

# A Fuzzy Analytic Hierarchy Process (FAHP) Approach for Optimal Selection of Low-carbon Energy Technologies

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In process systems engineering, one may encounter design problems that require choosing one among the predefined set of alternatives in a complex decision making environment. In most cases, the selection process involves multiple conflicting criteria that can be either qualitative or quantitative in nature. The use of multiple attribute decision making (MADM) technique thus has been proven effective in providing a transparent, systematic and rational approach to problem solving. In this study, a variant of the widely used MADM technique known as the Analytic Hierarchy Process (AHP) has been developed and applied for the selection of low-carbon technologies. This approach enables the selection of optimal alternative by incorporating not only the judgment of domain experts but also their degree of confidence through the use of fuzzy numbers for the pairwise comparison ratios in the AHP decision framework. The proposed fuzzy AHP then determines a set of crisp weights that maximizes the degree of consistency of all judgments taken together. The said approach can also derive crisp weights from an incomplete fuzzy pairwise comparative judgment matrix (PCJM). To illustrate the technique, a case study involving comparison of electricity storage technologies for renewable energy systems is considered. This selection problem is important due to the intermittent nature of many forms of renewable energy, which thus require cost-effective auxiliary energy storage subsystems for successful deployment.

## 1. Introduction

Climate change is indeed a global environmental concern as our planet already passed the 400 ppm mark of the atmospheric CO<sub>2</sub> level, which is well above what is regarded as the safe, pre-industrial level of 350 ppm (Rockström et al., 2009). Significant technological shifts on a massive scale will thus be required in a so-called carbon constrained world driven by the impacts of climate change. According to Pacala and Socolow (2004), the concept of making incremental reductions in CO<sub>2</sub> emissions relative to the “business as usual” (BAU) trajectory would rely on a set of technologies deployed strategically, instead of relying on a single major technological solution. In the deployment of low-carbon technologies, it is necessary to select the best technology for any given application. Selection in turn requires simultaneous consideration of different decision aspects (e.g., carbon footprint, cost, reliability, etc.), which need to be prioritized and balanced by the decision-maker. Likewise, many process engineering problems involve the selection from a predefined set of alternatives, often using multiple, potentially conflicting criteria. Furthermore, in the context of enterprise decision-making, different dimensions need to be considered by decision-makers, thus giving rise to complex problems that require systematic decision aids (Muñoz et al., 2013). Such cases require the systematic use of multiple attribute decision making (MADM) techniques to provide a rigorous and rational approach to problem-solving.

One of the most widely used MADM technique is the Analytic Hierarchy Process (AHP), which is originally developed by Saaty (1977) to derive ratio-scale priorities from pairwise comparative judgment matrix. Because of its intuitive appeal and flexibility, AHP has been applied in many problem domain areas as reported in Vaidya and Kumar (2006), and more recently in Sipahi and Timor (2010) that reviews its other extensions including the Analytic Network Process (ANP). This decision-aiding tool has been used for energy planning (Pohekar and Ramachandran, 2004) and process systems engineering such as process

safety (Arslan, 2009) and risk assessment (Řehák and Šenovský, 2014), among others. Development of AHP and its many variants is also continuously evolving as summarized in Ishizaka and Labib (2011). This includes the use of fuzzy judgments in pairwise comparisons because of the complexity and uncertainty involved in decision making, especially since AHP results are highly dependent on expert inputs (Promentilla et al., 2008).

In this paper, we demonstrate a recently developed fuzzy AHP methodology which was proposed in Tan et al.(2014) and then extended to group fuzzy ANP (Promentilla et al., 2014) for the selection of low-carbon technologies. In this study, the said MADM technique was applied in the prioritization of energy storage technologies. This aspect is important due to the intermittent nature of many forms of renewable energy, which thus require cost-effective auxiliary energy storage subsystems for successful deployment (Schoenung, 2011). It is assumed that criteria weights are established primarily through pairwise comparisons by domain experts, where subjective judgements are used to describe the intensity of importance of one over the other. The value judgment are expressed as triangular fuzzy numbers (TFN) whose spread signify ambiguity, or lack of confidence, of a given judgement.

**2. Methodology**

The procedure of the fuzzy AHP is described as follows:

Step 1. Decompose the decision problem in a linear hierarchical structure (e.g., see Figure 1). For example, the downward arrows in the digraph from Level 1 to Level 2 represent the priority weights of *n* criteria with respect to goal. The downward arrows in the digraph from Level 2 to Level 3 represent the priority weights of *m* alternatives with respect to each criterion.

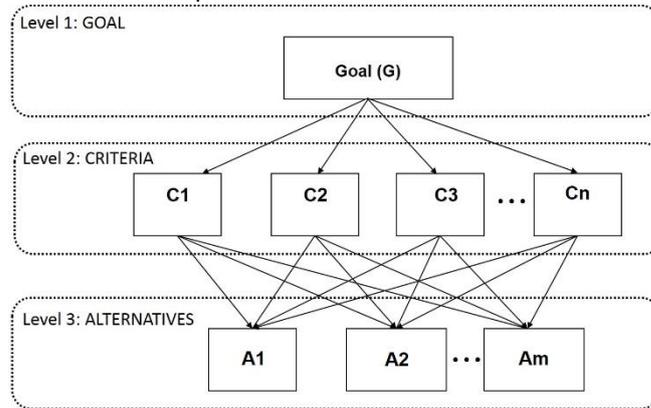


Figure 1: A hierarchical decision structure

Step 2. Elicit value judgment from stakeholder or expert for pairwise comparison matrices to derive the priority weights. The linguistic representations of AHP’s fundamental 9-point scale are modeled by triangular fuzzy numbers (TFN) to account for vagueness and decision maker’s degree of confidence (Tan et al., 2014). The fuzzy judgment  $\hat{a}_{ij} = \langle l_{ij}, m_{ij}, u_{ij} \rangle$  is defined in Table 1 where the triple  $\langle l_{ij}, m_{ij}, u_{ij} \rangle$  represents the lower bound, modal value and upper bound of the TFN respectively, and  $\delta$  is the degree of fuzziness or the spread of fuzzy number such that the spread shortens with increasing degree of confidence of the stakeholder or expert. For example, a  $\delta$  equal to 1 suggests high degree of confidence whereas a  $\delta$  equal to 3 suggests a low degree of confidence (Promentilla et al., 2008).

The  $\hat{a}_{ij}$  then used as entry to the reciprocal pairwise comparison matrix  $\hat{A}$  of order *n* (i.e., the number of elements to be prioritized in a cluster) such that:

$$\hat{A} = \begin{bmatrix} \langle 1,1,1 \rangle & \hat{a}_{12} & \dots & \hat{a}_{1n} \\ \hat{a}_{21} & \langle 1,1,1 \rangle & \dots & \hat{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{a}_{n1} & \hat{a}_{n1} & \dots & \langle 1,1,1 \rangle \end{bmatrix} \text{ where } \hat{a}_{ji} = \frac{1}{\hat{a}_{ij}} = \left\langle \frac{1}{u_{ij}}, \frac{1}{m_{ij}}, \frac{1}{l_{ij}} \right\rangle \tag{1}$$

The proposed nonlinear programming (NLP) formulation to determine the optimal **w** is as follows (Promentilla et al., 2014):

Table 1: Linguistic scale of the intensity of dominance of element  $i$  over element  $j$  and its corresponding fuzzy number

Fuzzy number $\hat{a}_{ij}$	Linguistic scale for comparison of Criteria	Linguistic scale for comparison of Alternatives
$\left\langle \frac{1}{1+\delta}, 1, 1+\delta \right\rangle$	More or less equally important	More or less equally preferred
$\langle 3-\delta, 3, 3+\delta \rangle$	Moderately more important	Moderately preferred
$\langle 5-\delta, 5, 5+\delta \rangle$	Strongly more important	Strongly preferred
$\langle 7-\delta, 7, 7+\delta \rangle$	Very Strongly more important	Very strongly preferred
$\langle 9-\delta, 9, 9+\delta \rangle$	Extremely more important	Extremely preferred

$$\max \lambda \quad (2a)$$

subject to:

$$a_{ij} - l_{ij} \geq \lambda(m_{ij} - l_{ij}) \quad ; \quad a_{ji} - l_{ji} \geq \lambda(m_{ji} - l_{ji}) \quad (2b)$$

$$u_{ij} - a_{ij} \geq \lambda(u_{ij} - m_{ij}) \quad ; \quad u_{ji} - a_{ji} \geq \lambda(u_{ji} - m_{ji}) \quad (2c)$$

$$\text{where } a_{ij} = \frac{w_i}{w_j} \quad ; \quad a_{ji} = \frac{w_j}{w_i} \quad \forall i = 1, \dots, n-1; j = 2, \dots, n; j > i \quad (2d)$$

$$\sum_{k=1}^n w_k = 1; \quad w_k > 0 \quad (2e)$$

In contrast to AHP eigenvector method, note that this method can derive crisp weights even from an incomplete pairwise comparative judgment matrix (PCJM).

Step 3. Compute the crisp priority vector  $\mathbf{w}$  from  $\hat{A}$  of order  $n$  by approximating the solution ratios ( $\mathbf{a}_{ij}$ ) that would maximize  $\lambda$ , i.e., the highest degree of membership in a membership function of triangular fuzzy numbers indicating the intersection of degree of satisfaction of all computed ratios that would satisfy the initial fuzzy judgments obtained from at least  $(n-1)$  out of the possible  $(n-1)(n-2)/2$  pairwise comparisons (Promentilla et al., 2014). A positive  $\lambda$  indicates a consistent fuzzy pairwise comparison matrix wherein a  $\lambda = 1$  suggests perfect consistency in preserving the order of preference intensities.

Step 4. Compute the global priority weights of these alternatives with respect to the goal. For example, given the decision structure in Figure 1, the global priority weights can be computed as follows:

$$W_{AG} = W_{AC}W_{CG} \quad (3)$$

where:  $W_{AG}$  = is matrix containing the global priority weights of alternative with respect to goal. In this study, it is a column vector of order  $m$  which corresponds to the number of alternatives in the decision structure. The  $W_{CG}$  is the matrix containing the importance weights of criteria with respect to goal. It is a column vector of order  $n$  which corresponds to the number of criteria in the decision structure. The  $W_{AC}$  is the matrix containing the priority weights of alternatives with respect to each criterion. It is an array of  $m$  rows and  $n$  columns representing the number of alternatives and criteria, respectively.

### 3. Case study: prioritization of energy storage technologies

Intermittency of many forms of renewable energy (e.g., solar and wind) results in the need for intermediate energy storage to balance the power supply with the corresponding loads. Such storage systems comprise a significant fraction of the actual costs of renewable energy systems, and thus optimal selection needs to be taken into consideration during planning (Schoenung, 2011). This case study thus considers the selection between eight energy storage technologies, namely: 1) Advanced lead-acid battery, 2) Sodium/sulphur battery, 3) Lead-acid battery with carbon-enhanced electrode, 4) Zinc/bromine battery, 5) Vanadium redox battery, 6) Lithium ion battery, 7) High-speed composite flywheel, and 8) Supercapacitor. The first six options are secondary batteries using electrochemical storage as basis for operation. On the other hand, the high-speed composite flywheel stores kinetic energy via a revolving mass directly coupled to a motor/generator; the supercapacitor stores energy as an electrostatic charge between two electrodes.

In a recent report (Schoenung, 2011), the characteristics of these options, as well as other energy storage systems (e.g., compressed air storage, pumped hydroelectric storage, etc.) are reviewed. In this work, we limit our analysis to technologies suitable for small-scale or mobile applications.

Selection of the best technology for any given application depends on both cost and performance parameters. Capital costs of storage systems are functions both of power capacity (i.e., maximum rate of discharge) and energy storage capacity. Storage efficiency, defined as the fraction of recoverable stored energy per cycle, is an important performance indicator with a strong influence on system operating cost. Also, system life, as measured in terms of typical limit for charge/discharge cycles, affects the amortization of fixed capital cost over the life of the system. These technologies are thus evaluated based on four criteria: a) cost per unit power, b) cost per unit energy storage, c) efficiency and d) life. Figure 2 shows the decision structure for this case study.

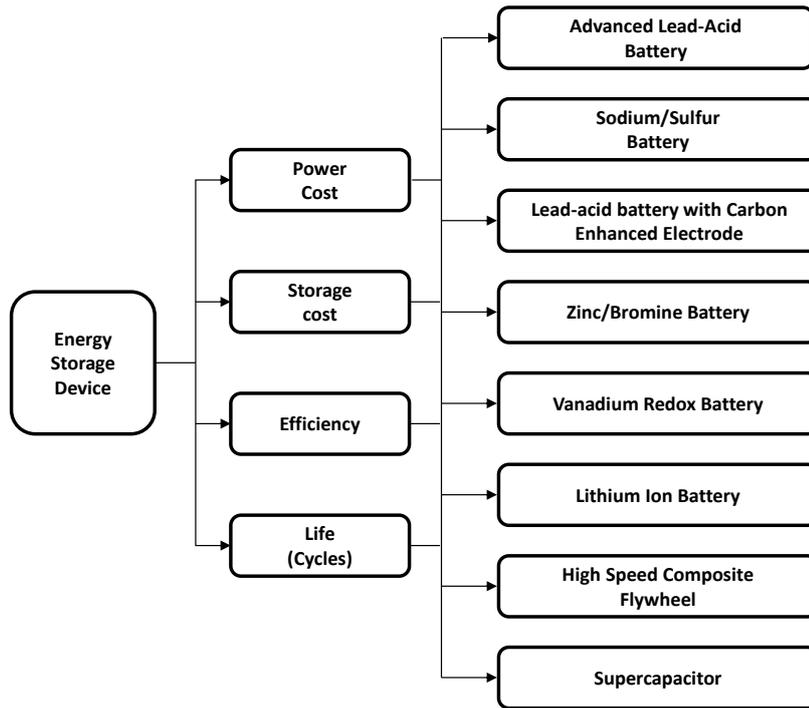


Figure 2: Decision structure for the prioritization of energy storage technologies

To quantify the relative importance of these criteria with respect to the goal of prioritizing these technologies, value judgments were elicited from a domain expert to fill up the pairwise comparison matrix as shown in Figure 3. Solving Eq(2) with LINGO 14.0 results in a fuzzy consistency index of  $\lambda = 0.116$ . The corresponding importance weights ( $W_{CG}$ ) of the four criteria are 0.386 for cost per unit power, 0.386 for cost per unit energy storage, 0.044 for efficiency and 0.184 for life.

	Power Cost	Storage Cost	Efficiency	Life
Power cost	$\langle 1,1,1 \rangle$	$\langle \frac{1}{3}, 1, 3 \rangle$	$\langle 5, 7, 9 \rangle$	$\langle 2, 3, 4 \rangle$
Storage cost	$\langle \frac{1}{3}, 1, 3 \rangle$	$\langle 1,1,1 \rangle$	$\langle 5, 7, 9 \rangle$	$\langle 2, 3, 4 \rangle$
Efficiency	$\langle \frac{1}{9}, \frac{1}{7}, \frac{1}{5} \rangle$	$\langle \frac{1}{9}, \frac{1}{7}, \frac{1}{5} \rangle$	$\langle 1,1,1 \rangle$	$\langle \frac{1}{6}, \frac{1}{5}, \frac{1}{4} \rangle$
Life	$\langle \frac{1}{6}, \frac{1}{5}, \frac{1}{4} \rangle$	$\langle \frac{1}{6}, \frac{1}{5}, \frac{1}{4} \rangle$	$\langle 4, 5, 6 \rangle$	$\langle 1,1,1 \rangle$

$$W_{CG} = \begin{bmatrix} 0.386 \\ 0.386 \\ 0.044 \\ 0.184 \end{bmatrix}$$

Figure 3: Sample fuzzy pairwise comparative judgment matrix and its computed priority vector

The performance of the eight technologies with respect to the four criteria as shown in Table 5 was used to derive the preference weights of the alternatives with respect to each criterion ( $W_{AC}$ ). These preference weights are normalized performance scores such that a higher performance value is more desired. The normalization is also conducted to make the scores comparable with each other. Since the first two criteria are associated with cost, the reciprocals of the first two criteria are used in the normalization, i.e., the lower the cost, the more preferred the alternative is. The normalized performance scores of the technologies are given in Table 6.

Table 5: Performance of alternatives with respect to each criterion (Schoenung, 2011)

Alternative	Power Cost (\$/kW)	Storage Cost (\$/kWh)	Efficiency (%)	Life (cycles)
1. Advanced lead-acid battery	400	330	80	2,000
2. Sodium/Sulfur battery	350	350	75	3,000
3. Lead-acid battery with carbon-enhanced electrode	400	330	75	20,000
4. Zinc/Bromide battery	400	400	70	3,000
5. Vanadium redox battery	400	600	65	5,000
6. Lithium-ion battery	400	600	85	4,000
7. High-speed composite flywheel	600	1,600	95	25,000
8. Supercapacitor	500	10,000	95	25,000

Table 6: Normalized performance scores with respect to each criterion and the overall score of alternatives

Alternative	Power Cost ( $w_1=0.386$ )	Storage Cost ( $w_2=0.386$ )	Efficiency ( $w_3=0.044$ )	Life ( $w_4=0.044$ )	Global priority
1. Advanced lead-acid battery	0.131	0.196	0.125	0.023	0.136
2. Sodium/Sulfur battery	0.150	0.185	0.117	0.034	0.141
3. Lead-acid battery with carbon-enhanced electrode	0.131	0.196	0.117	0.230	0.174
4. Zinc/Bromide battery	0.131	0.162	0.109	0.034	0.124
5. Vanadium redox battery	0.131	0.108	0.102	0.057	0.107
6. Lithium-ion battery	0.131	0.108	0.133	0.046	0.107
7. High-speed composite flywheel	0.088	0.040	0.148	0.287	0.109
8. Supercapacitor	0.105	0.006	0.148	0.287	0.102

The global priority weights of these technologies were then computed using Eq(3). This can be interpreted as the overall score which is analogous to additive weighting of normalized performance scores for all the criteria. As shown in the last column of Table 6, the lead-acid battery with carbon-enhanced electrode is the most preferred technology with a score of 0.174 while the supercapacitor is the least preferred having a score of 0.102. It can also be seen that the overall scores of the four least preferred options are very similar: supercapacitor (0.102), lithium-ion battery (0.107), vanadium redox battery (0.107) and high-speed composite flywheel (0.109). This trend shows the influence of the high priority placed on capital cost in this example, which puts these four alternatives at a distinct disadvantage compared to the more preferred technologies.

#### 4. Conclusion

An FAHP approach to decision-making for process engineering problems has been developed and applied to prioritization of low carbon technologies. This approach enables the selection of optimal alternatives based on multiple criteria which may be quantitative or qualitative in nature, based on the judgement of domain experts. In addition, the degree of confidence of the expert may be quantified through the spread of fuzzy numbers used for the pairwise comparison ratios in the AHP framework. The approach can also derive crisp weights from an incomplete fuzzy PCJM. The methodology then determines a set of crisp weights that maximizes the degree of consistency of all judgements taken together. A case study on the

prioritization and optimal selection of energy storage device have been solved to illustrate the technique. Although, as with conventional AHP, the approach developed here is still dependent on the inputs of the domain expert, it provides a more nuanced elicitation of judgements, by allowing confidence levels to be reflected for each pairwise comparison. Future work would account for inputs from multiple decision makers for group decision making incorporating also the degree of importance of each decision maker.

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