Study on Supply Chain Buy-Back Contract Model under Risk-averse Condition

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In this paper, supposing a supply chain composed of a risk-neutral supplier and a risk-averse retailer, the buy-back contract with a risk-averse retailer is designed and modelled, the strategy of providing the buy-back contract to increase revenue and coordinate the supply chain is analyzed. It uses downside risk restriction to measure the degree of risk the retailer is eager to assume. It also analyses the return policy to coordinate the supply chain under risk-averse condition. Finally, a simulation shows the accuracy and effectiveness of the return policy.

1. Introduction

Supply chain coordination is the key issue of supply chain management, and the contract is an important way which is widely used to achieve supply chain coordination. In the supply chain, proper information and incentives is provided to optimize the supply chain system, maximize the profits of the supplier and retailer and share the income and risk by the members. The common contracts include wholesale price contract, revenue sharing contract, buy-back contract, quantity flexibility contract and so on. Buy-back contract is one of the most common supply chain coordination mechanisms.

Much of the research on supply chain contract has assumed that the agents in the supply chain are risk neutral. They maximize their respective expected profits or minimize the expected cost, these contracts are not apply to the supply chain consisting of risk-averse partners. Because of the uncertainty of the market environment, the partner of risk-averse will choose to avoid the risk. In most cases, supply chain contract mechanism cannot be implemented effectively when ignoring the impact of risk. So, how to coordinate the supply chain when consider the risk preference is increasingly becoming one of the focus issues of supply chain management.

Yoo et al. (2015) reported pricing and return policies under various supply contracts in a closed-loop supply chain in which a supplier has more bargaining power than a retailer. Ruiz-Benitez and Muriel (2014) showed higher profits and better coordination are achieved when buyer and vendor acting in a decentralized fashion do not consider any information about consumer returns. Kulkarni et al. (2015) considered that perfect coordination of partners will help them improve their profits considerably. Amin-Naseri and Khojasteh (2015) developed a price competition model under a demand uncertainty environment between two leader-follower supply chains that each of them consists of one risk-neutral manufacturer and one risk-averse retailer. Glock and Kim (2015) indicated that the type of competition is of major importance for the structure of the supply chain after the merger, and that under certain conditions, the merger may benefit all parties involved, i.e. the vendor, the retailers, and the consumers. Chernonog and Kogan (2014) found that the risk-averse retailer does not necessarily order less than the risk-neutral one and may introduce a bias by choosing a specific demand distribution. Kim and Park (2014) questioned if each kind of risk management contract can improve the utility of all supply chain members compared to the utility without any of those, and how the conditions to achieve such improvements are different. Budde and Minner (2014) investigated a newsvendor-type retailer sourcing problem under demand uncertainty who has the option to source from multiple suppliers. Yoo (2014) considered the supplier's different risk attitudes, whether risk averse or risk neutral and indicated the optimal conditions for a generous return policy setting without quality enhancement. Guler and Keskin (2013) analyzed coordination in a supply chain with random yield and random demand (SCRYRD). Chung and Erfun (2013)
studied the case in which the supplier needs to account for both old and young units. Arcelus et al. (2012) evaluated the pricing and ordering policies of a retailer, facing a price-dependent stochastic demand, within a newsvendor framework, under different degrees of risk tolerance and under a variety of optimizing objectives. Takahashi et al. (2011) calculated the optimal delivery time to alleviate the risk. Adida and DeMiguel (2011) showed, unlike in the symmetric chain, the asymmetric chain efficiency depends on product differentiation and risk aversion because of the interaction between these features and the asymmetry of manufacturers and retailers. Oezen et al. (2010) focused on cooperation among the retailers—the retailers coordinate their initial orders and can reallocate their orders in the warehouse after they receive more information about their demand and update their demand forecasts. Fisher and Raman (1996) reported much of the research on supply chain contract has assumed that the agents in the supply chain are risk neutral.

This paper analyzes the impact of retailer’s risk preference on the strategy of supply chain collaboration by considering downside risk factor, and coordinate the supply chain through buy-back contract when considering salvage of the unsold products.

2. Decision-making model of the decentralized supply chain in the risk-neutral case

2.1 Background description

In this paper, the supply chain consists of a risk-neutral supplier and a risk-averse retailer. The market demand is random and is subject to a known distribution. We first introduce the following notations:

- \( w \): the per-unit wholesale price at which the supplier sells to the retailer
- \( c \): the per-unit cost of the product
- \( b \): the per-unit buy-back price of the unsold product from the retailer
- \( Q \): the order quantity of the retailer
- \( p \): the per-unit price at which the retailer sells the product
- \( r \): the per-unit salvage of product handled by the supplier
- \( v \): the per-unit salvage of product handled by the retailer
- \( \alpha \): the target profit of the risk-averse retailer
- \( \beta \): the probability of target profit of the risk-averse retailer
- \( D \): the random demand of the consumer, the density function \( f(x) \) and distribution function \( F(x) \) are known. \( F(x) \) is continuous, differentiable and reversible in \( x \), and assume \( w > c, r > v \).

2.2 Decentralized supply chain

In the centralized supply chain, whether the retailers are risk-averse or not, the market risk faced by the suppliers can fully be offset by buy-back contract. But in the decentralized supply chain, retailers and suppliers are independent from each other, they pursue their maximum benefits. Thus the retailer decides his optimal order with the following model:

\[
\max E[\pi^0_{\text{D, r}}(\min(Q, D))] = \min (\pi^0_{\text{D, r}}(D), \pi^0_{\text{D, r}}(Q))
\]

So the retailer’s optimal order quantity is:

\[
Q^0_{\text{D, r}} = \frac{F^{-1}(D - w)}{p - v}
\]

3. Buy-back contract model for the supply chain in the risk-adverse case

3.1 Description of the risk-averse retailers

Risk-averse decision-making preferences are the most common tool to analyze the behavior of supply chain management decision; there are many ways to describe risk-averse preference. This section will use the concept of risk-aversion to measure the risk-averse extent of the retailer. The probability of the retailer’s profit is less than or equal to his target profit, is defined as downside risk. Assuming that \( \alpha \) is retailer’s target profit, the downside risk can be described as \( P(\pi_{\text{D, r}}(Q, D) \leq \alpha) \).

The newsvendor wants to choose an order quantity \( Q \) so as to maximize his expected profit \( E[\pi_{\text{D, r}}(Q, D)] \), while specifying that his actual profit should not fall below his target profit level of \( \alpha \) with a probability exceeding a specified \( \beta \). Based on above, the model can be expressed as:
\[
\max_{Q \geq 0} E[\pi_r(\min(Q,D))] \\
\text{s.t. } P[\pi_r(\min(Q,D)) \leq \alpha] \leq \beta
\]
\[
E(\pi_r) = -w_Q + \int_0^Q pxf(x)dx + \int_0^Q v(Q-x)f(x)dx + \int_0^Q r(Q-x)f(x)dx
\]
\[
(3)
\]

### 3.2 The order decision of centralized supply chain

As in section 1.2, whether the retailers are not risk-averse, the market risk faced by the suppliers can fully be offset by buy-back. So the expected profit function is:
\[
E[\pi_r(\min(Q,D))] \\
\text{Max}
\]
\[
E[\pi_r] = -w_Q + \int_0^Q pxf(x)dx + \int_0^Q vQf(x)dx + \int_0^Q r(Q-x)f(x)dx
\]
\[
(4)
\]

The optimal order quantity of the supply chain is:
\[
Q^* = F^{-1}\left(\frac{p-c}{p-r}\right)
\]
\[
(5)
\]

### 3.3 The order decision of decentralized supply chain

In the decentralized supply chain, there are risk aversion preferences for the retailer, the restriction is obtained by the conception of downside risk and the risk aversion parameter \((\alpha, \beta)\), thus the retailer’s expected profit function is:
\[
\max_{Q \geq 0} E[\pi_{D(r)}(\min(Q,D))] \\
\text{s.t. } P[\pi_{D(r)}(\min(Q,D)) \leq \alpha] \leq \beta
\]
\[
E[\pi_{D(r)}] = -w_Q + \int_0^Q pxf(x)dx + \int_0^Q vQf(x)dx + \int_0^Q r(Q-x)f(x)dx
\]
\[
(6)
\]

Assume the retailer’s optimal order quantity is \(Q_{D(r)}\), and the critical order quantity is \(Q_0 = \frac{\alpha}{p-w}\), so

according to the difference of the interval, it can be classified as two cases:

1. when \(Q_{D(r)} < Q_0\),
   \[
   \alpha = (p-w)Q_0 \geq (p-w)Q_{D(r)} \geq \max \pi_{D(r)} \text{ is available from } Q_0 = \frac{\alpha}{p-w}, \text{ this } P[\pi_{D(r)}(Q,D) \leq \alpha] = 1, \text{ so now the downside risk is 1, which is greater than any } \beta, \text{ the retailer chooses no order because the downside risk doesn’t satisfy the restriction of the target function.}
   \]

2. when \(Q_0 \leq Q_{D(r)}\),

The retailer’s profile function is \(\pi_{D(r)} = p\min(Q,D) - wQ + V(Q)\), in which

\[
V(Q) = \begin{cases} 
  v(Q-D) & Q \geq D \\
  0 & Q \leq D
\end{cases}
\]

So \(P[\pi_{D(r)}(Q,D) \leq \alpha] = P[\pi_{D(r)}(Q,D) \leq \alpha] \cup [\pi_{D(r)}(Q,D) \leq \alpha]\).
and in which $P[\pi_{D(r)} \leq \alpha] = P \{ p \min(Q, D) - w Q \leq \alpha \} = P \{ (p - w) Q \leq \alpha \} = 0$ and

$P[\pi_{D(r)} \leq \alpha] = P \{ p \min(Q, D) - w Q + v(Q - D) \leq \alpha \} = P \{ D \leq \alpha + (w - v) Q \}$. Thus the retailer’s downside risk is $F(\frac{\alpha + (w - v) Q}{p - v})$ when $Q_0 \leq Q_{D(r)}$, and risk increasing monotonously in Q.

When $F(\frac{\alpha + (w - v) Q_{D(r)}}{p - v}) < \beta \leq 1$: the optimal order quantity $Q_{D(r)}$ satisfies the downside risk restriction, now the retailer’s current optimal order quantity is the optimal order quantity of the decentralized supply chain, see below:

$Q_D^0 = Q_{D(r)}^0 = F^{-1}(\frac{p - w}{p - v})$ (7)

When $F(Q_0) \leq \beta \leq F(\frac{\alpha + (w - v) Q_{D(r)}}{p - v}) \leq 1$: the $Q_{D(r)}$ cannot satisfy downside risk restriction of the target function.

When $Q_0 \leq Q_{D(r)}$, the downside risk $F(\frac{\alpha + (w - v) Q}{p - v})$ increases monotonously in Q, and the retailer’s expected profit function $E(\pi_{D(r)})$ increases monotonously in Q in the left of the equation, so the retailer’s optimal order quantity satisfies:

$F(\frac{\alpha + (w - v) Q}{p - v}) = \beta$ and the quantity is

$Q_D^0 = (\frac{p - v}{w - v}) F^{-1}(\beta) - \alpha$ (8)

### 3.4 The supply chain buy-back contract model with risk-averse condition

The ultimate goal of supply chain coordination is to maximize the expected profit of the decentralized supply chain; this makes the order quantity in the decentralized supply chain equal to the one in the centralized supply chain by buy-back contract. Assuming that buy-back price is b, the discussion according to the above conclusion is as follows:

1. When $F(\frac{\alpha + (w - v) Q_{D(r)}}{p - v}) \leq \beta \leq 1$ and $Q_{D(r)} \leq Q_0$, the quantity before implementing buy-back contract is

$Q_{D(r)} = F^{-1}(\frac{p - w}{p - v})$, the optimal order quantity after implementing contract is $Q_1$, so the buy-back price satisfies

$F^{-1}(\frac{p - w}{p - v}) = F^{-1}(\frac{p - b}{p - c})$, as a result, the buy-back price which can coordinate supply chain is

$b = \frac{w(p - r) + p(r - c)}{p - c}$ (9)

2. When $F(Q_0) \leq \beta \leq F(\frac{\alpha + (w - v) Q_{D(r)}}{p - v}) \leq 1$ and $Q_0 < Q_{D(r)}$, the quantity before implementing buy-back contract is

$Q_D^0 = (\frac{p - v}{w - v}) F^{-1}(\beta) - \alpha$, the optimal order quantity after implementing the contract is $Q_0$, so the buy-back price satisfies

$(\frac{p - v}{w - v}) F^{-1}(\beta) - \alpha = Q_0 = F^{-1}(\frac{p - b}{p - c})$, as a result, the buy-back price which can coordinate supply chain is

$b = \frac{w Q_0 + a - p F^{-1}(\beta)}{Q_0 - F^{-1}(\beta)}$ (10)

### 4. Simulation application

Supposing that the parameters of a supply chain are as follows: $p=2058$, $c=1625$, $w=1822$, $v=600$ and $r=900$ (Unit is supposed to be 1). The demand of the product follows normal distribution $D \sim N(11900, 1700^2)$.

Take the known condition into the equation (1)~(2), and get the optimal order quantity and maximum expected profit of the supply chain in the risk-neutral case. Table 1 shows the results.
Table 1: The expectation profile function of the centralized supply chain under the risk-neutral condition

<table>
<thead>
<tr>
<th>Risk-neutral</th>
<th>Centralized supply chain</th>
<th>Decentralized supply chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>1.1353e+004</td>
<td>1.0222e+004</td>
</tr>
<tr>
<td>E</td>
<td>4.4069e+006</td>
<td>2.2007e+006</td>
</tr>
</tbody>
</table>

As showed in the table 1, in the risk-neutral conditions, the optimal results of the decentralized supply chain are lower than which of the centralized supply chain. In the decentralized supply chain, the decision based on maximizing the retailer’s profit doesn’t achieve the supply chain optimization. Therefore, contracts need to be taken to achieve supply chain optimization, but more details are not involved in this paper.

When the retailer’s risk aversion parameter \((\alpha, \beta) = (2000000, 0.05)\), take the parameter into the buy-back contract model above, and get the buy-back price \(b = 1.4268e+003\), the retailer’s optimal order quantity and expected profit can be obtained from the model built above.

Table 2: The maximum expectation profit and the optimal order quantity of supply chain with the risk-averse condition

<table>
<thead>
<tr>
<th>Risk-averse</th>
<th>Centralized supply chain</th>
<th>Decentralized supply chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>1.1353e+004</td>
<td>9.9261e+003</td>
</tr>
<tr>
<td>E</td>
<td>4.4069e+006</td>
<td>1.9342e+006</td>
</tr>
</tbody>
</table>

As showed in the table 2, First, the optimal results of the centralized supply chain under the risk-averse condition are equal to those under the risk-neutral condition; this proves the correctness of the conclusions above. Second, the optimal results of the decentralized supply chain are lower than those of the centralized supply chain under the risk-averse condition. Last but not least, the retailer’s optimal order quantity of the decentralized supply chain achieve which of the centralized supply chain through the implementation of buy-back contract and coordinate the supply chain.

5. Conclusion and future research

This paper studies a two-stage supply chain consist of a risk-neutral supplier and a risk-averse retailer, the buy-back contract model enables to coordinate the supply chain and prove that the model is effective to realize win-win goals.

Certainly there are still many limitations in this paper; it can be extended in the following areas. Firstly, there are only one supplier and one retailer in the supply chain, in fact the supply chain structure is often more complex. Secondly, this paper does not take into account capacity constraints of the retailer. In future research, capacity constraints and risk aversion preferences can be taken into account when designing the contract coordination.

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References


