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Comparison of Robust Model-based Control Strategies Used for a Heat Exchanger Network

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The paper is focused on advanced control of the heat exchanger network (HEN). The HEN is used for cooling petroleum produced by distillation. The alternative robust model predictive control (RMPC) strategy was implemented to find the optimal control actions taking into account the boundaries on control inputs. The RMPC approach is also able to design a controller managing process uncertainties. The aim is to demonstrate that the robust model predictive control of HEN can be improved and the energy efficiency can be optimised using the nominal system optimization and the additional saturation of control inputs.

1. Introduction

The heat exchangers (HEs), as ones of the most energy demanding equipment in industry, attract high interest of specialist in chemical engineering and process control. The heat losses can rise up to 50 % (Čuček et al., 2013) and therefore it is necessary to implement advanced control strategies and to optimize operation of HEs. Model-based predictive control (MPC) (García et al.1989) is one of the most popular advanced control strategies with many practical applications. An optimal control action is calculated using the mathematical model of the controlled process subject to the constraints on control inputs and controlled outputs. Robust MPC (RMPC) (Bemporad and Morari, 1999) can be designed, when the mathematical model of the controlled process takes into account the process uncertainty. The uncertainty can be caused by the non-linear process behaviour, the time-varying parameters, or the external disturbances and measurement noises. Heat exchanger networks (HENs) are widely used in petroleum and petrochemical industries. Synthesis of HENs represents a heuristic approach that aims to optimize energy utilization. A novel strategy for obtaining feasible initial conditions of non-convex MINLP problem of HEN synthesis was proposed in Jongsuwat et al. (2014). The open-source software for creation, manipulation and optimization of parametric shell-and-tube HEs was presented in Létal (2014). The software represents an essential part in developing software for mechanical design or check of shell-and-tube HEs. The neural network predictive control with an auxiliary fuzzy controller was designed for non-linear HE in Vasičkaninová and Bakošová (2014). Simulation results confirmed improved control performance compared to the various well-tuned PID controllers. The temperature in a reaction distillation column was successfully controlled by MPC in Komkrajang et al. (2014). Generalized predictive control designed in Zhang et al. (2012) improved the control performance and disturbance rejection of a heat recovery process. RMPC has been used in various case studies and industrial applications up to now. Based on our previous research, we analysed various RMPCs for HENs, see e.g. Bakošová and Oravec (2014a) and references therein. Applying RMPC with additional control input saturation (ACIS) for HENs can increase the overall energy savings in comparison with the RMPC based just on the single Lyapunov function and the worst-case system optimization as it is shown in Bakošová and Oravec (2014b). Moreover, the ACIS-based RMPC assures better control performance with a smaller steady-state offset. The aim of this paper is to present the novel approach that increases the energy savings and improves the quality of control. The developed strategy is the RMPC based on the nominal system optimization (NSO) approach (Cao and Lin, 2005) and the ACIS approach (Wan and Kothare, 2003). Using the NSO-based strategy extends the set of feasible initial conditions. The optimization problem is transformed into the

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semidefinite programming (SDP) problem via linear matrix inequalities (LMIs). The optimization problem in the form of SDP is solved in each control step and has smaller computational burden than the original RMPC. The additional control input saturation enables to implement control inputs in a wider range. The ACIS-based procedure enables to increase the energy savings. The developed RMPC strategy is implemented for control of the HEN used in a refinery. The quality of the designed RMPC was analysed by simulations in the MATLAB programming environment, using the YALMIP toolbox and the solver MOSEK. Obtained results confirmed the significant energy savings in comparison with the original RMPC strategy and the standard control approach.

The paper is organized as follows. Section 2 describes the technological properties of considered HEN. Advanced RMPC design approaches are introduced in Section 3. Here, the proposed alternative RMPC design strategy is also described. Section 4 discusses the obtained simulation results. Finally, Section 5 formulates the main conclusions.

2. Heat exchanger network

The controlled plant was adopted from Bakošová and Oravec (2012) to obtain comparable results. The assumed HEN is composed of three identical counter-current shell-and-tube HEs in series. The feed of the HEN to be cooled down is the petroleum as a product of distillation in a refinery. Petroleum flows in the inner tubes and the cooling water in the shell of every heat exchanger. The tubes of the HEs are made from steel. The controlled variable is temperature of the outlet stream of petroleum from the 3rd HE. The control input is volumetric flow rate of the inlet cold water into the 3rd HE. The objective is to decrease the outlet temperature of the petroleum to the reference value 45 °C and to minimise the energy demands measured by the total consumption of cold water. Technological parameters and control conditions are the same as in Bakošová and Oravec (2014b) and are summarized in Table 1, where n is the number of HE's tubes, l is the length of the HE, $d_{\mathrm{in},1}$ is the inner diameter of the tube, $d_{\mathrm{out},1}$ is the outer diameter of the tube, $d_{\mathrm{in},2}$ is the inner diameter of the HE, A_{h} is the total heat transfer area, V is the volume, c_{p} is the thermal capacity, ρ is the density, T_{in} is the inlet temperature, q is the volumetric flow rate. The subscripts 1 and 2 refer to water and petroleum. The superscripts (1)–(3) denote individual HEs and the superscripts S and 0 denote the steady-state value and the initial value.

Table 1: Technological parameters and reference values of HEs

Parameter	Unit	Value	Parameter	Unit	Value
n	1	40	$T_{\mathrm{in},1}$	°C	20.0
l	m	6	$T_{\mathrm{in},2}$	°C	180.0
$d_{\mathrm{in},1}$	m	19×10^{-3}	$T_1^{(1),\mathrm{S}}$	°C	75.8
$d_{\mathrm{in},2}$	m	414×10^{-3}	$T_1^{(2),\mathrm{S}}$	°C	48.0
$d_{\mathrm{out},1}$	m	25×10^{-3}	$T_1^{(3),\mathrm{S}}$	°C	30.8
A_{h}	m^2	16.6	$T_2^{(1),\mathrm{S}}$	°C	113.0
V_1	m^3	91.2×10^{-3}	$T_2^{(2),\mathrm{S}}$	°C	71.3
V_2	m^3	716.5×10^{-3}	$T_2^{(3),\mathrm{S}}$	°C	45.3
q_2^{S}	$\mathrm{m}^3\ \mathrm{s}^{-1}$	5.8×10^{-3}	$T_1^{(1),0}$	°C	87.1
$c_{\mathrm{p},1}$	$\mathrm{J\ kg}^{-1}\ \mathrm{K}^{-1}$	4.186×10^3	$T_1^{(2),0}$	°C	55.7
$c_{\mathrm{p},2}$	$\mathrm{J\ kg}^{-1}\ \mathrm{K}^{-1}$	2.140×10^3	$T_1^{(3),0}$	°C	34.4
ρ_1	$\mathrm{kg\ m}^{-3}$	980.0	$T_2^{(1),0}$	°C	118.4
ρ_2	$\mathrm{kg\ m}^{-3}$	810.0±16.2	$T_2^{(2),0}$	°C	76.8
U	$\mathrm{J\ s}^{-1}\ \mathrm{m}^{-2}\ \mathrm{K}^{-1}$	482.3±9.7	$T_2^{(3),0}$	°C	48.7

Moreover, we consider that two technological parameters are uncertain, i.e., the heat-transfer coefficient U changes as the flow rate of the cooling medium changes, and the density of the petroleum ρ_2 depends on the temperature in the HEs (Table 1). The uncertainty of these parameters is represented via interval parametric uncertainty.

3. Advanced robust MPC design

For the robust MPC design, the mathematical model of the heat exchangers was derived using the enthalpy balances. The linearized time-invariant state-space model in the discrete-time domain is given by

$$
\begin{aligned}
& x(k+1) = Ax(k) + Bu(k),\ x(0) = x_0, \\
& y(k) = Cx(k), \qquad (1)\\
& [A,B] \in \Omega,\ \Omega = \mathrm{convhull}\left(\left[A^{(v)}, B^{(v)}\right], \forall v \in \{1,\dots,n_{\mathrm{v}}\}\right),
\end{aligned}
$$

where k represents the discrete time. The used sampling period was t_s = 25 s. Further, $x(k)$ is the vector of states represented by the temperatures $T_1^{(1)-(3)}$ and $T_2^{(1)-(3)}$ (Table 1), $u(k)$ is the control input represented by the volumetric flow rate of the cooling medium q_1, $y(k)$ is the vector of the system outputs. The matrices $A^{(v)}$, $B^{(v)}$, C have appropriate dimensions. The model in Eq(1) is an uncertain system with interval polytopic uncertainty. For the uncertain model of the HEN one can obtain four vertices computed as the combination of boundary values of uncertain parameters. Hence, the matrices $A^{(v)}$, $B^{(v)}$, v = 1,…,4, describe the vertex systems of the uncertain system Eq(1). The 5th considered system is the nominal system calculated for the mean values of the uncertain parameters (Table 1). Then the robust static state-feedback control problem in the discrete-time domain can be formulated as follows: find a state-feedback control law

$$u(k) = F_k x(k), \tag{2}$$

for the system described by Eq(1). The matrix F_k in Eq(2) represents the static state-feedback robust controller for the k-th control step.

Quality of the control performance can be described using the quadratic cost function

$$J = \sum_{k=0}^{n_k} \left(J_x(k) + J_u(k) \right) = \sum_{k=0}^{n_k} \left(x(k)^T W_x x(k) + u(k)^T W_u u(k) \right), \tag{3}$$

where n_k is the total number of control steps. For design purposes the infinity control horizon is assumed, and W_x, W_u are real square symmetric positive-definite weight matrices of the states $x(k)$ and the system inputs $u(k)$. The aim is to design the controller F_k that ensures robust stability of all considered vertex systems and minimizes the cost function J in Eq(3). The control performance can be improved by taking into account symmetric constraints on system outputs $y(k)$ and inputs $u(k)$ in the form

$$\|y(t)\|^2 \le y_{\max}^2, \|u(t)\|^2 \le u_{\max}^2, \tag{4}$$

Following conditions hold for the symmetric positively defined Lyapunov matrix P_k and the feedback controller F_k

$$P_k = \gamma_k X_k^{-1}, Y_k = F_k X_k, \Rightarrow F_k = Y_k X_k^{-1}, \tag{5}$$

where γ_k is the auxiliary optimization parameter, X_k is the symmetric positively defined matrix, and Y_k represents the auxiliary matrix enabling the evaluation of the robust feedback controller F_k (Cao et Lin, 2005).

Several strategies were used to investigate the robust MPC of HEN. $RMPC_1$ denotes NSO-based control strategy described in the paper (Wan et al., 2003). The robust stabilization problem can be solved as the robust MPC convex optimization problem based on the LMIs as follows

$$\min_{\lambda_k, X_k, Y_k} \gamma_k \tag{6}$$

subject to

$$\begin{bmatrix} 1 & x_k^{\mathrm{T}} \\ * & X_k \end{bmatrix} \ge 0, \tag{7}$$

$$\begin{bmatrix} X_k & \left(A^{(v)} X_k + B^{(v)} Y_k\right)^{\mathrm{T}} \\ * & X_k \end{bmatrix} > 0, \begin{bmatrix} X_k & \left(A^{(0)} X_k + B^{(0)} Y_k\right)^{\mathrm{T}} & X_k \sqrt{W_x} & Y_k^{\mathrm{T}} \sqrt{W_u} \\ * & X_k & 0 & 0 \\ * & * & \gamma_k I & 0 \\ * & * & * & \gamma_k I \end{bmatrix} \ge 0, \tag{8}$$

where v = 1,…, n_v. The symbol * denotes a symmetric structure of the matrix, and I, 0 are identity and zero matrices of appropriate dimensions. X_k is the symmetric positively defined matrix. The symmetric constraints on control inputs and outputs in the form of Eq(4) can be added to the optimization problem Eq(6) – Eq(7) in the following LMI form

$$\begin{bmatrix} u_{\max}^2 I & Y_k \\ * & X_k \end{bmatrix} \geq 0, \begin{bmatrix} X_k & \left(A^{(v)}X_k + B^{(v)}Y_k\right)^{\mathrm{T}} C^{\mathrm{T}} \\ * & y_{\max}^2 I \end{bmatrix} \geq 0, \tag{9}$$

where $v = 1,\ldots, n_{\mathrm{v}}$.

The second considered strategy, denoted as $RMPC_2$, is ACIS-based robust MPC approach presented in Cao et Lin (2005). The algorithm for the controller design by the $RMPC_2$ was presented in the paper Bakošová and Oravec (2014b).
The third strategy denoted by $RMPC_3$ is our developed approach based on the alternative formulation of strategies $RMPC_1$ and $RMPC_2$. The main idea is to adopt the advantages of both approaches to improve the control performance. Using NSO-based procedure reduces the overall computational effort, and ACIS-based strategy reduces the conservativeness of control input evaluation. In the optimization problem in Eq(6) – Eq(9) the LMIs presented in Eq(8) are replaced using

$$\begin{bmatrix} X_k & \left(A^{(v)}X_k + B^{(v)}\left(E_j Y_k + E_j^{-} U_k\right)\right)^{\mathrm{T}} \\ * & X_k \end{bmatrix} > 0, \tag{10}$$

$$\begin{bmatrix} X_k & \left(A^{(0)}X_k + B^{(0)}\left(E_j Y_k + E_j^{-} U_k\right)\right)^{\mathrm{T}} & X_k\sqrt{W_{\mathrm{x}}} & \left(E_j Y_k + E_j^{-} U_k\right)^{\mathrm{T}}\sqrt{W_{\mathrm{u}}} \\ * & X_k & 0 & 0 \\ * & * & \gamma_k I & 0 \\ * & * & * & \gamma_k I \end{bmatrix} \geq 0 \tag{11}$$

Instead of LMIs in Eq(9) the constraints are handled by following LMIs

$$\begin{bmatrix} u_{\max}^2 I & U_k \\ * & X_k \end{bmatrix} \geq 0, \begin{bmatrix} X_k & \left(A^{(v)}X_k + B^{(v)}\left(E_j Y_k + E_j^{-} U_k\right)\right)^{\mathrm{T}} C^{\mathrm{T}} \\ * & y_{\max}^2 I \end{bmatrix} \geq 0 \tag{12}$$

for $v = 1,\ldots, n_{\mathrm{v}}$, $j = 1,\ldots, n_{\mathrm{u}}$. The matrices E_j are the diagonal matrices with all variations of 1 and 0 on the principal diagonal and zeroes elsewhere; E_j^{-} are the complement matrices obtained as $E_j^{-} = I - E_j$. The idea of this extension is to take into account all variations of constrained and unconstrained control inputs. Then the algorithm for the $RMPC_3$ can be formulated in following eight steps.

The algorithm for $RMPC_3$ can be formulated in following eight steps:
Step 1: Set parameter $k = 0$.
Step 2: Set number of control steps N, initial conditions of states $x(0)$, values of the symmetric constraints on control input $u_{\max}$ and output $y_{\max}$.
Step 3: Set parameter $k = k + 1$.
Step 4: Set the values of states $x(k)$.
Step 5: Solve optimization problem described by Eq(6), Eq(7), Eq(10), Eq(11), Eq(12) to evaluate X_k, Y_k.
Step 6: Design the matrix F_k of the feedback controller using Eq(5).
Step 7: Calculate the control input $u(k)$ using the control law Eq(2).
Step 8: If the parameter $k < N$ then go to the *Step 3* else *Stop*.

4. Results and discussion

The designed robust MPC strategies $RMPC_1$ – $RMPC_3$ were investigated via simulations of control of the non-linear model of HEN using 2.8 GHz CPU and 4 GB RAM in the MATLAB-Simulink environment using the toolbox YALMIP (Löfberg, 2004) and the solver MOSEK. Alternatively, SeDuMi solver can be implemented as well. The robust state-feedback controllers were designed for the weight matrices W_{x}, W_{u} in the cost function described by Eq(3) in the form $\mathrm{diag}(W_x)=[0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.1,\ 0.1,]^{\mathrm{T}}$, $\mathrm{diag}(W_u)=[1{,}000]$, where diag denotes the diagonal matrix with the given elements on the principal diagonal and zero elsewhere. These weight matrices were used in all $RMPC_i$ algorithms to assure comparability of obtained results. The $RMPC_i$ strategies were compared according to the offset of the petroleum temperature $\Delta T_2^{(3)}$, and consumption of the cooling medium V_C. The aim of control was to cool down the petroleum temperature from 118.4 °C to 45.3 °C during 1,875 s. We designed also the well-known discrete-time LQ optimal controller (LQR, see e.g. Mikleš and Fikar, 2007, chap.8.2) to compare the results obtained by RMPC strategies. The gain matrix F_k, k = LQR, of the LQR controller in the feedback (2) was

$$F_{\mathrm{LQR}} = \begin{bmatrix} 105.1 & -29.7 & 1.6 & -20.1 & 1.0 & -8.1 \end{bmatrix} \times 10^{-6}. \quad (13)$$

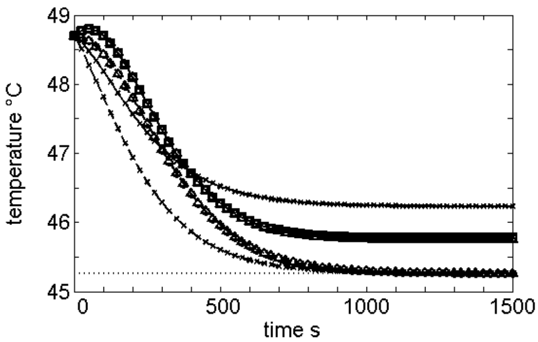

Figure 1: Control responses of the petroleum temperature assured by LQR (×), RMPC$_1$ (○), RMPC$_2$ (□), RMPC$_3$ (Δ) strategies in the worst-case (solid) and the best-case (dashed) scenarios.

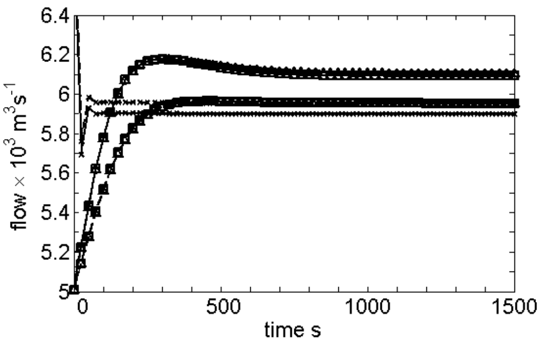

Figure 2: Flow-rates generated by LQR (×), RMPC$_1$ (○), RMPC$_2$ (□), RMPC$_3$ (Δ) strategies in the worst-case (solid) and the best-case (dashed) scenarios.

Figures 1, 2 show control responses only during 1,500 s to present the dynamics clearly. Figure 1 presents the control performances of the outlet petroleum temperature assured by *LQR* (×), *RMPC*$_1$ (○), *RMPC*$_2$ (□), *RMPC*$_3$ (Δ) strategies in the worst-case (solid) and the best-case (dashed) scenarios. The reference is denoted by the dashed-dotted line. The worst-case scenario represents the vertex system with the maximal value of criterion Eq(3). The best-case scenario considers the vertex system with the minimal value of analyzed criterion Eq(3). Figure 2 shows the associated control inputs. *LQR* control ensured the fastest convergence of temperature to the reference value in the best case scenario. On the other hand, the control response is the slowest with the largest steady-state offset in the worst-case scenario. These results confirm that LQR control is not proper strategy for processes with significant uncertainty. The control trajectories generated by RMPC approaches for the best-case and the worst-case scenarios are similar and close together and therefore it is hard to distinguish the matching lines visually.

Table 2: Results of LQ Control and RMPC

method	scenario	$\Delta T_2^{(3)}$ [°C]	J	$J_u \times 10^3$	t_{sol} [s]
LQR	*best case*	-0.01	54.7	0.866	0.05
	worst case	0.97	90.9	1.095	0.05
RMPC$_1$	*best case*	-0.01	140.1	2.723	0.29
	worst case	0.68	491.1	24.495	0.76
RMPC$_2$	*best case*	-0.01	140.1	2.724	0.66
	worst case	0.68	462.8	24.579	2.05
RMPC$_3$	*best case*	0.00	140.0	2.719	0.71
	worst case	0.56	492.7	24.574	1.71

Nevertheless, we analysed the other analytical quality criteria. In Table 2, method denotes implemented control strategy, $\Delta T_2^{(3)}$ is the steady-state offset of the controlled temperature, J is the value of quadratic criterion in Eq(3), J_u is the value of quadratic criterion in Eq(3) without taking states into account, and t_{sol} is the average computational time necessary for solving the controller design problem. LQR control ensured the best values of quadratic criterion, and has also the least computational effort. On the other hand, there is the maximal off-set in the worst-case scenario. Alternative NSO&ACIS-based RMPC$_3$ approach assured the minimal off-set. Moreover, RMPC$_3$ approach assured the best-case value of the criterion J_u that represents the differences in consumption of cooling medium, see Table 2. The worst-case value J_u of RMPC$_3$ was the second best, as RMPC$_1$ method ensured slightly better value. RMPC$_1$ strategy also needs the shortest computational time, compared to RMPC$_2$ that had the longest computational time between the described RMPC approaches. Alternative RMPC$_3$ approach is the compromise for the computational time between RMPC$_1$ and RMPC$_2$ (Table 2). Hence, RMPC$_3$ is the most suitable strategy for the temperature control in the HEN with significant uncertainty.

5. Conclusions

The paper demonstrates the possibility to implement various RMPC strategies for control of HEN with uncertainty. The obtained results were analysed according to the control responses, off-sets and computational time. LQR controller was designed as the reference controller. LQR control leads to the best results when the controlled process has no uncertain parameters. RMPC strategies assure better results when the controlled process has significant uncertainty. Between studied strategies, the alternative NSO&ACIS-based RMPC is the best choice as it assured sufficient control accuracy and utilization of cooling medium, and compromise computational effort.

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