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Robust Model Predictive Control of Heat Exchanger Network

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Heat exchanger optimal operation represents challenging task from the control viewpoint because of the system nonlinearities, varying process parameters, internal and external disturbances and measurement noise. Various robust control strategies were developed for overcoming all these problems. The robust model predictive control (RMPC) represents one of these approaches. It enables to design effective control algorithms for optimization of the control performance as well as to take process uncertainty into account. The possibility to use the RMPC for control of heat exchangers is presented in this paper and three serially connected counter-current heat exchangers represent the controlled process. The efficiency of the presented RMPC algorithm was verified by simulations. Simulations of control were done in the MATLAB-Simulink environment. The results confirmed that using the RMPC led to smaller consumption of the cooling medium in comparison with classical control approaches.

1. Introduction

Heat exchangers (HEs) belong to the standard equipment in the chemical and process industries and the heat exchanger networks (HENs) are the key processes in the petrochemical industry (González et. al, 2006). From the control viewpoint it is necessary to control the HENs operating in a wide range of temperature. The energy supply in the most industrial devices was recognized as the second highest operating cost after primary feedstock costs (Morrison et al., 2012). Moreover the energy prices tend to permanent increase (Morrison et al., 2012) and the optimized processing of HENs provides significant heat-recovery-based energy savings (Pan et al., 2012). It is necessary to improve the efficiency of energy utilization to reduce the energy consumption (Qian and Li, 2011). To optimize the control performance it is required to implement the advanced control strategies.
Uncertainty in the controlled process, disturbances, measurement noise and the constraints on the control inputs and the controlled outputs play an important role in finding the proper solution of the control problem (González et. al, 2006). The advantages of the RMPC originate from the possibilities to take into account the system constraints. Furthermore, the repetitive redesign of the controller in each control step can minimize the influence of neglected or unpredicted negative effects and improve the final control performance (Kothare et al., 1996).
It was shown in Bakošová and Oravec (2012) that the RMPC strategy leads to decrease of the energy consumption in the tubular and the jacketed HEs. The aim of this paper is to show that the RMPC can be successfully implemented also for HEN control and using the RMPC strategy leads to reduction of the cooling medium consumption. The designed RMPC is based on the formulation of the optimisation problem with constraints in the form of linear matrix inequalities and the obtained convex optimisation problem is solved using the semi-definite programming (Pólik, I., 2010). The optimisation problem is solved in the MATLAB programming environment using YALMIP toolbox (Löfberg, J., 2004) with

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SeDuMi solver (Pólik, I., 2010). Control performance of the designed RMPC was investigated using the case study of the counter-current shell-and-tube HEN control. For the simulation purposes the uncertain non-linear model was assumed.

2. Controlled heat exchangers

Three identical serially connected counter-current shell-and-tube heat exchangers (HEs) represent the controlled process (Figure 1), in which petroleum as a product of the distillation in a refinery has to be cooled.

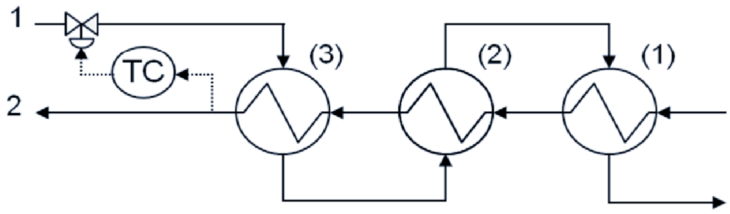

Figure 1: Scheme of the counter-current shell-and-tube heat exchanger network, where 1 is the cooling water, 2 is the hot petroleum, and (1) – (3) are the heat exchangers

Petroleum flows in the inner tubes and the cooling water in shell of every heat exchanger. The tubes of the HEs are made from steel. The controlled variable is the temperature of the outlet stream of the petroleum from the 3rd HE and the control input is the volumetric flow rate of the inlet stream of the cold water in the 3rd HE. The objective is to cool the outlet temperature of the petroleum to the reference value 45°C and to minimize energy consumption measured by the cold water consumption. Technological parameters and steady state values of the heat exchangers are summarised in Table 1, where n is the number of HE's tubes, l is the length of the HE, $d_{\text{in},1}$ is the inner diameter of the tube, $d_{\text{out},1}$ is the outer diameter of the tube, $d_{\text{in},2}$ is the inner diameter of the HE, A_h is the total heat transfer area, V is the volume, c_P is the thermal capacity, ρ is the density, T_{in} is the inlet temperature, q is the volumetric flow rate. The subscripts 1 and 2 refer to the water and petroleum, respectively. The superscripts (1) – (3) denote individual HEs and the superscripts S and 0 denote the steady state value and the initial value, respectively.

Table 1: Technological parameters and steady state values of HEs

Parameter	Unit	Value	Parameter	Unit	Value
n	1	40	q_2^S	m^3 s^{-1}	5.8×10^{3}
l	m	6	$T_1^{(1),S}$	°C	75.8
$d_{\text{in},1}$	m	19×10^{-3}	$T_1^{(2),S}$	°C	48.0
$d_{\text{out},1}$	m	25×10^{-3}	$T_1^{(3),S}$	°C	30.8
$d_{\text{in},2}$	m	414×10^{-3}	$T_2^{(1),S}$	°C	113.0
A_h	m^2	16.6	$T_2^{(2),S}$	°C	71.3
V_1	m^3	91.2×10^{-3}	$T_2^{(3),S}$	°C	45.3
V_2	m^3	716.5×10^{-3}	$T_1^{(1),0}$	°C	87.1
$c_{P,1}$	J kg^{-1} K^{-1}	4.186×10^{3}	$T_1^{(2),0}$	°C	55.7
$c_{P,2}$	J kg^{-1} K^{-1}	2.140×10^{3}	$T_1^{(3),0}$	°C	34.4
ρ_1	kg m^{-3}	980.0	$T_2^{(1),0}$	°C	118.4
$T_{\text{in},1}$	°C	20.0	$T_2^{(2),0}$	°C	76.8
$T_{\text{in},2}$	°C	180.0	$T_2^{(3),0}$	°C	48.7

Further, two uncertain parameters are considered in the HEs. The heat-transfer coefficient changes as the flow rate of the cooling medium changes and the density of the petroleum depends on the temperature in the HEs. The values of these parameters are given in the Table 2, where U denotes the heat transfer coefficient and ρ_2 represent the density of petroleum.

Table 2: Uncertain parameters of HEs

Parameter	Unit	Minimal value	Mean value	Maximal value
U	J s^{-1} m^{-2} K^{-1}	472.8	482.1	491.8
ρ_2	kg m^{-3}	793.8	810.0	826.2

3. RMPC

For control system design the mathematical model of the heat exchangers was derived from the enthalpy balances in the form of a discrete-time linear state-space system

$$\begin{aligned} x(k+1) &= A_v x(k) + B_v u(k),\ x(0) = x_0 \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where k is the discrete time and the sampling period is T_s = 25 s. Further, $x(k)$ represents the vector of states associated with the temperatures $T_1^{(1)-(3)}$ and $T_2^{(1)-(3)}$ (Table 1), $u(k)$ is the vector of control inputs represented by the volumetric flow rate of the cooling medium q_1, $y(k)$ is the vector of the system outputs. The matrices A_v, B_v, C have appropriate dimensions. The model in Eq(1) is an uncertain system with parametric polytopic uncertainty given in the Table 2. Matrices A_v, B_v, v = 1,…,4, describe our vertex systems of the uncertain system Eq(1). For the uncertain model of the HEN one can obtain four vertices computed as the combination of boundary values of uncertain parameters. The 5th considered system is the nominal system calculated for the mean values of the uncertain parameters. Then the robust static state-feedback control problem in the discrete-time domain can be formulated as follows: find the state-feedback control law

$$u(k) = F_k x(k) \tag{2}$$

for the system described by Eq(1). The matrix F_k in Eq(2) represents the static state-feedback robust controller for the k-th control step.
The quality of the control performance can be described using the quadratic cost function J

$$J = \sum_{k=0}^{N} \left(x(k)^T Q x(k) + u(k)^T R u(k) \right) \tag{3}$$

where N is the number of control steps. For the design purposes the infinity control horizon is assumed, and Q, R are the real square symmetric positive-definite weight matrices of the states $x(k)$ and system inputs $u(k)$, respectively. The aim is to design such controller F_k that ensures robust stability of all considered vertex systems and minimizes the quadratic cost function J in Eq(4).
The control performance can be improved by taking into account the symmetric constraints on the system outputs $y(k)$ and inputs $u(k)$ in the form

$$\|y(k)\|_2 \le y_{\max}, \quad \|u(k)\|_2 \le u_{\max}, \quad |u_j(k)| \le u_{j,\max}, \quad j = 1,2,\ldots,N_u \tag{4}$$

For the symmetric positively defined Lyapunov matrix P_k and the feedback controller F_k following conditions hold

$$P_k = \lambda_k X_k^{-1}, \quad Y_k = F_k X_k \ \Rightarrow\ F_k = Y_k X_k^{-1} \tag{5}$$

where λ_k is the auxiliary optimization parameter, X_k is the symmetric positively defined matrix and Y_k represents the auxiliary matrix enabling the evaluation of the robust feedback controller F_k (Kothare et al., 1996).
Using substitutions and Schur complement formula the robust stabilization problem can be transformed as the RMPC convex optimization problem based on the LMIs as follows (Kothare et al., 1996)

$$\min_{\lambda_k, X_k, Y_k} \lambda_k \tag{6}$$

subject to

$$\begin{bmatrix} 1 & x_k^T \\ * & X_k \end{bmatrix} \geq 0 \,, \quad \begin{bmatrix} X_k & X_k A_v^T + Y_k^T B_v^T & X_k \sqrt{Q} & Y_k^T \sqrt{R} \\ * & X_k & 0 & 0 \\ * & * & \lambda_k I & 0 \\ * & * & * & \lambda_k I \end{bmatrix} \geq 0 \,, \; v = 1,\ldots,N_v \tag{7}$$

where symbol * denotes a symmetric structure of the matrix, and *I*, *0* are identity and zero matrices of appropriate dimensions, respectively. The symmetric Euclidean norm and symmetric peak constraints on control inputs and outputs in the form of Eq(4) can be added to the optimization problem Eq(6) – Eq(7) in the following LMI form

$$\begin{bmatrix} u_{\max}^2 I & Y_k \\ * & X_k \end{bmatrix} \geq 0 \,, \quad \begin{bmatrix} U_k & Y_k \\ * & X_k \end{bmatrix} \geq 0 \,, \quad U_{j,j}(k) \leq u_{j,\max}^2 \,, \quad \begin{bmatrix} X_k & \left(A_v X_k + B_v Y_k\right)^T C^T \\ * & y_{\max}^2 I \end{bmatrix} \geq 0 \tag{8}$$

where $j = 1,\ldots,N_u$ and $v = 1,\ldots,N_v$.

The algorithm for the RMPC can be formulated in following eight steps (Kothare et al., 1996).

Step 1: Set parameter $k = 0$.

Step 2: Set number of control steps N, initial conditions of states $x(0)$, values of the symmetric constraints on control input $u_{\max}$ and output $y_{\max}$.

Step 3: Set parameter $k = k + 1$.

Step 4: Set the values of states $x(k)$.

Step 5: Solve optimization problem described by Eq(6) – Eq(7) to evaluate the matrices X_k and Y_k.

Step 6: Design the matrix F_k of the feedback controller using Eq(5).

Step 7: Calculate the control input $u(k)$ using the control law Eq(2).

Step 8: If the parameter $k < N$ then go to the *Step 3* else *Stop*.

4. Results and discussion

The robust model predictive control of HEs was investigated using simulations in the MATLAB-Simulink environment using 2.8 GHz CPU and 4 GB RAM. The obtained control performances were compared with the control performances assured by the discrete-time LQ optimal controller (Diaz, 2007) computed in the form $F_{LQ} = [110.4,\ 31.2, -1.7, 21.1, -1.0, 8.5]\times10^{-6}$. The gain matrix of the F_{LQ} controller was designed using the weight matrices Q, R of the cost function in Eq(3) in the form $diag(Q)=[100,100,100,100,100,100]^T$, $diag(R)=[100]$, where *diag* represents the diagonal matrix with the given elements on the main diagonal and with other zero elements. The same values of the weight matrices were used in the RMPC algorithm. The matrices Q, R were tuned to obtain satisfactory and comparable results. Both strategies were compared by evaluating the cooling medium consumption V_C, which was necessary for cooling the petroleum from 118.4°C to 45.3°C during control running 1250 s in $N = 50$ control steps. The consumption of V_C is presented in Table 3 for the nominal system and four vertex systems. These vertices were obtained for all combinations of two uncertain parameters (Table 2). In Table 3, $V_{C,RMPC}$ is the volume of the consumed cooling water using the RMPC control strategy and $V_{C,LQ}$ is the cooling water consumption during the LQ optimal control. The cold water consumption was smaller for all vertex systems when the RMPC was used.

Table 3 summarizes also other results. Here, *System* represents the controlled non-linear model of HEs, which is either the nominal or the vertex system. *Method* distinguishes the RMPC and the discrete-time LQ optimal control approaches. Comparison of methods is done by calculation

$$\Delta V_C = \frac{V_{C,LQ} - V_{C,RMPC}}{V_{C,LQ}} 100\% \tag{8}$$

As can be seen, the RMPC reduces consumption of cooling medium in all vertex systems, i.e. in the HEN with uncertainty. The LQ optimal control assures better result only for the nominal system, i.e.

HEN without uncertainty. Parameter $\Delta T_{2,\mathrm{RMPC}}^{(3)}$ is the offset, which is the least one for the nominal system controlled using the RMPC. J_{RMPC} is the value of the cost function in Eq(3) assured using the RMPC and J_{LQ} is the value assured using the optimal LQ control. The least values of J are again assured using the RMPC. The control performance of the nominal system (Table 3) assured using the LQ control and the RMPC is presented in Figure 2 and associated control inputs are shown in Figure 3. Control response obtained using the RMPC is faster and without overshoot.

Table 3: Results of RMPC and discrete-time LQ optimal control

Method	RMPC			LQ			Comparison
System	$\Delta T_{2,\mathrm{RMPC}}^{(3)}$ [°C]	J_{RMPC}	$V_{\mathrm{C,RMPC}}$ [m^3]	$\Delta T_{2,\mathrm{LQ}}^{(3)}$ [°C]	J_{LQ}	$V_{\mathrm{C,LQ}}$ [m^3]	ΔV_{C} [%]
nominal	0.01	1.410	7.330	0.02	1.652	7.269	-0.84
1st vertex	0.49	1.335	7.556	0.20	1.617	7.635	1.04
2nd vertex	0.55	3.559	6.756	0.20	3.447	6.804	0.71
3rd vertex	0.49	1.346	7.558	0.20	1.626	7.636	1.02
4th vertex	0.54	3.546	6.760	0.21	3.468	6.805	0.66

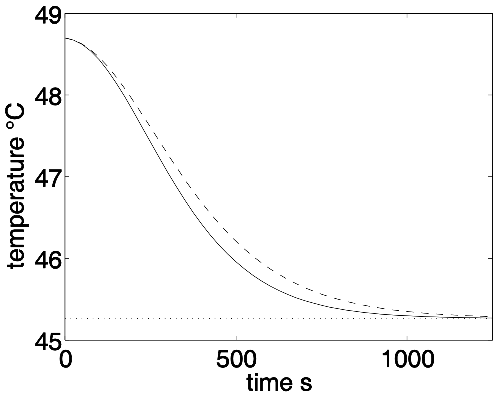

Figure 2: Control performance of the outlet temperature of the petroleum assured using the RMPC (solid) and the LQ optimal (dashed) controllers (nominal system)

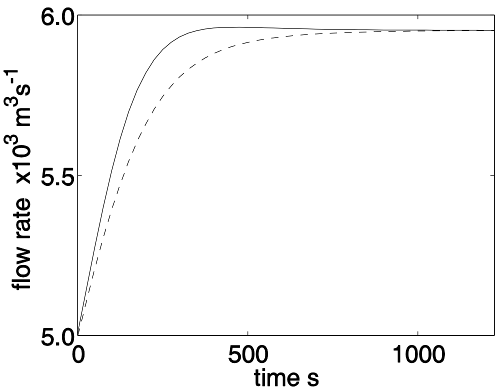

Figure 3: Control inputs generated by RMPC (solid) and LQ optimal (dashed) controllers (nominal system)

5. Conclusions

Simulation results confirmed the effectiveness of the proposed RMPC approach because of the smaller consumption of the water that is needed for cooling the petroleum in the studied HEN in the presence of uncertainty. The discrete-time LQ optimal controller ensured better control quality only in the nominal system that represents an ideal system. In the presence of uncertainty and boundaries on control inputs and controlled outputs, the robust feedback control approach increased the quality of the control performance. Consumption of cooling medium was reduced up to 1 % in about 20 min and could be even larger when more disturbances occurred. Therefore it can be stated, that using the RMPC strategy in practical implementations can lead to energy savings. The further research will be focused on the improvement of the RMPC algorithm so that the offset free control responses will be reached.

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References

Bakošová M., Oravec J., 2012, Robust Model Predictive Control of Heat Exchangers, Chemical Engineering Transactions 29, 1465-1470, DOI: 10.3303/CET1229245.

Diaz G., 2007, Controllability of cross-flow two phase heat exchangers, International Journal of Heat and Mass Transfer, 50, 4559-4567, DOI:10.1016/j.ijheatmasstransfer.2007.03.024.

González A.H., Odloak D., Marchetti J.L., 2006, Predictive control applied to heat-exchanger networks, Chemical Engineering and Processing, 45, 661-671, DOI: 10.1205/cherd05052.

Kothare M.V., Balakrishnan V., Morari M., 1996, Robust constrained model predictive control using linear matrix inequalities, Automatica, 32, 1361-1379.

Löfberg J., 2004, Yalmip: A toolbox for modelling and optimization in Matlab. Proc. of the CACSD Conference, Taipei, Taiwan, 284-289.

Morrison A.S., Atkins M.J., Walmsley M.R.W., 2012, Ensuring Cost-effective Heat Exchanger Network Design for Non-Continuous Processes, Chemical Engineering Transactions 29, 295-300, DOI: 10.3303/CET1229050.

Pan M., Bulatov I., Smith R., 2012, Retrofit Procedure for Intensifying Heat Transfer in Heat Exchanger Networks Prone to Fouling Deposition, Chemical Engineering Transactions, 29, 1423-1428, DOI: 10.3303/CET1229238.

Pólik I., 2010, Addendum to the SeDuMi user guide, version 1.1., <sedumi.ie.lehigh.edu> accessed 23.03.2013.

Qian X., Li Z., 2011, Analysis of entransy dissipation in heat exchangers, International Journal of Thermal Sciences, 50, 608-614, DOI:10.1016/j.ijthermalsci.2010.11.004.

Varbanov P.S., Klemeš J.J., Friedler F., 2011, Cell-based dynamic heat exchanger models-Direct determination of the cell number and size, Computers & Chemical Engineering, 35, 943-948, DOI: 10.1016/j.compchemeng.2011.01.033.