

Water and Heat Exchanger Network Design for Fixed-Flowrate System

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This work presents a new model of optimization-based approaches by mathematical programming for simultaneous water and heat exchanger network (WHEN) design of fixed-flowrate system. WHEN is designed by water network (WN) followed by heat exchanger network (HEN) in sequential step. First, WN is designed by mass balance from sources to sinks. Second, HEN is designed by matching hot waste streams and cold sink streams to satisfy the outlet temperature. The main objective is to minimize total annual cost (TAC) of freshwater cost, piping cost, investment cost of heat exchanger unit, and hot/cold utility cost using developing case study data. Mixed integer nonlinear programming (MINLP) is developed for simultaneous design by cascading calculation. Initialization technique is used because of the high non-convexity of MINLP problem.

1. Introduction

Water and hot/cold utilities are essential resources in industrial processes. Freshwater is usually used as separation agent in various units. Water network (WN) design problems are categorized into two kinds; fixed-load and fixed-flowrate problems. For fixed-load problem, contaminant load of each process is fixed, but inlet and outlet contaminant concentrations of stream change in the proper range where many researches are published in various scenarios later on. For fixed-flowrate problem, inlet and outlet streams of all units in process are categorized as sinks and sources, respectively. Without freshwater minimization, sink is intaken by freshwater and source is discharged as wastewater. For reuse method, source is combined with minimum freshwater to generate sink at desired flowrate and contaminant concentration. There are many techniques to target minimum freshwater required based on pinch technology. Water cascade table is one of famous technique proposed by Foo (2008). This technique targets both minimum freshwater flowrate required and minimum wastewater discharge, but it does not design the water network in one step. For heat exchanger network (HEN), the popular model is stage-wise superstructure model by Yee and Grossmann (1990). The objective of HEN design is to minimize hot utility, cold utility, and a number of exchangers. Mathematical programming is widely used to design optimal network with complex constraints. Mixed integer nonlinear programming (MINLP) is the most complex model, which is hard to reach the optimal result. Proper initialization values and bounding are required. Recently, Ahmetović et al. (2013) and Li et al. (2013) proposed optimal simultaneous water and heat exchanger networks of fixed-load problem by two-step approaches MINLP. First, they solve for the optimal water network to minimize freshwater and second solving step is overall water and HEN. Zhou et al. (2012) presented an optimal simultaneous water allocation and heat exchanger network by MINLP for multi-contaminant, fixed-flowrate problem where the objective is to minimize total annual cost. They developed multiscale state-space superstructure to solve their problem. A drastic complex model combines water and heat exchanger networks by distribution network. This work aims to present a new model of water and heat exchanger network design that can solve single-contaminant, fixed-flowrate problem in sequential-step approach where the objective is to minimize total annual cost, consisting of freshwater cost, piping cost, investment cost of heat exchanger unit, and hot/cold utility cost using collected and developed data.

2. Problem statement

The problem in this paper is stated as the WHEN design model as shown in Figure 1. WN is designed followed by HEN as sequential step. Index i is an index for the process source, j is an index for the sink, and k is an index for stages of HEN stage-wise superstructure. A set of process sources i for composition (CS_i), flowrate (FS_i), and temperature (TS_i) is addressed. A set of process sinks j for specific composition (CK_j), flowrate (FK_j), and temperature (TKL_j) is also given. Freshwater flowrate (FW_j) is determined by process data and its cost. Freshwater has constant temperature (TFW) and contaminant concentration (CFW). Piping allocation is determined by its cost. Wastewater must discharge at the temperature not over the limitation (T_w). HEN is determined by its investment cost and utility cost. All data are developed and adapted from literature by Foo (2008) and Ahmetović et al. (2013). The main goal of this paper is to generate optimal WHEN with lowest TAC.

The problem is solved by GAMS version 24.2.1 where solver are CPLEX v.12.6 as the LP solver, CONOPT v.3.15M as the NLP solver, and DICOPT as the MINLP solver on a PC machine (i7 2.4 GHz, 8 GB of RAM, 64-bit-operation system).

Assumptions;

1. Specific heat capacity (CP) is constant to $4.2 \text{ kJ}/(\text{kg } ^\circ\text{C})$.
2. Non-isothermal mixing occur when WN is designed.
3. After doing WN, Sinks must have lower temperature than sources for next-step HEN design (Sources are determined as hot streams and sinks are determined as cold streams).
4. Discharged waste concentration is not fixed.
5. Area cost equation, which is $8,000z_{i,j}+1,200(\text{Area})^{0.6}$, are taken from Ahmetović et al. (2013) and linearized to $(19,965.89+8,000)z_{i,j}+55.749(\text{Area})$.

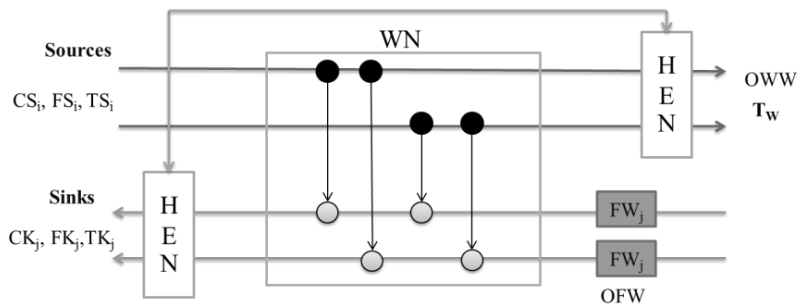


Figure 1: Water and heat exchanger network model

3. Model formulation

The model consists of 2 sets of equations; the mass balance constraints of WN design (Eqs. 1 - 11) and energy balance constraints of stage-wise HEN design (Eqs. 18 - 46). Both sets are mixed integer nonlinear programming (MINLP). Because of the presence of non-linear and non-convex terms of MINLP, it needs initialization technique that the WHEN model must be calculated for five steps sequentially with five objectives as shown in Figure 2. First, mixed integer linear programming is introduced to calculate minimum freshwater, transfer flowrate from sources to each sink, and minimum freshwater where the objective is to minimize freshwater cost. All calculated variables are used as initialization for second solving step, which is MINLP. Second, WN is designed where the objective is to minimize freshwater cost and piping annual cost. Piping allocation perform by its cost to the lower annual cost. Completed WN is designed with optimal sink flowrate and concentration value (FK_j and CK_j). Sinks temperature are changed by the non-isothermal mixing of freshwater and source streams to arbitrary value (TK_j). For the third solver, necessary variables consist of flowrate and temperature are linked between WN and HEN. HEN is calculated from sink streams (cold streams), where the inlet temperature is variable TK_j from second solver, and waste stream of source (hot streams) with the objective is to minimize hot/cold utility cost and exchanger area fixed cost in order to be initial values of next fourth solving step where the objective is to minimize hot/cold utility cost, and exchangers annual cost. Completed HEN is designed from previous solving step. From first through fourth steps, simple WHEN is generated. Furthermore, in the last step, all equations of WN and HEN are calculated simultaneously to design new WHEN by MINLP using initial variables of previous WHEN to develop lower TAC network. WHEN model equations are shown below.

WN mass balance;

$$\sum_i FS_i \cdot x_{i,j} + FW_j = FK_j \quad (1)$$

$$\sum_i FS_i \cdot x_{i,j} \cdot CS_i + FW_j \cdot CFW = FK_j \cdot CK_j \quad (2)$$

$$\sum_{i,j} x_{i,j} \leq 1 \quad (3)$$

$$x_{i,j} \cdot FS_i = F_{i,j} \quad (4)$$

$$WW_i = (1 - \sum_j x_{i,j}) \cdot FS_i \quad (5)$$

Non-isothermal heat balance;

$$\sum_i FS_i \cdot x_{i,j} \cdot TS_i + FW_j \cdot TFW = FK_j \cdot TK_j \quad (6)$$

Streams existing logic constraint;

$$F_{i,j} - \alpha \cdot y_{i,j} \leq 0 \quad (7)$$

$$FW_j - \alpha \cdot y_{fw_j} \leq 0 \quad (8)$$

$$WW_i - \alpha \cdot y_{ww_i} \leq 0 \quad (9)$$

WN cost;

$$FRcost = \sum_j FW_j \cdot FRC \cdot WH \quad (10)$$

$$PIcost = \sum_{i,j} (y_{i,j} \cdot FC1_{i,j} + F_{i,j} \cdot VC1_{i,j} \cdot WH) \quad (11)$$

$$+ \sum_j (y_{fw_j} \cdot FC2_j + FW_j \cdot VC2_j \cdot WH)$$

$$+ \sum_j (y_{ww_i} \cdot FC3_i + WW_i \cdot VC3_i \cdot WH) \quad (11)$$

Linking variables;

$$WW_i = FH_i \quad (12)$$

$$TS_i = TH_i^{in} \quad (13)$$

$$TW = TH_i^{out} \quad (14)$$

$$FK_j = FC_j \quad (15)$$

$$TK_j = TC_j^{in} \quad (16)$$

$$TKL_j = TC_j^{out} \quad (17)$$

HEN overall heat balance;

$$(TH_i^{in} - TH_i^{out}) \cdot FH_i \cdot CP = \sum_{j,k} q_{i,j,k} + qcu_i \quad (18)$$

$$(TC_j^{out} - TC_j^{in}) \cdot FC_j \cdot CP = \sum_{i,k} q_{i,j,k} + qhu_j \quad (19)$$

HEN stage heat balance;

$$(TH_{i,k} - TH_{i,k+1}) \cdot FH_i \cdot CP = \sum_j q_{i,j,k} \quad (20)$$

$$(TC_{j,k} - TC_{j,k+1}) \cdot FC_j \cdot CP = \sum_i q_{i,j,k} \quad (21)$$

$$TH_i^{in} = TH_{i,1} \quad (22)$$

$$TC_j^{in} = TC_{j,NOK+1} \quad (23)$$

Feasibility of temperature;

$$TH_i^{out} \leq TH_{i,NOK+1} \quad (24)$$

$$TC_j^{out} \geq TC_{j,1} \quad (25)$$

Hot and cold utility;

$$(TH_{i,NOK+1} - TH_i^{out}) \cdot FH_i \cdot CP = qcu_i \quad (26)$$

$$(TC_j^{out} - TC_{j,1}) \cdot FC_j \cdot CP = qhu_j \quad (27)$$

Exchangers existing logic constraint;

$$q_{i,j,k} - \omega z_{i,j,k} \leq 0 \quad (28)$$

$$qcu_i - \omega zcu_i \leq 0 \quad (29)$$

$$qhu_j - \omega zhu_j \leq 0 \quad (30)$$

Logic constraint for temperature difference;

$$dT_{i,j,k} \leq (TH_{i,k} - TC_{j,k}) + \gamma(1 - z_{i,j,k}) \quad (31)$$

$$dT_{i,j,k+1} \leq (TH_{i,k+1} - TC_{j,k+1}) + \gamma(1 - z_{i,j,k}) \quad (32)$$

$$dTcu_i \leq (TH_{i,NOK+1} - T_{cu}^{out}) + \gamma(1 - zcu_i) \quad (33)$$

$$dThu_j \leq (T_{hu}^{out} - TC_j^{out}) + \gamma(1 - zhu_j) \quad (34)$$

$$dT_{i,j,k}, dTcu_i, dThu_j \geq EMAT \quad (35)$$

Log mean temperature difference;

$$LMTD_{i,j,k} = \left[(dT_{i,j,k} \cdot dT_{i,j,k+1}) \cdot \frac{(dT_{i,j,k} + dT_{i,j,k+1})}{2} \right]^{1/3} \quad (36)$$

$$LMTDcu_i =$$

$$\left[dTcu_i \cdot (TH_i^{out} - T_{cu}^{in}) \cdot \frac{(dTcu_i + (TH_i^{out} - T_{cu}^{in}))}{2} \right]^{1/3} \quad (37)$$

$$LMTDhu_j =$$

$$\left[dThu_j \cdot (T_{hu}^{in} - TC_j^{out}) \cdot \frac{(dThu_j + (T_{hu}^{in} - TC_j^{out}))}{2} \right]^{1/3} \quad (38)$$

Exchangers area;

$$A_{i,j,k} = \frac{q_{i,j,k}}{U \cdot LMTD_{i,j,k}} \quad (39)$$

$$Acu_i = \frac{qcu_i}{U \cdot LMTDcu_i} \quad (40)$$

$$Ahu_j = \frac{qhu_j}{U \cdot LMTDhu_j} \quad (41)$$

Exchangers area cost;

$$Acost_{i,j,k} = (8000 + 1200(A_{i,j,k})^{0.6}) \cdot AF \quad (42)$$

$$Acucost_i = (8000 + 1200(Acu_i)^{0.6}) \cdot AF \quad (43)$$

$$Ahucost_j = (8000 + 1200(Ahu_j)^{0.6}) \cdot AF \quad (44)$$

Utility operating cost;

$$cuOP_i = qcu_i \cdot CUC \quad (45)$$

$$huOP_j = qhu_j \cdot HUC \quad (46)$$

Total annual cost;

$$TAC = FRcost + PIcost + \sum_{i,j,k} Acost_{i,j,k} + \sum_i Acucost_i + \sum_j Ahucost_j + \sum_i cuOP_i + \sum_j huOP_j \quad (47)$$

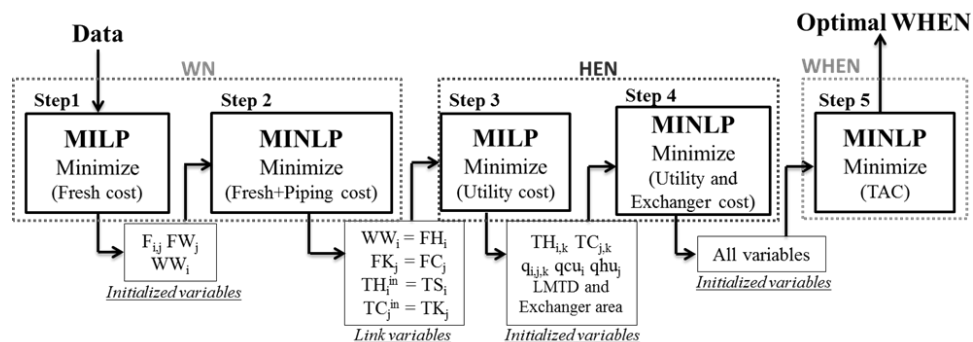


Figure 2: Sequential-step solvers flowchart

4. Example

The example is developed to illustrate the sequential WHEN model consists of five process sources (S) and five process sinks (D) with known concentration, flowrate, and temperature shown in Table 1. Cost data are shown in Table 2. Piping fixed-cost (FC) and variable cost (VC) of all possible streams are shown in Tables 3 and 4. For first-step and second solving step, minimum freshwater flowrate (FW_i), transfer flowrate ($F_{i,j}$), and minimum waste (WW_i) are found with appropriate make-up streams to complete WN with minimal freshwater and piping cost in annual. After doing WN, Sinks temperature ($TK_1 - TK_5$) are risen to 56.67 °C, 75 °C, 77.093 °C, 77.273 °C, and 55 °C, respectively. Residue sources (i_1, i_2, i_3) are used as hot stream (H) for next designing where sinks act as cold stream (C). Next, HEN is designed by solving step 3 and 4 to develop primitive WHEN. Up to this point, TAC is 488,160.62 \$/y. WHEN is designed twice at the last step where results are the same as previous WHEN. The result is shown in Table 5 and the overall WHEN and HEN are shown in Figures 3 and 4, respectively.

Table 1: Example sources and sinks data

Source i	FS _i (t/h)	CS _i (ppm)	TS _i (°C)	Sink j	FK _j (t/h)	CK _j (ppm)	TKL _j (°C)
1	9	130	120	1	10	20	100
2	9	108	100	2	4	20	100
3	9	70	130	3	12	20	100
4	9	44	140	4	8	20	100
5	4.5	22	80	5	6.5	20	100

Table 2: Cost and operating cost parameters

Parameter	Unit	Value
Freshwater cost (FRC)	\$/t	0.375
Cooling utility cost (CUC)	\$(kW·y)	189
Heating utility cost (HUC)	\$(kW·y)	377
Exchangers fixed cost	\$	8,000
Exchangers area coefficient cost	\$/m ²	1200
Cost exponent for exchangers		0.6
Overall heat-transfers coefficient (U)	kW/(m ² ·°C)	0.5
Working hours of plant per year (WH)	h	8,000
Annualize factor of investment cost (AF)	y ⁻¹	0.333
Inlet and outlet heating steam temperature ($T_{hu}^{in}, T_{hu}^{out}$)	°C	120
Inlet and outlet cooling water temperature ($T_{cu}^{in}, T_{cu}^{out}$)	°C	10, 20
Freshwater temperature (TFW)	°C	25
Wastewater temperature (Tw)	°C	30
Exchangers minimum approach temperature (EMAT)	°C	10

Table 3: Piping fixed-cost and variable cost of source to sink streams

Sink j \ Source i	1		2		3		4		5	
	FC (\$/y)	VC×10 ⁻³ (\$/t)	FC (\$/y)	VC×10 ⁻³ (\$/t)	FC (\$/y)	VC×10 ⁻³ (\$/t)	FC (\$/y)	VC×10 ⁻³ (\$/t)	FC (\$/y)	VC×10 ⁻³ (\$/t)
1	100	1.1	300	1.2	150	1.3	200	1.4	200	1.1
2	200	0.8	250	1.3	100	1.1	220	1.2	150	0.9
3	220	0.9	300	1.3	330	1.1	300	1.4	110	1.1
4	110	1.3	400	1.2	200	1.3	200	1.4	250	1.1
5	200	1.1	300	0.8	100	1.3	300	1.4	250	0.9

Table 4: Piping fixed-cost and variable cost of freshwater and wastewater streams

Source i or Sink j	1		2		3		4		5	
	FC (\$/y)	VC×10 ⁻³ (\$/t)	FC (\$/y)	VC×10 ⁻³ (\$/t)	FC (\$/y)	VC×10 ⁻³ (\$/t)	FC (\$/y)	VC×10 ⁻³ (\$/t)	FC (\$/y)	VC×10 ⁻³ (\$/t)
Fresh to Sink j	200	0.7	300	1.4	150	1.2	120	1.1	250	1.3
Source i to Waste	100	0.7	200	1.4	250	1.1	150	1.2	200	1.3

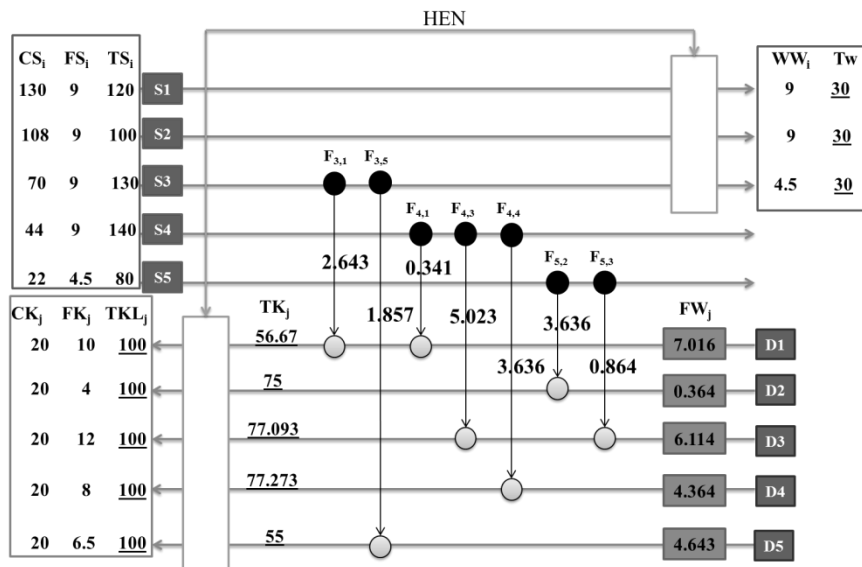


Figure 3: Optimal water and heat exchanger network by MINLP

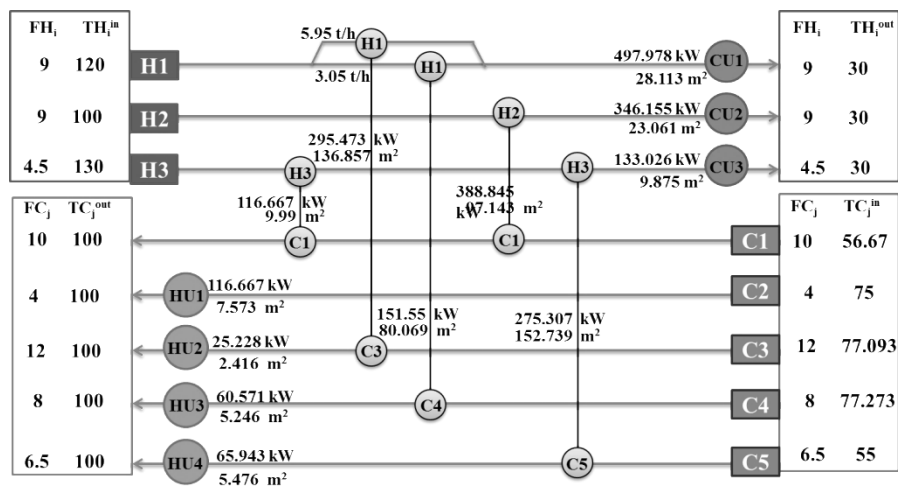


Figure 4: Heat exchanger network by MINLP

Table 5: Results of optimal water and heat exchanger network

Results	Unit	Value	Results	Unit	Value
Freshwater flowrate	t/h	22.5	Freshwater cost	\$/y	67,500
Wastewater flowrate	t/h	22.5	Piping annual cost	\$/y	3,353.68
Hot utility	kW	268.41	Hot utility cost	\$/y	101,190.24
Cold utility	kW	977.16	Cold utility cost	\$/y	184,683.07
Numbers of exchangers	Units	12	Exchangers total annual cost	\$/y	131,433.64
Exchangers total area	m ²	558.56	Total annual cost	\$/y	488,160.63

5. Conclusion

Sequential WN and HEN design is MINLP model using five cascading calculation steps. WHEN is primal designed from step 1 through step 4. Step 5 is simultaneous design to ensure the total WHEN result. This cascading initialization method can calculate MINLP without bounding technique requirement. Total annual cost is 488,160.63 \$/y. The computation time (CPU time) is 23.8 s. However, this model strategy does not guarantee that the result is global optimal result. The problem can be proved by other solvers to validate the result.

Nomenclature

FS_i	Source flowrate (t/h)	$LMTD_{i,j,k}$	Log mean temperature difference ($^{\circ}\text{C}$)
CS_i	Source concentration (ppm)	$LMTD_{cu_i}$	Cold utility log mean temperature difference ($^{\circ}\text{C}$)
FK_j	Sink flowrate (t/h)	$LMTD_{hu_j}$	Hot utility log mean temperature difference ($^{\circ}\text{C}$)
CK_j	Sink concentration (ppm)	$A_{i,j,k}$	Exchangers area (m^2)
$x_{i,j}$	Transfer fraction i to j	Acu_i	Cold utility Exchangers area (m^2)
$F_{i,j}$	Transfer flowrate i to j (t/h)	Ahu_j	Hot utility Exchangers area (m^2)
FW_j	Freshwater flowrate (t/h)	α	Single parameter for WN stream (100,000)
CFW	Fresh concentration (ppm)	ω	Single parameter for exchangers (100,000)
TFW	Fresh temperature ($^{\circ}\text{C}$)	γ	Single parameter for WN stream (100,000)
WW_i	Waste flowrate (t/h)	$EMAT$	Minimum temperature difference ($^{\circ}\text{C}$)
TKL_j	Sink temperature desire ($^{\circ}\text{C}$)	$y_{i,j}$	Binary variable of WN steam existing
TK_j	Sink temperature after WN ($^{\circ}\text{C}$)	yfw_j	Binary variable of fresh steam existing
FH_i	Hot stream flowrate (t/h)	yww_i	Binary variable of waste steam existing
FC_j	Cold stream flowrate (t/h)	$z_{i,j,k}$	Binary variable of exchangers existing
TH_i^{in}	Inlet temperature of hot stream ($^{\circ}\text{C}$)	zcu_i	Binary variable of cold utility existing
TH_i^{out}	Outlet temperature of hot stream ($^{\circ}\text{C}$)	zhu_j	Binary variable of hot utility existing
TC_j^{in}	Inlet temperature of cold stream ($^{\circ}\text{C}$)	WH	Working hour (h/y)
TC_j^{out}	Outlet temperature of cold stream ($^{\circ}\text{C}$)	AF	Annualize factor (y^{-1})
CP	Heat capacity $\text{kJ}/(\text{kg}\cdot^{\circ}\text{C})$	FC	Piping fixed-cost ($\$/\text{y}$)
T_{cu}^{in}	Cooling water inlet temperature ($^{\circ}\text{C}$)	(1) source to sink, (2) fresh to sink, (3) source to waste	
T_{cu}^{out}	Cooling water outlet temperature ($^{\circ}\text{C}$)	VC	Piping variable cost ($\$/\text{t}$)
T_{hu}^{in}	Steam inlet temperature ($^{\circ}\text{C}$)	(1) source to sink, (2) fresh to sink, (3) source to waste	
T_{hu}^{out}	Steam outlet temperature ($^{\circ}\text{C}$)	CUC	Cold utility cost ($\$/\text{kW y}$)
$q_{i,j,k}$	Heat transfer hot to cold stream (kW)	HUC	Hot utility cost ($\$/\text{kW y}$)
qcu_i	Cold utility heat transfer (kW)	$FRcost$	Freshwater annual cost ($\$/\text{y}$)
qhu_j	Hot utility heat transfer (kW)	$PIcost$	Piping annual cost ($\$/\text{y}$)
$TH_{i,k}$	Hot stream stage temperature ($^{\circ}\text{C}$)	$cuOP_i$	Cold utility annual cost ($\$/\text{y}$)
$TC_{j,k}$	Cold stream stage temperature ($^{\circ}\text{C}$)	$huOP_j$	Hot utility annual cost ($\$/\text{y}$)
$dT_{i,j,k}$	Hot and cold temperature difference ($^{\circ}\text{C}$)	$Acost_{i,j,k}$	Exchangers area annual cost ($\$/\text{y}$)
dT_{cu_i}	Cold utility temperature difference ($^{\circ}\text{C}$)	$Acucost_i$	Cold utility area annual cost ($\$/\text{y}$)
dTH_{u_j}	Hot utility temperature difference ($^{\circ}\text{C}$)	$Ahucost_j$	Hot utility area annual cost ($\$/\text{y}$)
		TAC	Total annual cost ($\$/\text{y}$)

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