Dynamic Output-Price Contract of Agricultural Supply Chain

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From the perspective of mechanism design, we analyzed the choices of the farmers' supply and prices of leading enterprise of the agricultural supply chain in the case of uncertainty supply of farmers and variable price of leading enterprise by the trading mechanisms including uniform price and discrimination price, and comparative analyzed the relevant properties of the two kinds of trading mechanism. The results showed that: For the uniform price trading mechanism, the quantity of farmers submit below its real supply at the low price, and above their true supply quantity at the high price. For the discrimination price trading mechanism, the quantity of farmers submit below their real supply at the low price, and equal their true supply quantity at the high price. Contrast to price discrimination auction, the more retailers participate into the trading mechanism, the less distortion of quantity submitted of farmers. From mechanism design point of view, leading enterprise tends to use uniform price trading mechanism for procurement of agricultural product.

1. Introduction

Agricultural modernization, industrialization and urbanization promote each other, which is the objective law of the modernization process has been proved. Agricultural industrialization is an important system innovation in China's rural areas, it is the foundation of the development of agricultural modernization, its essence is a benefit community formed by the association of agricultural multi agent, they follow the "risk sharing, benefit sharing" benefit distribution mechanism by contract as a link. Therefore, it is very important to establish a reasonable profit distribution mechanism for agricultural industrialization management. Contract determines the distribution of interests and risk sharing among the main bodies of the supply chain, according to the contract to take responsibility, to fulfill the power, to obtain benefits. The contract allows farmers and enterprise not only wouldn't change their operating independence, but also can ensure their interests within the organization through the contract.

There are three main types of agricultural industrialization in China. The first category is the primary "farmer + market", that is, the individual farmer enter the market to participate in the transaction of agricultural products, so that farmers, between farmers and agricultural enterprises are a pair of one-time contract relationship. The second category is "leading enterprise + farmer", that is, a professional enterprise (known as "leading enterprise") with a number of farmers signed a single or multi contract, leading enterprise centralized acquire agricultural products in accordance with the agreement of the price, and then processed and sold to the market. Leading enterprise sometimes provided with raw materials or technical guidance for the farmers at prenatal and production. Because the farmers grow production according to the order of the leading enterprise, so this mode is called "order agriculture". The third category is the "leading enterprise + farm", which is, leading enterprise integrates production, processing and sales, hire farmers to work on a farm or a production base and pay wages to them. Some other patterns are the derivative form based on the above three basic modes, the difference lies in the degree of relationship between enterprises and farmers. For example, "leading enterprise + cooperative+ farmers" is to introduce an intermediary organization among the leading enterprise and farmers, enterprise and farmers sign with the cooperative respectively. This pattern is not different from the "leading enterprise + farmers" in nature. Another mode is "leading enterprise + base + farmers", that is, leading enterprise connects with farmers by the base, and it is looser than "leading enterprise + farm model". Another mode is "professional market + intermediary organization + farmers". If we take the
professional market as a leading enterprise, then it is not essential to the "leading enterprise + farmers" (Huihua Nie, 2012).

At present, the most suitable agricultural industrialization management for our country is the leading enterprise and farmers to contract as a link. Under the current system of rural land, which is based on the household contract management system in China, commercial capital enter the agricultural sector with role of the leading enterprise, at the same time, they should bear the natural and market risks inherent in agricultural production. Leading enterprise can reduce the uncertainty of raw materials procurement by the way of contract agriculture, enterprise, at the same time, they should bear the natural and market risks inherent in agricultural production.

Leading enterprise plus farms, analyzed the effects of ownership, reputation, pledge and risk attitude compared for the allocation efficiency of three industrialization modes (farmers in market, leading enterprise plus farms), analyzed the effects of ownership, reputation, pledge and risk attitude on optimal agricultural contracts. Galioto et al. (2013) analyzed the optimal design of current tariff strategies with respect both to the actual regulator’s goals and the cost recovery objective of an ideal regulator driven by European Water Framework Directive principles and having full information based on the logic of a Principal-Agent model implemented as a mathematical non-linear programming model. Auction as a market trading mechanism is widely used in practice. Discriminate price auction, uniform price auction, and sequential auction are always used for homogeneous multi-item auction. In the existing literatures, the research mainly focus on fixed quantity of multi-item auction (Nautz and Wolfsteller (1997); Tenorio et al. (1999)). But with the development of electronic commerce, online transaction mode makes the supply change more attention in recent years. Hansen (1988) for the first time raised the issue of price determination of multiple items in the case of the auction problem. Mcadams (2007) through the analysis of the unified price auction, it is concluded that the price of the change in the supply of the auction and the auction of the social welfare are higher than the fixed supply. Damian (2010) analysis of the bidder in the unit demand auction, by comparing the uniform price auction and discriminatory price auction that uniform price auction can obtain higher selling price and trading volume for the seller, to obtain more income. Ausubel et al. (2014) established that such differential bid shading results generically in ex post inefficient allocations in the uniform-price. Akachi et al. (2014) examined the effect of varying the number of bidders and units on bid values in multi-unit auctions, the results suggest that the uniform-price auction is sensitive to demand reductions, however, increasing both number of bidders and units can lower the price, it can be seen from these references, although these references have analyzed variation the supply of multi-item trading mechanism model, but they are all analysis of the needs of buyers in the case of one unit.

Based on the above references, considering uniform price trading mechanism and discriminatory price trading mechanism, by means of the theory of relational contract (Baker et al., 2002), auction theory and incomplete contract theory (Grossman and Hart, 1986; Hart and Moore, 1990), study two acquisition contracts of the agricultural industrialization "leading enterprise + farmers" mode in the case of the variable supply of agricultural products and the purchase price, and comparative analysis the two contracts. First, some reasonable assumptions will be proposed. After that, I will analyze the optimal quantity of agricultural products submitted by the farmers and the optimal price of the leading enterprise of the uniform price trading
mechanism and discriminatory price trading mechanism. Finally, comparative analysis about the two trading mechanisms will be discussed. According to the results, I will put forward the optimal trading contract for the leading enterprise.

2. Model assumptions

There are a leading enterprise and \( n \) farmers in the form of contract for the purchase of agricultural products trading. First, the leading enterprise signs purchase contract with each farmer, there are two kinds of purchase price, the final price is determined according to the total submitted by all the farmers. Second, the farmers planting agricultural products according to the contracts, after the harvest of agricultural products, each farmer has two options, sell products to the leading enterprise in accordance with the contract prices or exist the trading. At the same time, the leading enterprise has two choices, to trade or exit.

Suppose purchase price of the leading enterprise are only two possible prices, \( p_l \) and \( p_h \), where \( p_h > p_l > 0 \). The trading is a two-stage model, in the first phase the farmers submit their supply quantity corresponding the prices the second stage enterprise chooses price and volume.

Assumption 1: Each farmer has a quasi-linear utility function \( u_i(q, z) = v_i(q) + \varepsilon \), where \( q \) indicates the supply products of farmer \( i \). \( v_i \) is the utility function of farmer \( i \), it is continuously differentiable, monotonically increasing and strictly concave function, and \( v'(0) > p_l \). Because farmers are price taker, so we can get the supply function from utility function: \( q = s_i(p) = \max[v_i^{-1}(0),0] \), and supply function is common knowledge. Since \( v_i \) is strictly concave and continuous, and therefore \( d_i \) is positive about the price decreasing and continuous function. \( v'(0) > p_l \) means \( s_i(p_l) > 0 \), namely any farmer has non-zero supply when price is \( p_l \), also it implies that for \( s_i(p_l) = 0 \), the farmer will be automatically excluded from the trading.

Assumption 2: the supply of each farmer is bounded, i.e. for any farmer \( i \), \( s_i(p) < \infty \).

In the first phase, farmer \( i \) submit two non-negative supply \( x_i \) and \( y_i \), \( i = 1, 2, \ldots, n \), corresponding denote the supply at low price and high price respectively, therefore \( y_i \geq x_i \). Unit production resell price \( \beta \) of the enterprise is propriety information, \( \beta \) obey \( G \) distributed random variables which is common knowledge for farmers.

Assumption 3: \( G(0) = 0 \). For any \( \beta > 0, \) \( G(\beta) > 0, G(p_l) < G(p_h) \) and \( \beta \) is continuously differentiable and strictly increasing in the interval \((0, p_h]\).

In the second stage, the enterprise chooses high price or low price, or cancel auction, the enterprise's strategy \( u \in \{ p_l, p_h, cancel \} \).

If the enterprise chooses \( p_l \), the farmer \( i \) supply \( x_i \) units; if the enterprise chooses \( p_h \), the farmer \( i \) supply \( y_i \) units; if the enterprise chooses cancel, there is no any farmer will supply products. Farmers’ payment depend on the form of the trading mechanism. The enterprise’s purpose is to maximize profits, its decision-making depends on its type (profit \( \beta \)) and total submit vector \((\sum x, \sum y)\).

3. Uniform price trading mechanism

In the uniform price trading mechanism, all farmers can obtain the same price for each unit of products. If the enterprise chooses low price, the farmer \( i \) payment is \( p_l x_i \), the profits of enterprise is \( \beta - p_l \) multiply of the total supply at low price. If the enterprise chooses high price, the farmer \( i \) payment is \( p_h y_i \), the profits of enterprise is \( \beta - p_h \) multiply of the total supply at high price.

Proposition 1: In the uniform trading mechanism, the optimal strategy of the leading enterprise is:

\[
\begin{cases} 
\text{cancel} & p_l < \beta \leq p_h \\
\text{cancel} & p_l < p_h \leq \beta \\
\text{cancel} & p_h < \beta
\end{cases}
\]

where \( \beta^* = \frac{(p_l \sum x - p_h \sum y) / (\sum x - \sum y)}{p_l \sum x - p_h \sum y} \) \( \sum x - \sum y < 0 \)

Proof: By assumptions we can know, only when \( \beta > p_l \), enterprise maybe can obtain non-negative profits, so when \( p_l \geq \beta \), the optimal strategy is to cancel the trading for the enterprise. In addition, because \( x_i \leq y_i \), therefore we have to discuss the two cases:

The first case is \( \sum x < \sum y \): If there is no difference to choose \( p_l \) or \( p_h \) for the enterprise, then we have \( (p_l - \beta) \sum x = (p_h - \beta) \sum y \), so we can obtain \( \beta^* = \frac{(p_l \sum x - p_h \sum y) / (\sum x - \sum y)}{p_l \sum x - p_h \sum y} \), and \( \beta^* > p_h \). Which means that \( (\beta - p_l) \sum x \leq (\beta - p_h) \sum y \) when \( \beta \geq \beta^* \). In this case the optimal choice for the enterprise is \( p_h \), otherwise \( p_l \).

The second case is \( \sum x = \sum y \): In this case the enterprise in order to maximize profits, because the supply at low price and high price are the same, \( p_l \) is optimal choice without doubt.
Proposition 1 characterizes the enterprise behavior. If and only if unit production profit is lower than \( p_2 \), the trading will be cancelled; if and only if unit production resell price more than the threshold value decided by Eq. (1), the enterprise will select high price. From Eq. (1) can be seen, \( \beta^* \) depending on farmers supply at different prices, which indicates that farmers are not entirely a price taker, they can change their supply to influence enterprise pricing options.

For farmers, they choose \((x_i, y_i)\) to maximize their expected utility. Farmers have known the enterprise production profits distribution \( g \), and aware that the enterprise determine \( \beta^* \) by the Eq. (1), so the probability of enterprise to choose low price is \( G(\beta^*) - G(p_1) \). To choose high price is \( 1 - G(\beta^*) \), the probability of the option to cancel the trading is \( G(p_2) \). So farmer \( i \) expected utility is determined by:

\[
\max \pi_i(x_i, y_i) = (G(\beta^*) - G(p_1))L_i(x_i) + (1 - G(\beta^*))H_i(y_i)
\]  

(2)

s.t. \( y_i \geq x_i \)

Where \( L_i(x_i) = p_ix_i - v_i(x_i) \), \( H_i(y_i) = p_3y_i - v_i(y_i) \). \( L_i(x_i) \) denotes the utility of farmer \( i \) in the case of supply \( x_i \) units when the enterprise chooses low price. \( H_i(y_i) \) denotes the utility of farmer \( i \) in the case of supply \( y_i \) units when the enterprise chooses high price.

Proposition 2: Under the hypotheses 1 and 3, the following proposition holds:

1. \( \partial_\beta \beta^* \leq 0 \) if and only if \( \sum x = 0 \) equation holds; \( \partial_\beta \beta^* > 0 \).
2. \( L_i(x_i) < H_i(y_i) \).
3. \( y_i > x_i \geq 0 \).

Proof: According to the hypotheses, we can get \( y_i \geq x_i \geq 0 \). According to Proposition 1, if \( y_i = x_i \), we can know \( \beta^* = \infty \), then the enterprise will choose low price at this time, therefore \( x_i < y_i \) are the optimal strategy for the farmers, thus (3) holds.

For Eq. (1) with respect to \( x_i \) and \( y_i \), respectively, partial derivative, we have:

\[
\partial_\beta \beta^* = \sum \frac{(p-a)\sum x}{(x-y)^2} \leq 0, \text{ if and only if } \sum x \text{ equation holds.}
\]

therefore (1) is established.

For the quasi-concave functions \( L_i(x_i) \) and \( H_i(y_i) \), we have \( H_i(0) = 0 \) and \( L_i(0) = 0 \). Because \( p_2a < p_3a \), so \( L_i(a) < H_i(a) \) for all \( a > 0 \). Since \( L_i(x_i) \) is quasi-concave function, so \( L_i(a) < 0 \) when \( a < s_i(p_2) \). \( L_i(a) > 0 \) when \( a > d_i(p_3) \), and \( L_i(a) \) reaches its maximum when \( a = s_i(p_3) \). Similarly, we can obtain \( H_i(z) > 0 \) when \( z > s_i(p_3) \). \( H_i(z) < 0 \) when \( z < d_i(p_3) \), and \( H_i(z) \) reaches its maximum when \( z = s_i(p_3) \) for all \( z > 0 \). We can know that trading equilibrium solutions exists, so the first-order condition of \( L_i(x_i) \) with \( x_i \) exist. Suppose \( x_i < s_i(p_2) \), then \( L_i(x_i) < 0 \) and \( L_i(x_i) < L_i(y_i) \). Because \( L_i(y_i) < H_i(y_i) \), so \( L_i(x_i) < H_i(y_i) \). Then \( \partial_\beta \pi_i(x_i, y_i) = G'(\beta^*)\partial_\beta \beta^*(L_i(x_i) - H_i(y_i)) + G(\beta^*)L_i'(x_i) < 0 \) this is inconsistent with the existence of equilibrium solutions, so \( x_i > s_i(p_3) \) and \( L_i(x_i) > 0 \), then \( L_i(x_i) < H_i(y_i) \), therefore (2) is established.

In the proposition 2, (1) shows that the impact of supply submitted by farmers on the enterprise’s choice, it can improve the probability of enterprise choose low price by reducing supply at low price and increasing supply at high price. (2) denotes that the utility of the farmer in the enterprise’s choice of low price is lower than that of the enterprise’s choice of high price. (3) shows that the supply of the farmer submitted in the high price is strictly higher than the low price.

Proposition 3: In the uniform price trading, \( x_i \leq s_i(p_1) \) if and only if \( \sum y = 0 \) equality holds, \( y_i \geq s_i(p_3) \) if and only if \( s_i(p_3) = 0 \) equality holds.

Proof: The first order condition for Eq. (2) with \( x_i; G'(\beta^*) [p_i - s_i^{-1}(x_i)] + G'(\beta^*)\partial_\beta \beta^* (L_i(x_i) - H_i(y_i)) = 0 \), then \( s_i^{-1}(x_i) = p_i + \frac{G'(\beta^*)}{\alpha G(\beta^*)}\partial_\beta \beta^* (L_i(x_i) - H_i(y_i)) < p_i \). Since \( s_i^{-1} \) is an increasing function, so we get \( x_i = s_i(p_i + \frac{G'(\beta^*)}{\alpha G(\beta^*)}\sum \frac{c'(\beta^*)}{(x-y)(x-y)} \sum x_i - s_i^{-1}(x_i) < p_i \). When \( \sum y = 0 \) it can obtain \( x_i = s_i(p_i) \). Similarly we can get \( y_i = s_i(p_3 - \frac{G'(\beta^*)}{\alpha G(\beta^*)}\sum \frac{c'(\beta^*)}{(x-y)(x-y)} \sum y_i - s_i^{-1}(y_i) < p_i \). When \( s_i(p_3) = 0 \), then \( y_i = 0 \). So if and only if \( s_i(p_3) = 0 \), \( y_i = s_i(p_3) \).

Proposition 3 shows that in the uniform price trading, the supply quantity submitted by farmer is lower than his actual supply at low price, the supply submitted by farmer is higher than his actual supply at high price. By the above analysis, it can be known that the equilibrium of the uniform price trading is always inefficient, and the supply is always distorted. There are two reasons. First, for the enterprise, different from the complete competition market, the unit production resell price is higher than the marginal cost. Secondly, farmers always cover up the true supply in different prices, which leads to distribution inefficiency. This can be seen from proposition 3.

4. Discrimination price trading

In the discrimination price trading, the enterprise adopts different retail price for different supply submitted by farmers. Compared with the uniform price trading, in addition to the payment of rules, the others are the same.
In this paper, the payment rules of discrimination price trading is: each farmer submit different supply \((x_i,y_i)\) for the two prices, and \(x_i < y_i\). If the enterprise chooses low price \(p_l\), the farmer supply \(x_i\) units item, obtain \(p_l\) per unit, the total payment is \(x_ip_l\); if the enterprise chooses high price \(p_h\), farmer obtain \(x_i\) units with price \(p_l\) per unit, the rest \(y_i-x_i\) units are paid \(p_h\) per unit, and then the total payment is \(x_ip_l + (y_i-x_i)p_h\). If the enterprise cancel the trading, the payment is 0.

Proposition 4: In the discrimination price trading, the enterprise’s optimal strategy is:

\[
u(\beta, \sum x, \sum y) = \begin{cases} 
\text{cancel} & p_l > \beta \\
\frac{p_l}{p_h} & p_l > \beta \geq p_l \\
\beta & \beta \geq p_h
\end{cases}
\]  

Proof: Suppose \(a = (\beta - p_l)\sum x, b = (\beta - p_h)\sum y - \sum x\). If the enterprise chooses low price, the profit is \(a\); if the price is high, the profit is \(a + b\). If \(\beta < p_l\), the profit of enterprise is negative regardless of his choice of high or low price, so at this time the optimal strategy for the enterprise is cancel the trading. If \(p_l \leq \beta < p_h\), then \(a > 0, b < 0\), the optimal strategy for the enterprise is low price. If \(\beta \geq p_h\),then \(a > 0, b > 0\), the optimal strategy for the enterprise is high price.

Proposition 4 shows that the optimal strategy of enterprise in discriminate price trading is only dependent on its own unit production resell price, which has nothing to do with the supply of the farmers.

Proposition 5: In the discrimination price trading, \(y_i = s_i(p_h), x_i \leq s_i(p_h)\), if and only if \(s_1(p_l) = 0\),the equation is established.

Proof: According to proposition 4 we can know, the probability of the enterprise chooses low price is \(G(p_h) - G(p_l)\), the probability of choosing the high price is \(1 - G(p_h)\). The farmer’s expected utility is:

\[
\max \pi_i(x_i,y_i) = (1 - G(p_h))(p_h(y_i-x_i) + p_lx_i - v_i(y_i)) + (G(p_h) - G(p_l))(p_lx_i - v_i(x_i))
\]  

The first order condition about Eq.(4) with \(y_i\): \((1 - G(p_h))\left(p_h - s_i^1(y_i)\right) = 0\), so \(y_i = s_i(p_h)\). According to the first order condition with \(x_i\),we get \((G(p_h) - G(p_l))\left(p_l - s_i^1(x_i)\right) = 0\). so \(x_i = s_i(p_l - (p_h - p_l))\), if and only if \(s_1(p_l) = 0\), the equation is established.

Proposition 5 shows that in the discrimination price trading, the farmer will choose to submit their true supply at high price, submit lower supply than their real needs at low price. And it also shows that discrimination price trading has also led to inefficient allocation of resources.

5. Comparative analysis

Proposition 6: The probability of choosing low price by enterprise in uniform price trading is higher than that in discrimination price trading; the probability of choosing high price by enterprise in uniform price trading is lower than that in discrimination price trading.

Proof: According to proposition 1 and 4, we can know that, when \(\beta^* = (\sum x - \sum p_h) / (\sum y - \sum x)\), \(\beta^* > p_h\) then the probability of choosing high price by enterprise in uniform price trading is \(1 - G(\beta^*)\), which is lower than the probability \(1 - G(\beta_h)\) in discrimination price trading. The probability of choosing low price by enterprise in uniform price trading is \(G(\beta^*) - G(p_l)\) higher than \(G(p_h) - G(p_l)\) in discrimination price trading.

As shown in Figure 1.

**Figure 1: price selection probability in uniform price trading and discrimination price trading**

Corollary 1: In the uniform price trading, the more participating farmers, the less supply distortion. In the discrimination price trading, the supply distortion has nothing to do with the number of farmers.

Proof: According to proposition 4 and 5, the equilibrium strategy of the enterprise is only related to the unit production resell price in the discrimination price trading, has nothing to do with the number of farmers.

Suppose there are \(k\) farmers for farmer \(i\) type in the uniform price trading, then there are \(kn\) farmers participate in the trading, we can get \(\sum x = \sum_{j=1}^{k}\sum_{i=1}^{n} x_{ij}/k, \sum y = \sum_{j=1}^{k}\sum_{i=1}^{n} y_{ij}/k, \) so \(\delta x, \beta^* = (\sum x - \sum p_h) / (\sum x - \sum y), \delta y, \beta^* = (\sum y - \sum p_h) / (\sum x - \sum y)\).

When \(k \to +\infty\), \(\delta x, \beta^* \to 0\), \(\delta y, \beta^* \to 0\), and \(x_i = \lim_{k \to +\infty} s_i( p_l + \frac{g(\beta^*)}{\delta y, \beta^*} (y_i - H_i(y_i)))\to d_i(p_l), \) \(y_i = \lim_{k \to +\infty} s_i( p_h - \frac{g(\beta^*)}{\delta x, \beta^*} (x_i - H_i(x_i)))\to d_i(p_h).\)
Thus it can be seen that in the uniform price trading, the more farmers, the more close to the real supply. Therefore, from the perspective of mechanism design, if more farmers involved in the trading, compared to discrimination price trading, enterprise would tend to use the uniform price trading.

6. Conclusion

In this paper, from the perspective of mechanism design, we analysed the choices of the farmers’ supply and prices of leading enterprise of the agricultural supply chain in the case of uncertainty supply of farmers and variable price of leading enterprise by the trading mechanisms including uniform price and discrimination price, and comparative analysed the relevant properties of the two kinds of trading mechanism. The results showed that: For the uniform price trading mechanism, the quantity of farmers submit below its real supply at the low price, and above their true supply quantity at the high price. For the discrimination price trading mechanism, the quantity of farmers submit below their real supply at the low price, and equal their true supply quantity at the high price. The optimal strategy of enterprise in discriminate price trading is only dependent on its own unit production resell price, it has nothing to do with the supply quantity of the farmers. But the optimal strategy of enterprise in uniform price trading connects with the supply quantity of the farmers. Contrast to price discrimination auction, the more retailers participate into the trading mechanism, the less distortion of quantity submitted of farmers. From mechanism design point of view, leading enterprise tends to use uniform price trading mechanism for procurement of agricultural product. This paper only discussed the two price trading, if there are more than two prices for the manufacture to choose, the result is how? This is what needs to be further studied in the future.

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