

An MILP Model for Distributed Energy System Optimization

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Investigations regarding means to achieve savings in primary energy use and to decrease anthropogenic CO₂ emissions have shown that improved energy efficiency has potential to offer a significant contribution to this cause. Considering energy networks and systems, improvements can be achieved by optimizing the structure of the energy chains and distribution networks. For instance, in the case of district heating, it is possible to find an optimal choice of energy supply and design of piping networks to provide the energy required by the customers. Additionally, heat storages and local small-scale energy contributions from e.g. heat pumps could provide benefits and should be taken into consideration. The complexity of the optimization problem increases when the choices of technologies and capacities as well as the network topologies need to be considered. Furthermore, a level of intricacy is added by the large differences in energy demand and temperature conditions between day and night and different seasons. A mixed-integer linear programming model for optimizing distributed energy systems, which takes into account the above mentioned aspects, is being developed and is presented herein. The model is tested with a case study concerning district heating in a locality in southern Finland.

1. Introduction

In the current day and age, it is nigh on impossible to pass a day without hearing or reading about the urgency of uncovering new solutions for filling the energy needs of modern society. Even as renewable energy sources continue their trend of rising popularity and potency, fossil fuels keep holding on to the lion's share of the energy markets. In order to slightly reduce the alarming rate at which energy consumption increases effort is put into improving the energy efficiency of the technologies that society is readily employing. The European Union has even set its aim at reducing the use of primary energy by 20 % by 2020 (EU 2012), and recent IEA reports (IEA 2012) highlight the need for improved energy efficiency in the course of unveiling more sustainable energy solutions, with the projection that the increase in primary energy demand could be halved with economically viable efforts to improve energy efficiency alone. Improving energy efficiency can take place at a multitude of levels, from equipment to system design, and each level has its part to play. When it comes to designing energy systems, optimization techniques can provide useful information to support the design task.

A distributed energy system can be imagined as a network of energy suppliers and consumers connected by e.g. electric lines or water pipes for transporting the energy flows between the network components. In the case of district heating, the energy suppliers could be combined heat and power (CHP) plants, boilers or heat pumps, connected to different types of buildings and industry by a pipe network. The pipe network would consist of both a feed line for the high-temperature water and a return line, creating a closed loop. Additionally, the district heating network could include heat storages – water tanks capable of storing some of the heat produced for use in times of greater need.

The design of an energy network is challenging, as there are hundreds, if not thousands, of parameters and design options to consider and reconsider, and telling which options are better than others – or the best ones – is arduous, if not impossible. Performing this task could be greatly assisted by revelatory optimization models capable of calculating and comparing the different plausible combinations of energy supplies and distribution technologies, pipe network topologies, pipe diameters and water flow parameters, in short, the entire structure of the heating network such that the energy needs are satisfied at a cost, ecological and economical, as low as possible. This article will describe one approach for such a model.

2. Optimization model

The type of model chosen here is based on the modelling principles described by Söderman and Pettersson (2006). It mainly consists of energy balances and physical models of the network components, in addition to binary variables representing the existence of the different components as well as different discrete choices to be made, such as the pipe diameters. Due to the effectiveness of linear programming algorithms, the equations are all chosen as linear relations, thus forming a mixed-integer linear programming (MILP) problem to be solved.

Initially, situation-based parameters are specified. These include economical parameters, temperatures, coordinates for the different sites and pipe locations, characteristics for available technologies and limits for the variables. Some of these could be included in the optimization as variables, or in a master optimization similar to the one explained by Weber et al. (2007), but for now they are simply stated in advance as specified by the case to be optimized. For the sake of capturing the fluctuations in circumstances between day and night and different seasons, the model includes the possibility of dividing the year into several periods, each possibly having individual parameters.

Variables in the model include heat flows \dot{Q} between different components, component capacities S , storage volumes V_R , electrical power supplies and demands P , water velocities v , binary variables for the choice of pipe diameters d_r and binary existence variables y for the optional components. Suppliers, consumers, storages, pipeline parts and heat pumps are denoted with S , C , R , L and HP , respectively, and the sets I , J , K , M and P denote the sets of indices for the suppliers, consumers, storages, pipeline parts and periods.

2.1 Objective function

At the current stage of development, the model is optimized based on economic factors – the annualized investment costs and running costs of each network part. For instance, the investment costs of a new combined heat and power plant would be calculated as

$$C_{inv,Si} = a_{CHP} [c_{fix,CHP} y_{Si} + c_{SD,CHP} (S_{Si,Q} + S_{Si,P})] \quad \forall i \in I \quad (1)$$

taking into account a fixed cost $c_{fix,CHP}$ if the plant exists and a size-dependent cost depending on the total capacity of the plant. The result is moreover multiplied by an annuity factor a_{CHP} . Other network components also have a cost based on a fixed part and a size-dependent part:

$$C_{inv,HPj} = a_{HP} (c_{fix,HP} y_{HPj} + c_{SD,HP} S_{HPj}) \quad \forall j \in J \quad (2)$$

$$C_{inv,Rk} = a_R (c_{fix,R} y_{Rk} + c_{VD,R} V_{Rk} + c_{ED,R} S_{Rk}) \quad \forall k \in K \quad (3)$$

$$C_{inv,Lm} = a_L \sum_{r=1}^{n_d} c_{L,r} d_{r,m} l_m \quad \forall m \in M \quad (4)$$

Worth noting is that the investment costs of storages $C_{inv,R}$ in Eq(3) are dependent both on the storage volumes and heat output capacities (Söderman et al. 2005). In Eq(4), for the investment costs of pipeline parts $C_{inv,L}$ the choice of pipe diameter r will give the corresponding binary variable d_r , the value 1, while the others remain at 0 (cf. Eq(22)). Thus the cost is determined by the diameter and the length of the pipeline part. The running costs of different components depend on their delivery in each period, so

$$C_{oper,Si,p} = c_{S,p} P_{Si,p} t_p + c_{S,Q} Q_{Si,p} t_p \quad \forall i \in I, \forall p \in P \quad (5)$$

$$C_{oper,HPj,p} = c_p COP_{HP} Q_{HPj,p} t_p \quad \forall j \in J, \forall p \in P \quad (6)$$

$$C_{oper,Rk,p} = c_R Q_{Rk,p} t_p \quad \forall k \in K, \forall p \in P \quad (7)$$

$$C_{oper,Lm,p} = c_p P_{Lm,p} t_p \quad \forall m \in M, \forall p \in P \quad (8)$$

where, for instance, $C_{oper,Lm,p}$ stands for the operating cost of pipeline m in period p . COP_{HP} stands for the coefficient of performance of the heat pumps. Pumping power varies depending on both the water velocities and the pipe diameters (cf. Eq(16) – (20)), so they are effectively also functions of the heat flows through them, albeit in a slightly more complex manner.

All in all, the objective function is the sum of all the individual costs from each period, written as

$$C_{tot} = \sum_{i=1}^{n_S} (C_{inv,Si} + C_{oper,Si}) + \sum_{j=1}^{n_C} (C_{inv,HPj} + C_{oper,HPj}) + \sum_{k=1}^{n_R} (C_{inv,Rk} + C_{oper,Rk}) + \sum_{m=1}^{n_L} (C_{inv,Lm} + C_{oper,Lm}) + \sum_{j=1}^{n_C} C_{P,Cj} \quad (9)$$

where $C_{P,Cj}$ is the cost of the electricity bought from the grid by consumer j . Naturally, the cost coefficients c in the above equations vary largely depending on the situation and greatly affect the optimization results.

2.2 Constraints

While the costs in the objective function are to be kept as low as possible, the whole purpose of the network is to deliver sufficient amounts of energy to the consumers, considering the physical restrictions of the system. Each network component has a certain capacity which the flows in or out cannot exceed. For instance, the total heat output of a supplier cannot exceed its capacity limit, so

$$\sum_{m=1}^{n_m} \dot{Q}_{Si,Lm,p} \leq S_{Si,Q} \quad \forall i \in I, \forall p \in P \quad (10)$$

where $\dot{Q}_{Si,Lm,p}$ signifies the heat flow from supplier i to pipeline part m in period p . Some suppliers may have additional restrictions in certain periods, such as heat flows from industries which are limited or shut down part of the year. These restrictions have to be stated separately. Corresponding equations such as Eq. (10) are formulated for the provision of electrical power from the suppliers, except the connections are stated as being directly from suppliers to consumers, without the pipelines in between.

For each component in the network an energy balance has to hold, such as

$$\dot{Q}_{Cj,p} = \sum_{m=1}^{n_L} \dot{Q}_{Lm,Cj,p} + \dot{Q}_{HPj,p} - \sum_{m=1}^{n_L} \dot{Q}_{Cj,Lm,p} \quad \forall j \in J, \forall p \in P \quad (11)$$

which describes the balance of heat flows at consumer nodes. The consumer heat demand has to equal the external heat supply and the heat from an optional heat pump, taking into account that a certain amount of heat can still be transferred further on from the node. In the balance equations for the pipeline parts the heat losses have to be considered, so

$$(1 - Q_{loss,L} l_m) \left(\sum_{n=1}^{n_L} \dot{Q}_{Ln,Lm,p} + \sum_{i=1}^{n_S} \dot{Q}_{Si,Lm,p} + \sum_{j=1}^{n_C} \dot{Q}_{Cj,Lm,p} + \sum_{k=1}^{n_R} \dot{Q}_{Rk,Lm,p} \right) = \sum_{n=1}^{n_L} \dot{Q}_{Lm,Ln,p} + \sum_{i=1}^{n_S} \dot{Q}_{Lm,Si,p} + \sum_{j=1}^{n_C} \dot{Q}_{Lm,Cj,p} + \sum_{k=1}^{n_R} \dot{Q}_{Lm,Rk,p} \quad \forall m \in M, \forall p \in P \quad (12)$$

where part of the inflows is lost as heat losses, expressed by the factor $(1 - Q_{loss,L} l_m)$, in which l_m is the length of pipeline part m and Q_{loss} is a coefficient which in this study was adapted from data on the energy networks in Finland (Finnish Energy Industries 2011), with $Q_{loss} \approx 8 \cdot 10^{-6}$. Essentially, the heat loss would also vary with different insulation and pipe diameters, as well as with the outside temperature, which may be added in further models. For now, the data average of the length-dependent heat losses was deemed an adequate enough approximation for this system-level model. The entire sum of the inflows to a pipeline part will hereby be denoted by $\dot{Q}_{Lm,in,p}$.

Heat storages were modeled according to the principle that their heat content should stay constant over any day-night period. This will result in

$$(1 - Q_{loss,R}) \left(\sum_{m=1}^{n_L} \dot{Q}_{Lm,Rk,p} + \sum_{m=1}^{n_L} \dot{Q}_{Lm,Rk,p+1} \right) = \sum_{m=1}^{n_L} \dot{Q}_{Rk,Lm,p} + \sum_{m=1}^{n_L} \dot{Q}_{Rk,Lm,p+1} \quad \forall k \in K, \forall p \in P_1 \quad (13)$$

where P_1 is the set of odd-numbered periods, i.e. the ones representing daytime periods in the present model. Contrary to the pipeline heat losses, which are modelled as length-dependent, the storage heat losses were chosen to be at a constant 1 % rate of the inflows of heat. Again, more detailed modelling can be applied depending on the scope of the system being modelled. The storages are also constrained by their volumes. No net volumetric water flow into any storage during any day or night can exceed the storage volume, which is expressed as

$$\left(\sum_{m=1}^{n_L} \dot{Q}_{Lm,Rk,p} - \sum_{m=1}^{n_L} \dot{Q}_{Rk,Lm,p} \right) \frac{t_d}{\rho_w c_{p,w} \Delta T} \leq V_{Rk} \quad \forall k \in K, \forall p \in P \quad (14)$$

where t_d equals 12 hours, ρ_w is the water density $c_{p,w}$ is the specific heat capacity of water and ΔT is the temperature difference between the feed and return water in the network.

Much of the complexity of the model stems from the problem of deciding which components to include in the network. The existence of components is expressed with binary variables, with $y = 1$ if a component exists and $y = 0$ otherwise. These variables are controlled by specific equations. For instance, if a supplier is to have a capacity for delivering heat or electrical power, its existence variable has to be unity, which can be written as

$$S_{Si,Q} + S_{Si,P} - M_S y_{Si} \leq 0 \quad \forall i \in I \quad (15)$$

where M_S is a sufficiently large number. Similar equations are formulated for pipeline parts, heat pumps and storages. Selecting the pipe diameters also involves several considerations and affects many other properties. Firstly, the velocity of the water running through a pipeline part is both a function of the heat flow and the pipe diameter. With discrete diameter choices, the proper constraints for the water velocity can be chosen according to

$$v_{Lm,p} \geq \frac{4 \dot{Q}_{Lm,in,p}}{\pi \rho_w c_{p,w} \Delta T D_r^2} - M_v (1 - d_{m,r}) \quad \forall m \in M, \forall p \in P, \forall r \in R \quad (16)$$

where $v_{Lm,p}$ is the water velocity in pipeline part m in period p , while D_r and $d_{m,r}$ are the diameter size r and its corresponding binary choice variable, respectively. Essentially turning the above equations (16) into equalities active in the case that $d_{m,r} = 1$, the matching "less than" inequalities are also included in the constraints. Water velocities are in practice restricted, with wider pipes supporting higher velocities (Sipilä et al. 2000). Thus for each possible diameter there is an equation

$$v_{Lm,p} \leq v_{max,r} + M_v (1 - d_{m,r}) \quad \forall m \in M, \forall p \in P \quad (17)$$

where $v_{max,r}$ stands for the above mentioned upper velocity limits for each pipe size.

The operational costs associated with pipeline parts are in direct relation to the pump power required to keep the water flowing through the pipes. If the pump power is stated as a function of water velocity, then

$$P_{Lm,p} = \frac{\Delta p_{Lm,p} \pi D_{Lm}^2}{4 \eta_{pump}} l_m v_{Lm,p} \quad \forall m \in M, \forall p \in P \quad (18)$$

$$\Delta p_{Lm,p} = \frac{f_D \rho v_{Lm,p}^2}{2 D_{Lm}} \quad \forall m \in M, \forall p \in P \quad (19)$$

$$f_{D,Lm} = \{1.8 \cdot \log_{10} [\frac{6.9}{Re} + (\frac{k/D_{Lm}}{3.7})^{1.11}]\}^{-2} \quad \forall m \in M \quad (20)$$

where $P_{Lm,p}$ is the required pump power for pipeline part m , in period p . The pressure loss in a pipeline $\Delta p_{Lm,p}$ in its turn depends on the friction factor $f_{D,Lm}$ which was approximated using the Haaland equation (Brkić 2011). Now, with the actual pump power essentially being proportional to the cube of the water velocity, the relation had to be linearized for the model. After considering more complicated solutions, simple overestimating linearization between the points of minimum and maximum velocities for each pipe size was deemed adequate. Thus the linearized pump power will be slightly larger than Eq(18) –Eq(20) would suggest. With this linearization, the constraints formed for the model are

$$P_{Lm,p} \geq k_{p,r} v_{Lm,p} - M_p (1 - d_{Lm,r}) \quad \forall m \in M, \forall p \in P, \forall r \in R \quad (21)$$

where $k_{p,r}$ is the linear coefficient for pipe diameter r , calculated according to the above description. The inclusion of discrete pipe diameters increases the problem complexity considerably, but nonetheless, continuous diameters would leave out an important aspect of the actual problem and would make the linearization of the pump power more difficult. The last constraints to be included are

$$\sum_{r=1}^{n_d} d_{Lm,r} = y_{Lm} \quad \forall m \in M \quad (22)$$

which simply state that a diameter must be chosen for an existing pipeline part.

3. Case study

As a model such as the one presented above may or may not be functional and practical, a study of the district heating system of a town area was conducted. Besides residential areas spread out across town the area contains two industrial sites, in connection with which most of the boilers are located. Dividing the year in 12 periods, six two-month stretches with separate day and night periods, the typical consumer behaviour was assessed to follow the Finnish weather conditions, with a high consumption of heat during the cold winter months, a consumption which gradually lessens towards the summer. With three diameter options for each pipeline part, the initial problem formulation had over 2500 binaries, which was reduced to a little over 1,000 by lumping together nearby consumer sites. This resulted in having a total of 25 consumers and seven suppliers, as well as two possible storages and 330 different potential pipeline connections. Most of the winter heat demands at the consumer sites stay in the range of 0 – 2 MW, with one industry site requiring 50 – 70 MW. Another industry site requires approximately 130 MW of steam each period, which is provided for by an adjacent plant. In summertime, consumer demands mostly stay below 0.5 MW, and the smaller industry site demanding 8 – 20 MW.

The optimization was conducted for three cases. The first one corresponded to a real situation where six plants existed and the seventh was being planned at a specific location. An existing pipe network connecting part of the consumers existed as well. In the second case, no pre-existing pipe network was assumed, to let the optimizer propose an optimal piping network to compare the existing one with. Additionally, a third case where none of the suppliers or pipes pre-existed was investigated, to search for an overall optimal design solution for the network.

The solutions in all three cases were similar, as each included all seven suppliers, one storage and almost identical pipeline networks. The solution of the second case is shown in Figure 1, where circles represent supplier sites and squares represent consumer sites, while the single diamond shape is the storage site. Essentially, in addition to the network structure shown in Figure 1, the optimization also gives the characteristics of the different graph components, such as the supplier and storage operations. In the studied case, the supplier at coordinates (1.1, 1.1) has to run at close to full capacity throughout the year to provide the steam required by the adjacent industry, while outputs of the other suppliers, denoted with S2 – S7, vary according to the values in Figure 2. During summertime, most of the heating needs are satisfied by the newly built heating plant S7 at (7, 8.1), and S5 at (7.6, 8.6), which represents surplus heat from an industrial process distributed to the district heating network. In practice they would most likely supply all of the heat, since the other heating plants would not be run at such small rates. Other parameters suggested by the optimization are pipe diameters and the power required to transport the water through the pipes. In this case, no local heat pumps were suggested in the solution. A heat storage, to be in use throughout the year, is suggested at the northern industrial area at coordinates (7.5, 8.5), where most of the energy supplies are located.

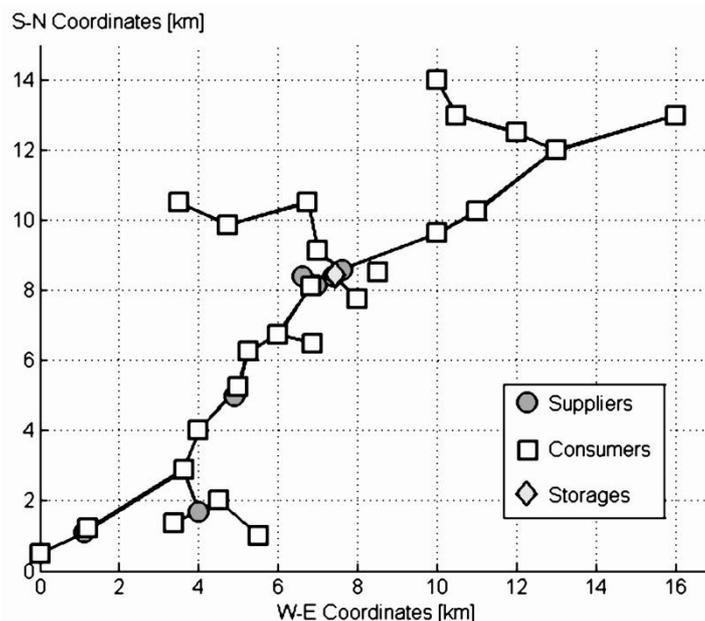


Figure 1: Optimal network configuration in the case study

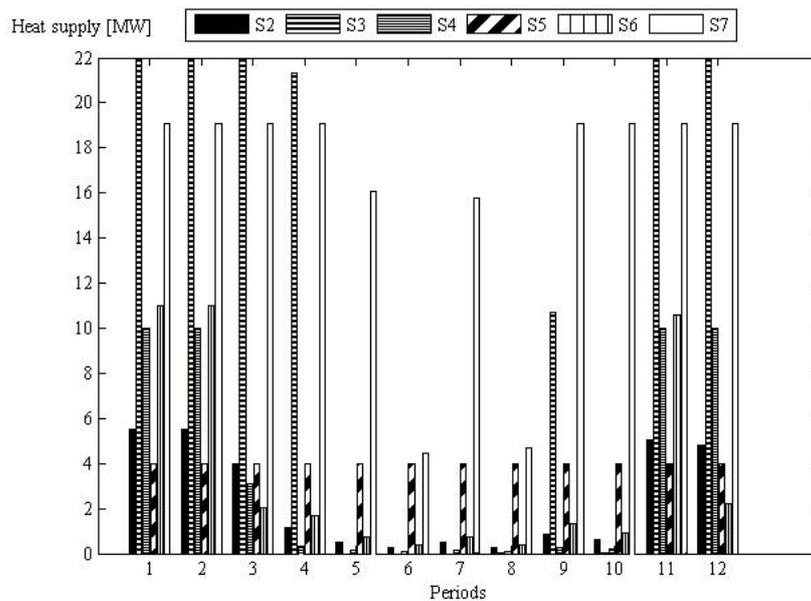


Figure 2: Optimal supplier operation in the case study. During summer periods (5 – 8) two suppliers take care of the entire heat supply

4. Conclusions

With still many additional aspects to take into account, the model already shows itself as a useful aid when designing and analysing district energy networks. In a constant act of balancing the aspects of accuracy and practicality, the chosen level of discretization may prove to be too computationally demanding as additional functions are added to the model. But the principles work adequately, and algorithms for careful pre-evaluation of the design problem and the network components can make the optimization more effective. Thus the model is deemed worthy of further development and more rigorous tests to fully evaluate and hone its capabilities. The efficiency might be improved by exploring ways of dividing the optimization problem into several parts, and new test cases will be devised and analysed.

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