Analysis and Evaluation of Relative Efficiency of Warehousing and Distribution Operations Based on Mixed DEA Model

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It is crucial that performance evaluations of transport and distribution be accurate and objective. Therefore, we should not only consider such enterprises from the perspective of cost and maximum benefit, but also from the broader angle of external costs and sustainable development. Ergo, with the aid of a mixed DEA model, this article will build a series of evaluation indexes. By solving the model, we have established a comprehensive performance evaluation which can reflect the quality of services, industry trends, external issues, along with providing a logistics warehouse on a more complete evaluation of socio-economic benefits evaluation system. These measures have certain scientific and practical values.

1. Introduction

Correct and objective evaluations of comprehensive performance in warehousing and distribution operation can not only can help related departments master the basic information, but also help managers to learn from experience and to improve their operation performance. Due to the significance of evaluation in improving and perfecting systems, logistics system performance evaluation has become a hotspot in the research of foreign academic circles.

Schefczyk (1993) reported he had integrated with the two evaluation technologies "productivity" and "data envelopment analysis (DEA)", and the results showed that there is a negative relation between the overall scale of warehouse facilities and their productivity. Hollingsworth (1995) reported that he wanted to establish a model to depict storage behavior, and obtained a better result by using data envelopment analysis. Hackman et al (2001) reported that he had established an evaluation index system, the input of which is labor, space and equipment, and the output of which is movement, storage and collection of cargo. The conclusions are as follows: (1) Small warehouses are often more efficient than large warehouses, (2) The low degree of an automation warehouse tends to be more efficient. And established the evaluation index system (Charnes A, Cooper W W, Rhodes E (1978)), McGinnis (2002) reported that large a warehouse is not more effective, at least in terms of the equipment and labor force, resulting in the majority of the warehouse being at or below 50% of the system operating efficiency. The Mercer Management Consulting firm (2012) reported constructing an Evaluation System from seven aspects, such as: the transport timeliness, damage rate control, timeliness of delivery, order fulfillment rate, transport accuracy, project completion rate, and inventory controlling. The above considered evaluation of costs and benefits is from the perspective of the enterprise, which can only reflected pure economic efficiency of enterprises in the production process. So we should consider the perspective of national or regional economic interests and the society as a whole, taking into account social contribution, evaluation trends and external costs, and so on. In order to fully investigate the business activities of the logistics industry, this paper will attempt to evaluate more comprehensively by using mixed DEA model from another perspective.
2. Evaluation indicators of warehousing and distribution operations

The production and business operation of any business or unit basically relies on the indicators of its facilities’ performance, environmental influences and cost, and so on. The output, meanwhile, is usually measured by the indicators of working efficiency, and is made more efficient by working class indicators, resources, and quality and efficiency of service. Minimum investment and maximum output should both be in line with social norms, which is to get the most comprehensive output. Most of these indicators have considered the factors of political, economic, technological and ecological environments.

In general, the input of warehousing and distribution operations needs a variety of resources and elements. This input is extremely broad, including input from human resources, facilities, equipment, and be also some input in other areas. This article will take these external costs into consideration. All in all, these are reflected in human, financial and physical factors, which can each be divided into several categories. As to the output indicators, we need to focus on their overall performance, fully reflecting their social value, such as resource efficiency, contribution to the city’s commodity circulation and their promote of the overall level of the industry. The main work is as follows: the quantity of services, the quality of service and the efficiency of service.

Considering the availability of data, and in order to get an accurate and reasonable evaluation, this article selected the following nine input and output indicators: (1) External costs (EC), (2) Concentrated quantity of goods (AQ), (3) Effective fulfillment of order forms (WF), (4) Efficient use of storage capacity (ES).

3. Mixed DEA model and its solution methods in terms of the evaluation of warehousing and distribution operation

The DEA model can be divided into two orientations: “input” and “output” planes. In general, the input elements are under the control of evaluation, yet the output is controlled by the market, and not by any internal factors. Thus this study wants to adopt “input-oriented” efficiency evaluation models. Suppose there are N decision making units, each DMU has five kinds of input and four outputs. The input and output vector of j-th decision making unit is \( X_j = (x_{1j}, \ldots, x_{mj}) \) \( \geq 0 \) and \( Y_j = (y_{1j}, \ldots, y_{nj}) \) \( \geq 0 \), \( j = 1, \ldots, n \) respectively, DMU; \( x_{ij} \) indicates the j-th input and the i-th inputs. \( y_{ij} \) indicates the j-th DMU and the i-th inputs. DMU\(_0\) being valued is the \( j_0 \)-th DMU. (For simplicity, denoted DMU as DMU\(_0\), the same as below). To have a better understanding of the factors affecting the performance of integrated warehousing and distribution operations, this article combined the DEA model with the Tobit regression method. The detailed explanation of the two phrase is as follows:

Phrase I: DMU\(_0\) evaluation model based the relative effectiveness of input Production possibility set:

\[
T = \left\{ (x_{1j}, \ldots, x_{mj}, y_{1j}, \ldots, y_{nj}) | \sum_{j=1}^{n} WS \lambda_j \leq WS, \sum_{j=1}^{n} LT \lambda_j \leq LT \right. \\
\left. \sum_{j=1}^{n} CC \lambda_j \leq CC, \sum_{j=1}^{n} EC \lambda_j \leq EC, \sum_{j=1}^{n} ECC \lambda_j \leq ECC, \sum_{j=1}^{n} TQ \lambda_j \geq TQ, \sum_{j=1}^{n} AQ \lambda_j \geq AQ \right. \\
\left. \sum_{j=1}^{n} WF \lambda_j \geq WF, \sum_{j=1}^{n} ES \lambda_j \geq ES, \lambda_j \geq 0, j = 1, 2, \ldots, n \right\} 
\]

(4-1)

In order to analyze the efficiency of the evaluation and decision making unit, and consider the ease of operation and some specific purpose, we can utilize the form of the dual integrated DEA model as shown below:

\[
\begin{align*}
\min & \quad \theta \\
\text{s.t.} & \quad \sum_{j=1}^{n} WS_j \cdot \lambda_j \leq \theta \cdot WS_0 \\
& \quad \sum_{j=1}^{n} CC_j \cdot \lambda_j \leq \theta \cdot CC_0 \\
& \quad \sum_{j=1}^{n} EC_j \cdot \lambda_j \leq \theta \cdot EC_0 \\
& \quad \sum_{j=1}^{n} LT_j \cdot \lambda_j \leq \theta \cdot LT_0 \\
& \quad \sum_{j=1}^{n} ECC_j \cdot \lambda_j \leq \theta \cdot ECC_0 \\
& \quad \sum_{j=1}^{n} TQ_j \cdot \lambda_j \geq TQ_0 \\
& \quad \sum_{j=1}^{n} AQ_j \cdot \lambda_j \geq AQ_0 \\
& \quad \sum_{j=1}^{n} ES_j \cdot \lambda_j \geq ES_0 \\
& \quad \sum_{j=1}^{n} WF_j \cdot \lambda_j \geq WF_0
\end{align*}
\]

(4-2)
\[ \delta_i \left( \sum_{j=1}^{n} \lambda_j + \delta_2 \cdot (-1)^{n+1} \lambda_{n+1} \right) = \delta_i \]

\[ \lambda_j \geq 0, \quad j = 1, 2, \ldots, n, n+1, \theta \in E^1 \]

WS: stands for the input value of the storage area of the j-th DMU;
CC: stands for the input value of the construction cost of the j-th DMU;
EC: stands for the input value of the facility cost of the j-th DMU;
LT: stands for the input value of the work hour of the j-th DMU;
ECC: stands for the input value of the external cost of the j-th DMU;
TQ: stands for the output value of the cargo throughout the j-th DMU;
AQ: stands for the output value of pickup and delivery of the j-th DMU;
WF: stands for the output value of order fulfillment of the j-th DMU;
\( \theta \): stands for the overall effectiveness of each sample logistics unit;
\( \lambda_j \): stands for the combination rate of the j-th DMU to re-construct a DMU group.

To evaluate a decision-making unit \( j = \{1, \ldots, n\} \), ECC is an unexpected index among all the indicators in the evaluation warehousing and distribution operations. Generally, the cost index have been handled as output indicators in the DEA model, characterized as small as possible, so this article will deal with them as input indicators (Gu J X, et al. (2010)). We also note that the input indicators include storage areas (WS), labor hours (LT), construction costs (CC), the equipment cost (EC), and external costs (ECC). All are absolute indicators. The output indicators, due to the market research data and indicators set are concentrated quantity of goods (AQ), Effective fulfillment of Order forms (WF), efficient use of storage capacity (ES) are merely relative indicators, yet the cargo throughput (TQ) is an absolute indicator. Further, when WS, LT, CC, EC, ECC increases their \( r \) times, accordingly, TQ also increases its \( r \) times, yet AQ, WF, and ES remained unchanged. The first two indicators are called as cone index, and the third is called the non-cone indicator.

Based on the documents (Hackman S T et al. (2001)) we can get the form of the production possibility based on the invalid and minimal criterion:

\[
T - \left\{ (WS, LT, CC, EC, ECC, TQ, AQ, WF, ES) \left| \sum_{j=1}^{n} \lambda_j \leq WS, \sum_{j=1}^{n} LT \lambda_j \leq LT \right. \right. \\
\left. \sum_{j=1}^{n} CC \lambda_j \leq CC \right. \left. \sum_{j=1}^{n} EC \lambda_j \right. \left. \sum_{j=1}^{n} ECC \lambda_j \leq ECC \right. \left. \sum_{j=1}^{n} TQ \lambda_j \right. \left. \sum_{j=1}^{n} AQ \lambda_j \right. \right. \\
\left. \sum_{j=1}^{n} WF \lambda_j \right. \left. \sum_{j=1}^{n} ES \lambda_j \right. \left. \sum_{j=1}^{n} \lambda_j \geq 0, j = 1, 2, \ldots, n \right\} \right. \\
(4-3)
\]

Theoretically, the C²R model constantly returns to scale, and is used to evaluate the effectiveness of the decision-making unit. BC² (also known C²GS) model is variable and returns to scale. If we use them to assess, we may ignore some important factors. To ensure a scientific and objective evaluation, we need to build a mixed structure DEA model which could handle cone and non-cone indicators simultaneously, and consider the form of the dual model of equation (with slack variables and a non-Archimedean infinitesimal \( \delta \)). Based on the production possibility set \( T \), the input-oriented mixed DEA model is as follows:
\[
\min \theta - \varepsilon \left( s_{WS} + s_{LT} + s_{CC} + s_{EC} + s_{ECC} + s_{TQ} + r_{AQ} + r_{WF} + r_{ES} \right) \\
\text{s.t.} \sum_{j=1}^{n} \lambda_j \cdot WS_j + s_{WS} = \theta \cdot WS_n \\
\sum_{j=1}^{n} \lambda_j \cdot LT_j + s_{LT} = \theta \cdot LT_n \\
\sum_{j=1}^{n} \lambda_j \cdot CC_j + s_{CC} = \theta \cdot CC_n \\
\sum_{j=1}^{n} \lambda_j \cdot EC_j + s_{EC} = \theta \cdot EC_n \\
\sum_{j=1}^{n} \lambda_j \cdot ECC_j + s_{ECC} = \theta \cdot ECC_n \\
\sum_{j=1}^{n} \lambda_j \cdot TQ_j - s_{TQ} = \theta \cdot TQ_n \\
\sum_{j=1}^{n} \lambda_j \cdot AQ_j - r_{AQ} = \gamma \cdot AQ_n \\
\sum_{j=1}^{n} \lambda_j \cdot WF_j - r_{WF} = \gamma \cdot WF_n \\
\sum_{j=1}^{n} \lambda_j \cdot ES_j - r_{ES} = \gamma \cdot ES_n \\
\sum_{j=1}^{n} \lambda_j = \gamma, \lambda_j \geq 0, \gamma \geq 0, j = 1, 2, \ldots, n \\
\text{s.t.} s_{WS}, s_{LT}, s_{CC}, s_{EC}, s_{ECC}, s_{TQ}, r_{AQ}, r_{WF}, r_{ES}, \lambda^*_j \geq 0 \]

\text{are called slack variables, while } s_{WS}, s_{LT}, s_{CC}, s_{EC}, s_{ECC}, s_{TQ}, r_{AQ}, r_{WF}, r_{ES}, \lambda^*_j \text{ stands for overload input, } s_{WS}, s_{LT}, s_{CC}, s_{EC}, s_{ECC}, s_{TQ}, r_{AQ}, r_{WF}, r_{ES}, \lambda^*_j \text{ stands for overload output. } \gamma \text{ means the index number met the cone condition, while } \theta', \gamma', s_{WS}', s_{LT}', s_{CC}', s_{EC}', s_{ECC}', s_{TQ}', r_{AQ}', r_{WF}', r_{ES}', \lambda^*_j \text{ are the optimal solution of dual programs.}

From an economic sense, the effective decision-making unit obtained from the above the model shows both technical effectiveness and scale efficiency. SchefczykM (1993) can proved the following theorem.

If \( \theta' > 1 \) and \( s_{WS} + s_{LT} + s_{CC} + s_{EC} + s_{ECC} + s_{TQ} + r_{AQ} + r_{WF} + r_{ES} = 0 \), DMU0 is DEA efficiency. It means that the input has reached optimal output in an evaluation of objects.

If \( \theta' < 1 \) and \( s_{WS} + s_{LT} + s_{CC} + s_{EC} + s_{ECC} + s_{TQ} + r_{AQ} + r_{WF} + r_{ES} > 0 \), DMU0 is weak DEA efficiency. It means that if the input \( X_0 \) is reduced by \( s \), yet the output \( Y_0 \) remains unchanged, or if the \( X_0 \) remains unchanged, the output could increase by \( s \).

Step 1 solving the following model to obtain \( \theta' \):

\[
\min \theta \\
\text{s.t.} \sum_{j=1}^{n} \lambda_j \cdot WS_j + s_{WS} = \theta \cdot WS_n \\
\sum_{j=1}^{n} \lambda_j \cdot LT_j + s_{LT} = \theta \cdot LT_n \\
\sum_{j=1}^{n} \lambda_j \cdot CC_j + s_{CC} = \theta \cdot CC_n \\
\sum_{j=1}^{n} \lambda_j \cdot EC_j + s_{EC} = \theta \cdot EC_n \\
\sum_{j=1}^{n} \lambda_j \cdot ECC_j + s_{ECC} = \theta \cdot ECC_n \\
\sum_{j=1}^{n} \lambda_j \cdot TQ_j - s_{TQ} = \theta \cdot TQ_n \\
\sum_{j=1}^{n} \lambda_j \cdot AQ_j - r_{AQ} = \gamma \cdot AQ_n \\
\sum_{j=1}^{n} \lambda_j \cdot WF_j - r_{WF} = \gamma \cdot WF_n \\
\sum_{j=1}^{n} \lambda_j \cdot ES_j - r_{ES} = \gamma \cdot ES_n \\
\sum_{j=1}^{n} \lambda_j = \gamma, \lambda_j \geq 0, \gamma \geq 0, j = 1, 2, \ldots, n \\
\text{s.t. } s_{WS}, s_{LT}, s_{CC}, s_{EC}, s_{ECC}, s_{TQ}, r_{AQ}, r_{WF}, r_{ES} \geq 0
\]
\[
\begin{align*}
\text{max} & \quad \left( s_{w_0} + s_{w_1} + s_{w_2} + s_{w_3} + s_{w_4} + s_{w_5} + s_{w_6} + s_{w_7} + s_{w_8} + s_{w_9} + s_{w_{10}} + s_{w_{11}} + s_{w_{12}} \right) \\
\text{s.t.} & \quad \sum_{j=1}^{n} \beta_j \cdot WS_j + s_{w_0} = \theta \cdot WS_n \\
& \quad \sum_{j=1}^{n} \beta_j \cdot LT_j + s_{w_0} = \theta \cdot LT_n \\
& \quad \sum_{j=1}^{n} \beta_j \cdot CC_j + s_{w_0} = \theta \cdot CC_n \\
& \quad \sum_{j=1}^{n} \beta_j \cdot ECC_j + s_{w_0} = \theta \cdot ECC_n \\
& \quad \sum_{j=1}^{n} \beta_j \cdot TQ_j + s_{w_0} = \theta \cdot TQ_n \\
& \quad \sum_{j=1}^{n} \beta_j \cdot AQ_j + s_{w_0} = \theta \cdot AQ_n \\
& \quad \sum_{j=1}^{n} \beta_j \cdot WF_j + s_{w_0} = \theta \cdot WF_n \\
& \quad \sum_{j=1}^{n} \beta_j \cdot ES_j + s_{w_0} = \theta \cdot ES_n \\
& \quad \sum_{j=1}^{n} \beta_j = \gamma_j \geq 0, \gamma_j \geq 0, j = 1, 2, \ldots, n \\
& \quad s_{w_0}, s_{w_1}, s_{w_2}, s_{w_3}, s_{w_4}, s_{w_5}, s_{w_6}, s_{w_7}, s_{w_8}, s_{w_9}, s_{w_{10}}, s_{w_{11}}, s_{w_{12}} \geq 0
\end{align*}
\]

Step 2 put \( \theta \) to the following model and get \( \gamma^*, s_{w_0}^*, s_{w_1}^*, s_{w_2}^*, s_{w_3}^*, s_{w_4}^*, s_{w_5}^*, s_{w_6}^*, s_{w_7}^*, s_{w_8}^*, s_{w_9}^*, s_{w_{10}}^*, s_{w_{11}}^*, s_{w_{12}}^* \) is the solution of the original mixed DEA model.

Phrase II: Different external environments have produced different strategic decisions and so on that can influence efficiency. Considering the external environment of different evaluation objects, the initial DEA value is set as explanatory variables, and the environment variables are set as explanatory variables. Then we construct an appropriate regression model to test whether the environment variable is significant to the influence of comprehensive performance.

After the first phrase of mixed DEA model, we get the efficiency values of each storage unit. The value is between 0 and 1. If we use the least squares regression directly, it is estimated the results will be biased and inconsistent, due to the following data interception problem: less than or equal to 0 and greater than 1. Therefore, we consider to use the limited dependent variable model proposed by economist Tobin. The efficiency evaluation's results are regarded as a dependent variable, and the environment variable is regarded as the independent variable. Then we can carry out the influencing factors analysis of DEA efficiency values.

In the Tobit regression model, the data should be reviewed, all the negative value of dependent variables are defined as a value of 0, rather than simply removed from the sample. Thus, it is also known as examining a return model. The basic structure of the Tobit model can be expressed as follows:

\[
y_i = X_i \beta + u_i, \quad \text{if } X_i \beta + u_i > 0
\]
\[
y_i = 0, \quad \text{if } X_i \beta + u_i \leq 0
\]

Where \( N \) is the sample's quantity, \( y_i \) is a dependent variable, \( x_i \) is an explanatory variable, \( \beta \) is a coefficient and is the error term of the independent variable. Assuming that \( u \sim N(0, \sigma) \), the parameter estimations are \( \hat{\beta} \) and \( \hat{\sigma} \) and thus \( y_i \sim N(x_i \hat{\beta}, \hat{\sigma}) \).

We can review the parameters of the regression model with a great likelihood estimated, \( u \sim N(0, \sigma) \). Assuming that \( f \) and \( F \) is \( f^* \) and \( F^* \) respectively is \( f = f(X, \beta, \sigma) \) and \( F = F(X, \beta, \sigma) \). When \( Y = 0 \), the independent variables are \( X_i, \beta + \varepsilon_i < 0 \) the occurrence rate of event \( \{ Y=0 \} \) is:

\[
P(Y=0) = P(\varepsilon_i < -X_i \beta) = P\left( \varepsilon_i < -\frac{X_i \beta}{\sigma} \right) = 1 - F_i
\]

If \( Y > 0 \), the odds of this event still obey normal distribution. When \( E(Y) = X_i \beta, Var(Y) = Var(\varepsilon_i) = \sigma^2 \), thus

\[
P(Y > 0) = \frac{1}{\sqrt{2\pi \sigma^2}} \int_{-\infty}^{0} e^{-\frac{(x-0)^2}{2\sigma^2}} \, dx
\]

Assuming that there are \( n \) samples, samples meet the condition that \( Y=0 \) samples also meet the condition that \( Y > 0 \). Then we can get the likelihood function of all the samples:
\begin{align*}
I &= \prod_{i=1}^{n} (1-F_i) \prod_{i=1}^{n} \left(2\pi \sigma^2 \right)^{\frac{i}{2}} \exp \left[ -\frac{(Y_i - X_i \beta)^2}{2\sigma^2} \right] \\
\text{Its logarithmic function is:} \\
L &= \ln l = \sum_{i=1}^{n} \ln (1-F_i) - \frac{1}{2} \sum_{i=1}^{n} \left( \frac{(Y_i - X_i \beta)^2}{2\sigma^2} + \ln \sigma^2 + \ln \left( 2\pi \right) \right)
\end{align*}

(4-10)

After solving the above function about $\beta, \sigma^2$, we can get the maximum value, that is, the estimate parameters (Jinjin Shi (2013)).

4. Conclusions

In this paper, we constructed warehousing and delivery logistics sub-processes and economic evaluation networks in the DEA model. This was based on the establishment of a building, storage, distribution, and selected the corresponding input and output indicators before finding solving methods, or steps, respectively, based on estimates of the data results collected. Furthermore, the paper constructed a "DEA-Tobit evaluation model" and introduced the mixed DEA model. It showed how the mixed DEA model, to some extent, overcomes the limitations of traditional models, and how its handle and non-tapered cone index indicators have greater effects, including the establishment of quality of service to reflect industry trends. Finally, the paper detailed how comprehensive Performance Evaluation of external issues provides a set of socio-economic benefits, before showing how evaluation on logistics and warehousing in a more comprehensive evaluation system, is both scientific and practical.

References

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