Research on Productivity for Multi-stage Fracturing of Horizontal Wells

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Hydraulic fracture technology has been widely used to exploit the tight and low permeability reservoir to enhance well productivity. In this paper, the multi-stage fracturing of horizontal wells productivity model is established based on Green's Function. Varying pattern of productivity can be obtained under the conditions of water flooding. The calculation result shows that production ratio of different fractures are same on the earlier stage by using elastic drive. And the production ratio of fractures increased near by the flooding boundary and decreased far from the flooding boundary in the later stage. The result of the production forecast model of multi-stage fracture of horizontal well with water flooding shown that, even in the condition of water flooding, production of multi-stage fracture horizontal well mainly depend on formation energy.

1. Introduction

Tight reservoirs are defined as reservoirs that can’t be recovered at profitable flow rates or produce economic volumes of hydrocarbons except a special technique is applied to stimulate reservoir. In such reservoirs, the industry applies enormous hydraulic fracture technology, horizontal wellbore or multilateral wellbores to increase flow rates and enhance the recovery efficiency in the reservoir. Hydraulic fracture technology which can generate high conductivity fracture network is one of the most effective and practice ways in the development of unconventional reservoirs. And multi-stage fracture technology of horizontal wells is an effective technology for the development of low permeability and tight reservoirs, which had been widely used in oil field. Currently, the research of the horizontal well hydraulic fracture is focusing on the techniques, although the relevant theoretical study is not enough.

Giger et al. (1984) proposed quick analytical model to predict the productivity index for horizontal wells. Soliman et al. (1990) discussed the fracture orientation in a horizontal wellbore, located a horizontal well to optimize fracture height, number of fractures. The mechanism of fluid flow in a horizontal well with hydraulic fractures was also discussed in their work. Larsen and Hegre (1991) reported general solutions for multi-fractured horizontal wells. Laplace transformed, semi-analytical solutions for the pressure transient behavior of finite conductivity fractures in horizontal wells for transverse and longitudinal fractures were presented.

Wan and Aziz (2002) presented a semi-analytical solution for hydraulic fractured horizontal wells. Wei and Economides (2005) presented a practical model for predicting well performance of transverse hydraulic fractures in horizontal wells both oil and gas reservoirs. Guo et al., (2009) presented a simplified analytical model to predict the production performance of Multi Fractured Horizontal well for oil and gas reservoirs. Their proposed model is applied for evaluation as well as optimization of multi-fractured horizontal wells.

Xiaoquan Wang (2009) presented the rule of oil well's cumulative production change in different fracture parameters by using Eclipse software. Chunqin Li (2011) presented the fractured horizontal well is equivalent to some vertical well by using the method of equivalent horizontal well radius. After running this method, it could give a formula about steady state fracturing productivity. Khan et al., (2014) presented a correlation that can predict the skin during the study of pre-fracture analysis, to see the effect of production increase as a result of Hydraulic fracturing in tight reservoirs for vertical wells.

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Most researchers have optimized the fracture parameters to analysis the production capacity, but none of them has researched the adaptation of development method by mathematical analysis method. In this paper, based on the Green's function method, it researched the analytical method on the multiple stage fracture of horizontal well production. Using the changing rule of Multiple stage fracture horizontal well production capacity under water flooding condition to discuss the development method of low permeability reservoirs. This research conclusion can also be used to observe the effect of the rational development of Tight and low permeability oil field that will help to increase the outcome from hydraulic fracture.

2 Mathematical model

Tight and low permeability oil reservoir is a typical dual media system made of nature or induced fracture and matrix, in which fracture is the main flow channel and matrix is the main storage space. The schematic diagram of tight oil horizontal well multi-stage fracture theoretical model is showing in Fig. 1:

![Schematic diagram of tight oil horizontal well multiple-stage fracture theoretical model](image1)

By multiple-stage fracture technology to generate some hydraulic fractures which is independent around horizontal well-bore. Assume the formation length, width, respectively is $X_e$, $Y_e$, formation thickness is $h$, fracture is parallel to the X direction, fracture length is $X_f$, fractures are infinite conductivity, regardless of the fracture width, constant pressure boundary is parallel to the X axis, closed boundary is parallel to the Y axis. As shown in Fig. 2.

![The Multi-stage fracture horizontal well seepage physical model](image2)

2.1 Single fracture pressure drop expression

According to the method of Green's function and Newman method, a single fracture of instantaneous source function has the following forms of expression:

$$G(x_n, y_n, x, y, t) = X(x_n, x, t) \cdot Y(y_n, y, t)$$  \hspace{1cm} (1)

Where:
According to relationship between the pressure drop and the instantaneous source function:

\[ p_0 - p = \frac{ds}{\varphi c} G(x, y, t) \quad (2) \]

Where: \( ds \) is the instantaneous source strength of unit length fracture, unit the thickness of the formation:

\[ ds = Q(\tau) \frac{1}{h} \frac{1}{x_f} \cdot d\tau \]

Under the condition of continuous source of pressure drop:

\[ p_0 - p(x, y, t) = \frac{1}{\varphi c} \frac{1}{h} \int_{\tau_1}^{\tau_2} Q(\tau) G(x, y, t) d\tau \quad (3) \]

### 2.2 Multiple fracture pressure drop expression

The pressure generated in the fracture \( i \) that it is the pressure drop in fracture \( j \) can be written as the following:

\[ \Delta p_0 = \frac{1}{\varphi c} \frac{2}{h} \frac{1}{x_f y_e} \int_{0}^{t} X(x_i, x_j, t) \cdot Y(y_i, y_j, t) \cdot d\tau \quad (4) \]

Where:

\[ X(x_i, x_j, t) = 1 + \frac{4x_e}{x_f \pi} \]

\[ \sum_{n=1}^{\infty} \frac{1}{n} \exp\left(-\frac{n^2 \pi^2 \eta t}{x_e^2}\right) \sin\left(\frac{n\pi x_f}{2x_e}\right) \cos\left(\frac{n\pi x_w}{x_e}\right) \cos\left(\frac{n\pi x}{x_e}\right) \]

\[ Y(y_i, y_j, t) = \sum_{n=1}^{\infty} \exp\left(-\frac{n^2 \pi^2 \eta t}{y_e^2}\right) \sin\left(\frac{n\pi y_f}{2y_e}\right) \sin\left(\frac{n\pi y_w}{y_e}\right) \sin\left(\frac{n\pi y}{y_e}\right) \]

The pressure generated by \( n \) fractures that their pressure drop in fracture \( j \) the can be written as:

\[ p_0 - p_w = \frac{1}{\varphi c} \frac{2}{h} \frac{1}{x_f y_e} \int_{0}^{t} \sum_{i=1}^{n} G_i dt \quad (5) \]

For \( n \) fractures, there are \( n \) equations that can make a following equation set:
\[
\begin{align*}
\rho_0 - \rho_v &= \frac{1}{\varphi c} \frac{1}{h} \frac{2}{x_y y_e} \int_0^t \sum_{i=1}^\infty Q_{iG1} dt \\
\rho_0 - \rho_v &= \frac{1}{\varphi c} \frac{1}{h} \frac{2}{x_y y_e} \int_0^t \sum_{i=1}^\infty Q_{iG2} dt \\
\cdots \\
\rho_0 - \rho_v &= \frac{1}{\varphi c} \frac{1}{h} \frac{2}{x_y y_e} \int_0^t \sum_{i=1}^\infty Q_{iGn} dt 
\end{align*}
\] (6)

According to the above equations (such as Eq(4), Eq(5), Eq(6)), Solving the system of equations, can get the all fracture's production rate.

3. Case calculation and analysis

3.1 Reservoir basic data

The basic data of fractured wells and formation of a tight oil reservoir in Changqing Oilfield are as the following:

The tight oil reservoir development by horizontal well with five spot water flooding injection-production pattern (Fig. 3). The length of the horizontal well production unit \( (Ye) \) is 600 m, and the width \( (Xe) \) is 400 m. There are 5 fractures distribute uniform in the production unit. The fractures interval \( (L) \) is 100 m, fracture length \( (Xf) \) is 400 m. The fractures are symmetrically distributed along the wellbore. Fluid viscosity \( (\mu) \) is 1.01 mPa·s, formation thickness \( (h) \) is 7.1 m, supply boundary pressure \( (Pe) \) is 16.8 MPa. Assume the bottom-hole flowing pressure is equal to the formation saturation pressure \( (Pw) \) is 7.25 MPa.

Figure 3: Schematic diagram of tight oil horizontal well with five spot water flooding injection-production pattern

Table 1 and Table 2 show the production percentage of each fracture contributing to the total production of horizontal well with the various \( K0 \) values are 0.01 mD and 0.05 mD.

Table 1: The percentage of each fracture production \((K0 =0.01 \text{ mD})\)

<table>
<thead>
<tr>
<th>Time (day)</th>
<th>( Q_1/Q )</th>
<th>( Q_2/Q )</th>
<th>( Q_3/Q )</th>
<th>( Q_4/Q )</th>
<th>( Q_5/Q )</th>
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<tbody>
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<td>20</td>
<td>20</td>
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</tr>
<tr>
<td>60</td>
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<td>19.8</td>
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<td>19.48</td>
<td>19.72</td>
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</tr>
<tr>
<td>150</td>
<td>20.73</td>
<td>19.54</td>
<td>19.46</td>
<td>19.54</td>
<td>20.73</td>
</tr>
</tbody>
</table>
Table 2: The percentage of each fracture production (K0 = 0.05 mD)

<table>
<thead>
<tr>
<th>Time (day)</th>
<th>Q1/Q</th>
<th>Q2/Q</th>
<th>Q3/Q</th>
<th>Q4/Q</th>
<th>Q5/Q</th>
</tr>
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<tbody>
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<td>6.93</td>
<td>1.46</td>
<td>6.93</td>
<td>42.34</td>
</tr>
</tbody>
</table>

3.2 Results Analysis
According to the above data (Table 1 and Table 2), the results are as follows:
The percentage of the fracture production which was near the supply boundary increased, on the other hand, the percentage of the fracture production which was far away from the supply boundary decreased, with time increasing.

(2) Initially, the production of each fracture was at the same percentage. This is due to the formation fluid seepage in early unstable seepage, pressure wave did not propagate to the boundary, the influence of supply boundary is very small, almost every fracture's output were rely on the formation energy; At the same time, the more closer to the supply boundary, the higher production percentage is. This is because the pressure wave propagate to the supply boundary, the influence will increase. Because of the blocking effect from the fracture, only fracture 1 and 5 can fully effect, fracture 2, 3 and 4 are not near to the supply boundary, so they could not get the supplement from it.

(3) The higher the permeability was, the conduction velocity of pressure wave became faster, and the differentiation of fracturing production appeared faster.

4. Conclusions
With around 80 billion barrels reserves and recent increase in oil demand in China, there is no doubt that there would be tremendous effort on the development of low permeability reservoirs or tight oil in the next decades. The conclusions of this study can be summarized as followings:

(1) The mathematical model established can be used to calculate and analysis the production capability of multiple-stage fractured horizontal wells in this paper.

(2) Although multiple-stage fracturing can result in a complicated fracturing network, but only the fractures near the supply boundary can be fully affected by injection water. The supply boundary has less impact on the fractures far away from the supply boundary. These fractures which far away from the supply boundary will no longer provide production after the elastic energy of formation is fully released.

(3) Therefore, the tight oil reservoirs are developed by multiple-stage fractured horizontal wells should be developed by natural energy rather than water flooding.

(4) As it’s showing in the production forecast model of multi-stage fracture horizontal well with water injection, even in the condition of water injection, the variation of daily output per multi-stage fracture horizontal well mainly depend on formation energy.

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Nomenclatures

- P0——initial formation pressure, MPa
- Pw——bottom hole pressure, MPa
- Ye——the length of production unit, m
- Xe—— the width of production unit, m
- h——formation thickness, m
- K0——initial permeability, mD
- μ—— fluid viscosity, mPa·s
- c——composite compressibility, Mpa⁻¹
φ——porosity
Q——productivity of Horizontal well, m³/d
Qi——productivity of the Fracture i, m³/d

Reference
Wei Y., Economides M., 2005, Transverse Hydraulic Fractures from a Horizontal Well, Presented at SPE Annual Technical Conference and Exhibition, Dallas, 9-12 October, SPE-94671-MS, DOI: 10.2118/94671-MS.