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# On Robust Maintenance Scheduling of Fatigue-prone Structural Systems Considering Imprecise Probability

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Crack propagation in metallic mechanical components subject to cyclic loading may lead to loss of serviceability or even collapse. Often, these undesirable events may be prevented by performing appropriate maintenance activities. Nonetheless, the scheduling of these activities is highly involved due to the inherent uncertainty associated with crack propagation. In this contribution, a framework for optimal scheduling of maintenance activities within the theory of imprecise probabilities is presented. In this manner, effects of uncertainty are considered in a rational way. The proposed approach is implemented in a general purpose software for stochastic analysis. A numerical example demonstrates the applicability of the proposed framework as well as the importance of considering the effects of uncertainty.

## 1. Introduction

Maintenance activities can be extremely cost-effective for mitigating damage accumulation in fatigue-prone metallic structures (e.g. Faber et al. 1996). In fact, inspection and repair activities may prevent loss of serviceability or even collapse of structure (Zio et al. 2012). However, scheduling maintenance activities is a challenging task due to the monetary cost associated with inspection and eventual repair that need to be pondered against the cost of failure (i.e. loss of serviceability) and due to unavoidable uncertainties arising in crack propagation, inspection and repair activities. In this scenario, reliability-based optimization (RBO) becomes a most valuable tool, as it offers a systematic and robust approach for taking decisions under uncertainty. For applying the RBO methodology, uncertainties should be described by means of probability theory. However, classical probability theory may not provide a conclusive answer to the challenges posed by diverse uncertainty forms, which include variability, imprecision, incompleteness, vagueness, ambiguity, dubiety, subjective experience and expert knowledge. One way of overcoming this limitation consists in considering lower and upper expectations, also known as imprecise probability (see e.g. Beer et al. 2013). Imprecision allows not introducing artificial or unrealistic assumptions.

In this paper a methodology for designing robust maintenance strategies for metallic structures subject to cyclic loading is presented. Reliability metric is redefined within the imprecise probability framework. In this manner, a maintenance strategy, which is both optimal - from an economical viewpoint - and robust with respect to the uncertainties, is determined. The efficiency and applicability of the proposed approach is demonstrated by means of a numerical example involving a simplified model of a bridge structure where the performance of a welded connection prone to fatigue damage is of interest. Uncertainties are considered both on the crack propagation phenomenon - by means of a fracture mechanics approach - and on inspection activities. These uncertainties are reflected on imprecise probabilities of repair and failure as well as on imprecise expected costs associated with inspection, repair and eventual failure. The proposed strategy is implemented in a general purpose software for uncertainty quantification and risk management (see e.g. Patelli and De Angelis 2012). The software has been designed to provide the maximum flexibility in combining different components and source of uncertainties thanks to its modular structure. Furthermore, thanks to the possibility to interface to any 3rd party software, it allows the applicability of the proposed solution strategy to large-scale models and systems of practical interest (Patelli et al. 2012).

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## 2. Description of the Problem

### 2.1 General remarks

Due to cyclic loading, metallic components tend to develop fatigue cracks. As these cracks propagate, the structural system accumulates damage that may lead to loss of serviceability or collapse. One possible approach to control damage accumulation is scheduling inspection activities followed by eventual repair. In this context, maintenance activities offer a cost-effective means for mitigating damage accumulation caused by crack propagation. This is due to the fact that costs associated with inspection and repair are often much lower than the costs associated with failure. However, their scheduling is quite challenging due to the uncertainties in the crack propagation. Furthermore, during the inspection activities a crack may be detected or it may be missed, increasing the complexity of the identification of an optimal maintenance strategy. In this contribution, the design of an optimal maintenance strategy involves identifying the time at which inspection (and eventual repair) is performed. Time of inspection is so fundamental for scheduling maintenance activities that if it is selected in an inappropriate manner it might even result pointless.

### 2.2 Crack propagation

The crack propagation phenomenon is modelled using the Paris-Erdogan law (Paris and Erdogan 1963).

$$\frac{da}{dN} = C\left(\Delta K\right)^{m} \tag{1}$$

In the above equation, $a$ represents the crack length, $N$ the number of load cycles, $C$ and $m$ are material properties and $\Delta K$ is the stress intensity factor amplitude, which depends on the crack length, crack geometry and applied alternating stress. This equation is integrated with respect to the number of load cycles until the failure condition is reached or the target lifetime is fulfilled. Note the failure condition is given by the stress intensity factor exceeding the material's toughness; this condition can be expressed alternatively as the crack length exceeding a critical value $a_c$. The Paris-Erdogan law is appropriate for characterizing crack growth under constant amplitude cyclic loading, small scale yielding (i.e. yielding ahead of the crack tip) and long cracks. For those cases where these conditions are not met, the Paris-Erdogan law may not be appropriate and alternative models should be considered. In particular, note that the Paris-Erdogan law cannot model the crack initiation stage. Throughout this contribution, it is assumed that structures possess initial cracks of length $a_0$.

### 2.3 Maintenance Activities and Events during Life Time

As a means of controlling damage associated with crack propagation, maintenance activities are performed. These activities consist of non-destructive inspection (NDI) and repair. NDI refers to procedures used to detect cracks in a structure without introducing additional damage. As inspection activities are not perfect, NDI techniques have associated a probability of detection (POD), (see e.g. Zheng and Ellingwood, 1998). In this contribution, the POD is modelled as shown below.

$$POD\left(a\right) = \left(1-p\right)\left(1-e^{-\lambda a}\right) \tag{2}$$

In this equation, $p$ is the probability of not detecting a large crack while $\lambda$ is a constant depending on the specific NDI technique applied. Note the probability of detection is calculated based on two factors: the first one $(1-p)$ measures the probability of detecting a very large crack while the second factor $\left(1-e^{-\lambda a}\right)$ can be interpreted as a weight between 0 and 1 that depends on the crack length. Once a crack is detected, a decision can be made on whether the structure may either or not be repaired depending on the level of damage. Hence, during the target lifetime of the structure the following events may occur:

- The crack length reaches a critical value and fracture occurs at a time before inspection takes place.
- The structure survives until NDI is carried out. Two possible outcomes may result. In the first one, inspection may not detect any cracks and no repair is carried out. In the second outcome, one or more cracks are detected. In case the crack exceeds a certain threshold level that endangers the structure it is decided to perform repair. Note that for both outcomes, the structure may either or not survive until the target lifetime.

### 2.4 Uncertainty Quantification

As discussed above, both the crack propagation process and the inspection activities are subject to inherent uncertainties. While uncertainties in inspection are modelled considering the POD (see equation 2), the uncertainties in crack propagation are modelled by introducing uncertainty in the length of the initial crack $a_0$. More precisely, the expected value of the initial crack length is modelled as a fuzzy variable with a triangular membership function while the crack length is modelled considering a lognormal random variable. Hence, $a_0$ is actually a fuzzy random variable (Beer et al. 2013). The latter issue implies that all relevant variables associated with the model (e.g. probability of failure, total costs) become fuzzy random variables as a result.

**2.5 Optimal Maintenance Scheduling**

In this contribution, the selection of an optimal maintenance schedule refers to determining the time of inspection $t_I$ at which inspection activities and eventual repair should take place. For the sake of simplicity, this paper considers a single inspection. Although the proposed methodology could be extended in order to consider several inspections, this is outside the scope of this contribution. The function to be minimized is the total cost of operation $c_T$ , which is expressed as the summation of the costs of inspection, repair and failure (Kupfer and Freudenthal 1977). As the cost of operation depends on the input variables of the model (POD and initial crack length), it is actually a fuzzy random variable. Hence, in order to solve the optimization problem, a deterministic substitute problem is defined (Youn et al. 2007). This problem consists in minimizing the expected cost of operation for a particular level of its membership function. More specifically, it is sought to minimize the average between the lower and upper values of the expected cost of operation for the membership function equal to $\alpha$ =0.5. In mathematical terms, this is defined as:

$$\min_{t_I \in \left[t_I^L, t_I^U\right]} \frac{1}{2}\left(E\left[c_{T,\alpha=0.5}^L\right] + E\left[c_{T,\alpha=0.5}^U\right]\right) \tag{3}$$

where $E\left[c_{T,\alpha=0.5}^L\right]$ and $E\left[c_{T,\alpha=0.5}^U\right]$ are the minimal and maximal value that the expected cost function assumes for a membership function equal to $\alpha$ =0.5, while $t_I^L$ and $t_I^U$ are the bounds for selecting the inspection time. As described above, the total cost function is the summation of inspection, repair and failure costs. The inspection cost $c_I$ represents the resources spent on performing NDI. In principle, this cost is an uncertain variable, as the structure may fail before inspection occurs. However, it can be assumed that failure is a rare event and hence, $c_I$ is approximated as a deterministic, fixed parameter. In order to describe the costs associated with repair and failure, assume first the optimal maintenance schedule problem involves random variables only. Then, the expected costs of repair and failure can be described as proportional to the probability of occurrence of these events (Kupfer and Freudenthal, 1977). That is, $c_R p_R\left(t_I\right)$ and $c_F p_F\left(t_I\right)$ represent the expected costs of repair and failure; $c_R$ and $c_F$ are the costs associated with repair and failure while $p_R\left(t_I\right)$ and $p_F\left(t_I\right)$ are the probabilities of repair and failure, respectively. In this contribution, the optimal maintenance schedule problem involves not only random variables but also fuzzy variables. As a consequence, the probabilities of repair and failure become imprecise probabilities, i.e. each one of these has associated a membership function. Note in the above formulation, the discount rate has been omitted from the calculation of cost for the sake of simplicity. However, it could be certainly considered within the framework proposed.

## 3. Solution Strategy: Theoretical Aspects

The solution of the optimization problem posed in equation 3 could be regarded as a double-loop problem. In the outer loop, different values of the inspection time are explored in order to determine the one that leads to a minimum of the objective function. In the inner loop and for a fixed value of the time of inspection, it is necessary to determine the minimum and maximum values that the expected cost function can assume within the intervals of the fuzzy variables for a value of the membership function equal to $\alpha$ =0.5. This means one must assess several probabilities of repair and failure for different combinations of time of inspection (design variable) and mean value of the initial crack length (fuzzy variable) in order to determine an optimal maintenance strategy. This task can become daunting from a numerical viewpoint. Hence, appropriate approaches must be employed in order to render the optimization problem tractable.

The strategy suggested in this contribution for solving the optimization problem in equation 3 consists in constructing a Response Surface meta-model of the probabilities of repair and failure as an explicit function of the design variable and the fuzzy variable. Specifically, these probabilities are approximated using an exponential function with a quadratic polynomial as argument (Gasser and Schuëller 1997).

$$p_x\left(t_I, E[a_0]\right) = \exp\left(b_0 + b_1 t_I + b_2 E[a_0] + b_3 (t_I)^2 + b_4 t_I E[a_0] + b_5 \left(E[a_0]\right)^2\right) \tag{4}$$

In the above equation, the subscript $x$ represents either repair ($R$) or failure ($F$) while $b_j$, j=0,…,5 are real, constant coefficients. The latter coefficients are determined in a least square sense by evaluating the probabilities at a grid of points of the inspection time and expected value of the initial crack length. Such grid of points is selected using an appropriate design of experiments scheme, e.g. full factorial, Latin hypercube sampling, etc. In order to estimate the probabilities of repair and failure at these grid points, importance sampling approach using design points is applied (Schuëller and Stix, 1987). Numerical experience indicates the meta-model in the above equation is appropriate within the scope of this contribution. A possible means for verifying its accuracy consists of testing it with some validation samples (i.e. probabilities of repair and failure are calculated exactly at some points and compared with the results of the meta-model). Once the meta-models of the probabilities have been constructed, the solution of the aforementioned double-loop problem becomes straightforward. This is due to the fact that the evaluation of the meta-model is computationally inexpensive.

## 4. Solution Strategy: Practical Implementation Using a General Purpose Software

In this section, the general purpose software OpenCossan is used to carry out the challenging task of solving the double-loop optimization problem (Patelli and De Angelis, 2012). This software is available as open-source computational framework. Designed and coded in a object-oriented fashion within Matlab®, it provides a modular and user-friendly programming environment. The problem is addressed by combining the Reliability, the Optimization and the Meta-Model toolboxes, in order to evaluate a suited approximation of the expected values of cost and to compute the extreme values of cost respectively. Toolboxes are collections of interconnected objects and methods to perform the tasks they are designed for. Figure 1 shows the implementation of the proposed approach in OpenCossan. The proposed strategy consists in reducing the degree of complexity and computational cost of the double-loop by separately performing reliability and optimisation analysis for the expectations of cost.

```
% ==========================================================================
% OPENCOSSAN (http://www.cossan.co.uk) General purpose matlab toolbox for risk and uncertainty quantification
% Authors: Edoardo Patelli and Marco de Angelis
% ==========================================================================
%% Definition of the Input
% Definition of the Design Variable (load cycles)
timeInspection = DesignVariable('Sdescription','Time of Inspection','lowerBound', 0.4e6,'upperBound',1.6e6);
% Definition of the Fuzzy variable
meanInitialCrack  = FuzzyVariable('VsupportPoints',[0.5  1  1.5],'ValphaLevels',[0  0.5  1]);
% Collect DesignVariable and Fuzzy Variable in the Input Object
MyInput = Input('XdesignVariable',timeInspection,'XfuzzyVariable',meanInitialCrack);
%% Definition of the Probabilistic Model
% MyEvaluator provides the link to the deterministic model
MyModel = Model('Xinput',MyInput,'Xevaluator',MyEvaluator);
% Definition of the Performance Function
MyPerformanceFunction = PerformanceFunction('Sdemand','crackLength','Scapacity','criticalValue','Soutputname','Vg');
% Definition of the Probabilistic Model
MyProbabilisticModel = ProbabilisticModel('Xmodel',MyModel,'XperformanceFunction',MyPerformanceFunction);
%% Definition of the Reliability Based Optimization
% Definition of the Reliability solver
MySimulator = ImportanceSampling('Nsamples',100);
% Definition of the Objective function (Expected value of cost)
MyObjectiveFunction = ObjectiveFunction('Sscript','expectedCost=pf*Cf+pr*Cr');
% Definition of the RBO problem and consequent mapping in design space
MyRBOproblem = RBOProblem('XProbabilisticModel',MyProbabilisticModel,'Xsimulator',MySimulator, ...
'XobjectiveFunction',MyObjectiveFunction,'SfailureProbabilityName','pf','alphaCut',0.5,'intervalMeasure','centralValue');
% Definition of the Optimization solver
MyOptimizer = Cobyla('NmaxModelEvaluations',60);
%% Performing RBO analysis
Xoptimum = MyRBOproblem.optimize('Xoptimizer',MyOptimizer);
```

*Figure 1: Pseudo-code for the Reliability-based Optimization with Fuzzy Variables*

With this method the reliability analysis is invoked only once, in order to feed the approximated form of probability (equation 4) with those values needed for performing a proper regression analysis. Such procedure is usually called meta-model calibration (or training) and comes out with the values of the polynomial coefficients. Once the meta-models are calibrated, they fully replace the computational costly original models with analytic functions easy to be evaluated and consequently speed-up the optimization procedure. The key issue is that the meta-model must be calibrated in order to provide set-based (or interval-based) values of probabilities. For this reason the calibration must take into account both design and fuzzy variables that define a bounded domain to seek the approximated extreme values in. Effectively a fuzzy variable for a given alpha-cut may be seen as a bounded design variable, which tells the optimizer to search for both maximum and minimum value of objective function. The solution strategy for this specific problem shall be designed to include this particular type of variable; therefore the three methods just mentioned, such as Reliability, Optimisation and Meta Model, will be invoked in a specific sequence of instructions. The key for accessing the solution of this challenging problem resides in designing the solution sequence for properly combining the methods. Firstly the reliability analysis is performed according to a predefined scheme of points. Then, the MetaModel toolbox is invoked for calibration. The Design of Experiment method is considered as tool for producing full factorial combinations of values. This is done by crossing values in the space of the uncertain variables (both design and fuzzy), in order to eventually build up a global approximation of the objective function. The object “RBOproblem” provides the natural environment for solving optimisation problems involving uncertain variables. Thus, in this case it reveals to be particularly effective in dealing with fuzzy variables. It embeds all the required tools and it allows directly calling the meta-model calibration. Finally, the reliability-based optimization is performed - within the object “RBOproblem” by invoking the method “optimize” (see e.g. Figure 1). This action returns an object containing the optimal (robust) solution.

## 5. Example and Results

In order to illustrate the application of the proposed approach, consider the girder of a bridge. More specifically, the welded connection between a web stiffener and the girder’s flange is studied (see figure 2). It is assumed due to stress concentration, a crack appears at the toe of the weld. This example has been studied previously in (Lukić and Cremona, 2001). The aspect ratio between the crack’s width and depth is assumed to be $a/c$ =0.6. The mean value of the initial crack length $a_0$ is modelled as fuzzy variable of triangular membership function with values as {0.5, 1, 1.5} mm. In addition, the initial crack length is modelled by means of a fuzzy lognormal random variable with standard deviation of 0.4 mm. The critical crack length $a_c$ is set as 15 mm. The parameters of the Paris-Erdogan law are taken as $m$ =2.4 and $C$ =2x10$^{-10}$ mm/cycle(N/mm$^{1.5}$)$^{2.4}$ while the amplitude of the alternating stress applied is 30 MPa. The parameters associated with the POD are modelled such that $p$ =0.02 and $\lambda$ =0.1 mm. The target lifetime is 2 million load cycles and the inspection time can be chosen within the interval of 0.4 and 1.6 million load cycles. The costs associated with inspection, repair and failure are set as $c_I$ =50, $c_R$ =300 and $c_F$ =2x10$^5$; all these costs are expressed in monetary units. In order to train the meta-model for the probabilities of repair and failure, a full factorial design of experiments is used comprising a total of 35 grid points. The resulting approximations are shown in the figures below: figure 3 illustrates the probability of repair while figure 4 illustrates the probability of failure. Once the meta-model for the probabilities has been constructed, it becomes straightforward to assess the membership function of the fuzzy expected cost function in terms of the time of inspection. Such a plot is shown in detail in figure 5. It is interesting to note the behaviour of the expected cost function. When inspection is performed early, the cost tends to increase. This is due to the fact that by the time inspection takes place the crack is still too small to be detected; therefore a little probability of detecting and repair is expected from computations. Around the middle of the life span, inspection becomes more effective in detecting the crack, repair is more likely to take place and the probability of failure is reduced. This implies the total expected costs to become minimal. However, when inspection is performed late within the lifetime of the component, the total expected costs increase again as repair becomes too frequent (thus increasing the costs). The solution of the optimization problem posed in equation 3 indicates that the optimal time for performing inspection for the example at hand is equal to 0.74 million load cycles. This value is given along with interval measures of robustness against the initial crack size. Therefore, such procedure allows judging the sensitivity of the optimum point with respect to the uncertain variables.

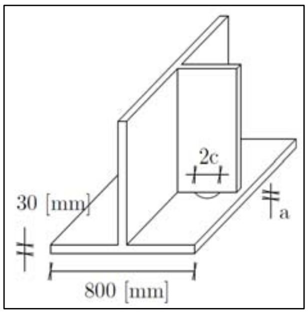


*Figure 2: Illustration of structural detail and crack*

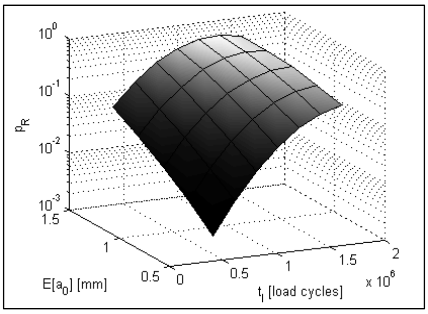


*Figure 3: Probability of repair as a function of expected value of initial crack length and time of inspection*

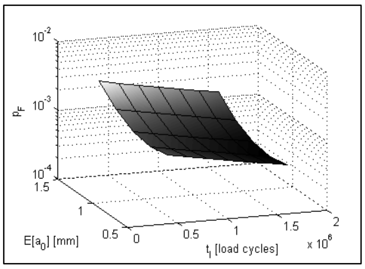


*Figure 4: Probability of failure as a function of expected value of initial crack length and time of inspection.*

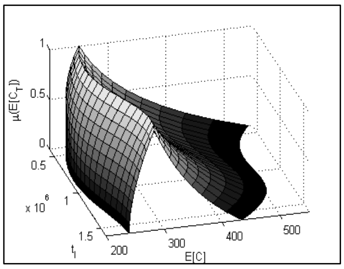


*Figure 5: Membership function of expected total cost as a function of the time of inspection.*

## 6. Conclusions

The results herein presented point out the relevance of considering uncertainty in the scheduling of maintenance activities for fatigue-prone metallic components. As shown by the numerical example, the optimal time for performing maintenance is a compromise between repair activities and the negative consequences of failure. The results obtained indicate that the effects of uncertainty – both aleatory and epistemic – cannot be ignored as they all affect the structural performance. Such effects lead to set-based measures of probabilistic expectations that in the specific problem quantify the cost of operation. Calculating such measures and using them to perform optimisation it is not a simple task; thus a solution strategy is proposed. This strategy computes upper and lower expectations at different alpha-levels by means of a global approximation constructed using advanced meta-modelling technique. By means of the described procedure it is possible to obtain set-based measure of probabilities. This allows creating models capable of more rationally processing the uncertainties that are no longer based solely on probability, and provides the final user with self-contained measures of robustness.

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