 + \operatorname{Re}^2)}, \quad (7)$$

$$k_i = \frac{\omega(T_i^2 \omega^2 + 1)a - 2\omega b}{2(\operatorname{Im}^2 + \operatorname{Re}^2)(\operatorname{Re} T_i \omega - \operatorname{Im})}, \quad (8)$$

$$k_d = \frac{(T_i^2 \omega^2 + 1)^2 a}{2\omega(\operatorname{Im}^2 + \operatorname{Re}^2)(\operatorname{Re} T_i \omega - \operatorname{Im})}, \quad (9)$$

where

$$a = ((\operatorname{Re} \operatorname{Re}' \omega + \operatorname{Im} \operatorname{Im}' \omega - \operatorname{Im}^2) \cos(\varphi) - (\operatorname{Re} \operatorname{Im}' \omega - \operatorname{Re} \operatorname{Im} - \operatorname{Re}' \operatorname{Im} \omega) \sin(\varphi)),$$

$$b = (\operatorname{Re} T_i \omega - \operatorname{Im})(\operatorname{Im} \cos(\varphi) - \operatorname{Re} \sin(\varphi)),$$

$$\operatorname{Re} = \frac{d \operatorname{Re}}{d \omega_c}, \operatorname{Im} = \frac{d \operatorname{Im}}{d \omega_c}, \omega = \omega_c, \varphi = \varphi_m.$$

In the book (Aström and Hägglund, 2006) there is information, if IE is minimal, k_i will be maximum. According to this, suppose, that k_i has a maximum in stability area. One of the properties of an extrema is zero value of the first derivation, i.e.

$$\left. \frac{dk_i}{d\omega} \right|_{\omega=\omega_c} = 0, \quad (10)$$

Eq.(10) could be used for finding filter constant. Authors got an analytical solution of the Eq(10) with MATLAB software. Solution is not written in the article, because it is too long. But from Eq(8) and Eq(10) could be seen that the maximum degree of Eq(10) by T_i is the third. It means that there are two types of solution: 2 complex roots and 1 real root or 3 real roots. Complex roots are improper, because filter constant should be real. Additional requirement to T_i is bigger than zero. If all three root fulfil conditions, it have to be checked and most proper solution must be chosen.

3. Comparison study

Let us consider the following controlled plants (see Table 1). Comparisons provided with conventional PID tuning methods, which are widespread for industrial purpose: AMIGO (Aström and Hägglund, 2006), Ziegler-Nichols method (Ziegler and Nichols, 1942). Filter constant was chosen according to recommendation in book (Visioli, 2006):

$$T_i = \frac{k_d}{N}, \quad (11)$$

where N ranges from 2 to 20. Let us assume that N=10 for AMIGO and Ziegler-Nichols method.

Table 1. PID controller parameters and IAE comparisons

Plant	PID tuning method	k_p	k_i	k_d	T_i	IAE
$\frac{1}{(15s+1)} e^{-3s}$	Proposed method	4.68	0.03	-134	38.6	20.13
	AMIGO	2.45	0.28	3.47	0.35	19.66
	Ziegler-Nichols method	2.5	1	9	0.9	22.57
$\frac{(s^2 + 2s + 1)}{(3s^3 + 4s^2 + 5s + 1)} e^{-s}$	Proposed method	1.96	0.12	-56.1	37.4	8.88
	AMIGO	1.1	0.65	0.49	0.05	8.55
	Ziegler-Nichols method	0.96	1.17	1.22	0.12	14.77

For the FOPTD example phase margin equals to 69° and crossover frequency equals 0.1 rad/s. For this data filter constant has one real positive root and two complex roots (are ignored). Controller was specially tuned to provide smaller overshoot. Step and disturbance responses are shown on the Figure 2. The comparison

shows that the controller tuned by means of the proposed method is characterized by a smaller overshoot at the step response. Unfortunately, disturbance attenuation has bigger overshoot and much bigger settling time.

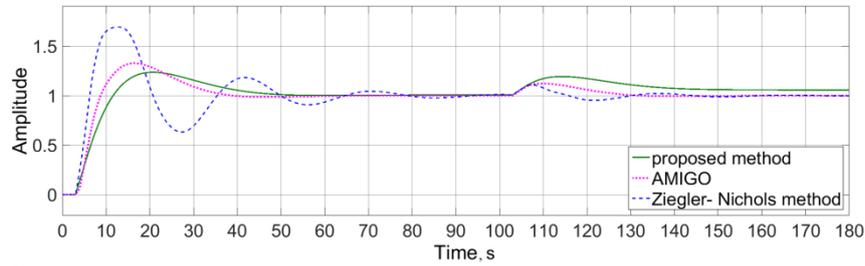


Figure 2: Step and disturbance responses for the FOPTD plant

For the second example phase margin equals to 80° and crossover frequency equals 0.15 rad/s. For this data filter constant have three real roots, but two of them are negative, so, we could ignore it. Controller was also specially tuned to provide smaller overshoot. Step and disturbance responses are shown on the Figure 3.

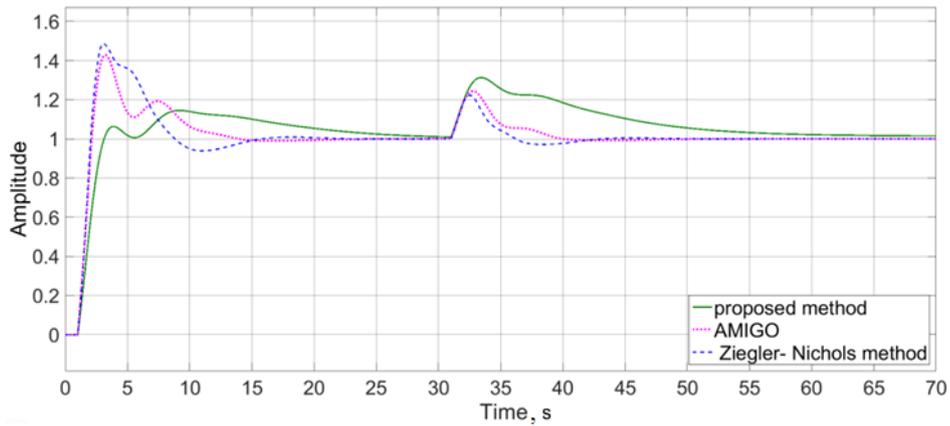


Figure 3: Step and disturbance responses for the third order plant

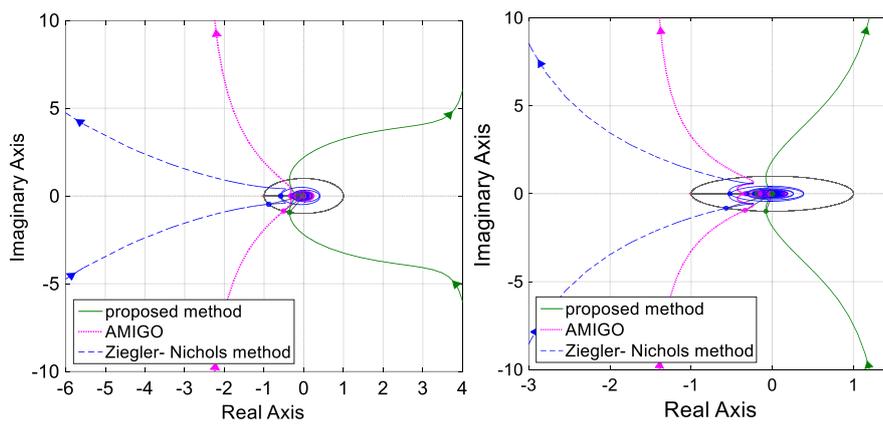


Figure 4: Nyquist curves for the both examples (for the first example at the left side, the second – at the right side)

The comparison shows that the controller tuned by means of the proposed method is characterized by a smaller overshoot at the step response. The Nyquist curves of proposed, AMIGO and Ziegler-Nichols methods for both examples are presented on Figure 4.

Obviously, PID controller in both examples could be tuned to another phase margin and crossover frequency which will provide smaller settling time or other quality indicators.

4. Development of control system for the block of centrifugal extractors

Centrifugal extractors are widely used in chemical and atom industries (Vedantam et al., 2012). Chain of extractors allows providing high extraction level of the target components. Extractor description is presented in details (Vedantam et al., 2012). The developing control system is a small part of global Russian project of nuclear reprocessing.

Extractant is tributyl phosphate kerosene mixture. The mathematical model of extraction column, described in the article (Goryunov and Mikhaylov, 2012), could be easily adapted for modelling of centrifugal extractor by changing the cell of column to annular mixing zone of centrifugal extractor. Separator could be approximate by an ideal mixing model. The model contains 12 centrifugal extractors that are connected by backflow mode. Target component is uranium. Control provided by valve position. Sensor is uranium concentration meter. Sample time of sensor is equal to half minute. The control system is presented in Figure 5. Disturbance is changing uranium concentration in inlet flow.

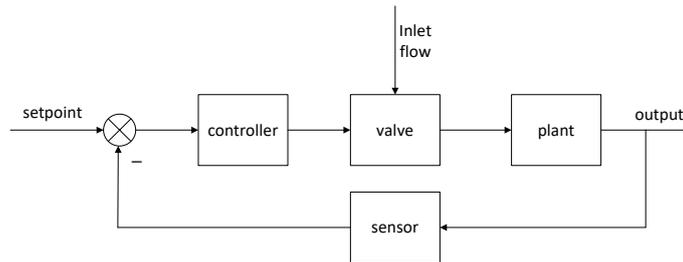


Figure 5: Structure of extractors control system

Plant identification (with valve and sensor) result near the working point (40 - 60 % of sensor scale) is transfer function:

$$G(s) = \frac{175.62}{(900s + 1)} \quad (12)$$

In practice controllers have digital form, so the discrete transfer function is

$$C(z) = \left(k_p + \frac{k_i T_s}{z-1} + \frac{k_d (z-1)}{T_s (z-1) + T_s} \right), \quad (13)$$

where T_s is the sampling time. Because of sample time of sensor is 30 s, T_s is also equal 30 s.

The control system has to provide minimum overshoot for step response, i.e. the system should be conservative. For this reason crossover frequency should be small. We propose to choose $\omega_c = 0.01$ rad/s. Phase margin is 80° . PID controller parameters are $k_p = 5.39 \times 10^{-2}$, $k_i = 4.41 \times 10^{-5}$, $k_d = -1.2$, $T_i = 43.5$.

Modelling results are presented in Figure 6. Concentration is shown in percentages of sensor scale.

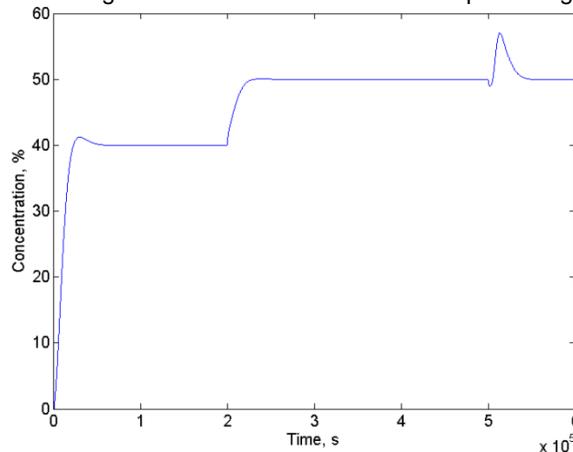


Figure 6: Step response and disturbance attenuation of the developed control system for block centrifugal extractors

As seen of Figure 7, developed PID controller provides small overshoot for step response and disturbance of changing uranium concentration in inlet flow. In addition, developed control system allows to control straight after start-up, despite of the plant in start-up mode is quasi-linear.

5. Conclusions

The proposed method allows to find PID controller with first order filter for derivative part and provides desired phase margin. Crossover frequency is used in the method as a tuning parameter. The article presents clear mathematical description of proposed method. Advantages are possibility to provide PID tuning for high order plants with pure delay, provides desired time domain characteristics by vary of crossover frequency. Control system for block of centrifugal extractors is developed: presents plant identification and PID controller was tuned. According to modeling results, developed system provides small overshoot and acceptable disturbance attenuation.

Reference

- Moradi H., Vossoughi G., 2015, Robust control of the variable speed wind turbines in the presence of uncertainties: A comparison between H^∞ and PID controllers, *Energy*, 90, 1508-1521
- Kristiansson B., Lennartson B., 2006, Evaluation and simple tuning of PID controllers with high frequency robustness, *Journal of Process Control*, 16(2), 91-103, DOI: 10.1016/j.jprocont.2005.05.006.
- Sanchis R., Romero J.A., Balaguer P., 2010, Tuning of PID controllers based on simplified single parameter optimisation, *International Journal of Control*, 83(9), 1785-1798, DOI: 10.1080/00207179.2010.495162.
- Vadutov O.S., 2014, Design of PID controller for delayed systems using optimization technique under pole assignment constraints, *Bulletin of the Tomsk Polytechnic University*, 325(5), 16-22 (in Russian).
- Aström K.J., Panagopoulos H., Hägglund T., 1998, Design of PI Controllers based on Non-Convex Optimization, *Automatica*, 34(5), 585-601, DOI: 10.1016/S0005-1098(98)00011-9.
- Mikhalevich S.S., Baydali S.A., Manenti F., 2015, Development of a tunable method for PID controllers to achieve the desired phase margin, *Journal of Process Control*, 25, 28-34.
- Mikhalevich S.S., Rossi F., Manenti F., Baydali S.A., 2015, Robust PI/PID controller design for the reliable control of plug flow reactor, *Chemical Engineering Transactions*, 43, 1525-1530.
- Aström K.J., Hägglund T., 2005, *Advanced PID Control* in Research Triangle Park, NC 27709, ISA - Instrumentation, Systems, and Automation Society, North Carolina, USA..
- Goryunov A.G., Mikhaylov V.S., 2012, The automatic control system of a multi-component nonequilibrium extraction process in the pulse column, *Journal of Process Control*, 22(6), 1034-1043, DOI: 10.1016/j.jprocont.2012.04.009.
- Ziegler J.G., Nichols N.B., 1942, Optimum Settings for Automatic Controllers, *Transactions of the A.S.M.E.*, 115, 759-768.
- Vedantam S., Wardle K.E., Tamhane T.V., Ranade V.V., Joshi J.B., 2012, CFD Simulation of Annular Centrifugal Extractors, *International Journal of Chemical Engineering*, 2012, 2-31, DOI: 10.1155/2012/759397