

VOL. 33, 2013



DOI: 10.3303/CET1333137

Failure Prediction of Oil Wells by Support Vector Regression with Variable Selection, Hyperparameter Tuning and Uncertainty Analysis

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In order to comply with safety requirements and due to the increasing demand on oil production, failure prediction of the related systems (e.g. oil wells) has become an important task. An analytical modelling of the reliability behaviour of these systems is often impractical and this justifies the use of data-driven learning methods like Support Vector Regression (SVR). The paper proposes a comprehensive failure prediction framework based on the combination of Particle Swarm Optimization (PSO) and bootstrap methods with SVR. The PSO portion of the methodology is responsible for the simultaneous selection of SVR hyperparameters' values and the choice of the most relevant influencing variables. The adjusted SVR model feed bootstrap methods, which provide point and interval estimates of the response variable. The bootstrapped PSO + SVR is applied in the context of the Brazilian oil industry and the obtained results suggest that it is a valuable tool in the support of maintenance-related decisions.

1. Introduction

Given the increasing demand on oil exploration and production, the proper functioning and availability of oil wells becomes a must. In addition to unattended market share, the occurrence of failures on these systems can provoke loss of production, accidents, environmental damages as well as increase of costs due to corrective maintenance actions and replacement of damaged components. In this way, the failure prediction of oil wells becomes an important task as it enables the implementation of preventive actions in order to reduce or avoid the undesired failure effects.

The reliability behaviour of these systems is often dependent on a number of factors related to operational and environmental conditions as well as to aging and previous maintenance effects. The relationship between these influencing factors and the response variable (e.g. Times Between Failures – TBFs) can present nonlinearities and interactions, which result in intricate models that are difficult to be analytically established. In these situations, the adoption of a model-free learning method such as Support Vector Regression (SVR) becomes attractive, given that it does not require previous knowledge about the function or process that maps the influencing variables into the response variable. SVR is a variant of Support Vector Machines (SVMs – Vapnik, 2000) that were primarily developed to tackle classification problems. The training phase of SVMs involves the resolution of a convex and quadratic optimization problem for which the Karush-Kuhn-Tucker (KKT) first order conditions are necessary and sufficient for global optimality (Boyd and Vandenberghe, 2004). Also, the associated objective function to be minimized is two-fold as it entails empirical errors and the errors related to the model capacity in predicting unseen data. Indeed, these characteristics are advantages of SVM over other learning methods such as Artificial

Neural Networks (ANNs) that can be trapped into local optima and only consider the minimization of empirical errors (Schölkopf and Smola, 2002).

The performance of SVM depends on a set of hyperparameters from the training optimization problem that have to be previously set. The adjustment of these parameters has been performed by a number of methods: grid and pattern search (Momma and Bennett, 2002), gradient-based (Chapelle et al., 2002; Ito and Nakano, 2003; Chang and Lin, 2005), Genetic Algorithms (GAs - Pai, 2006; Chen, 2007), Particle Swarm Optimization (PSO - Lin et al., 2008; Fei et al., 2009; Lins et al., 2012a). Also, in some practical cases, among the various factors that are supposed to influence the response variable, only a subset of them may be important to describe the response variable behaviour (Baraldi et al., 2012). Thus, a variable selection procedure can be applied to identify such a subset of variables so as to improve the prediction performance of predictors (e.g. SVMs), to construct faster predictors and also to provide better understanding of the underlying process that might have generated the response variable (Guyon and Elisseeff, 2003). Rakotomamonjy (2003) performs variable selection for SVM classification by means of a backward elimination strategy based on relevance criteria originated from SVM theory (weight vector and upper bound of the generalization error), but hyperparameter tuning is not considered. The training set is modified whenever a variable is included or excluded from the model, thus the hyperparameters' values may also change for each specific case. In order to tackle this issue, a PSO algorithm is used by Lins et al. (2011) to simultaneously adjust the SVR hyperparameters and select the most important input variables.

Once the SVR model is adjusted, it is able to provide point estimates of the response variable but for the same set of regressors' values, the estimated SVR model gives the same response value no matter how many times it is calculated. However, in practice, there is uncertainty about the response value due to non-controllable effects. Given that SVR does not require any hypothesis about the distribution of the error term, the central limit theorem enables the approximation of confidence and prediction intervals when large data sets are available (Brabanter et al., 2011). On the other hand, for small numbers of data points, the intervals based on bootstrap (Efron, 1979; Efron and Tibshirani, 1993) tend to be more accurate, given that they do not rely on asymptotic results but on the construction of the limit distribution from the available data. The coupling of bootstrap methods (based on pairs and residuals samplings) with SVR is analyzed by Lins et al. (2012b).

This paper proposes a comprehensive prediction framework based on the combination of PSO and bootstrap methods with SVR, which provides point and interval estimates of the response variable via SVR models accurately adjusted over the most relevant input factors and with the appropriate hyperparameters. In this way, this work is an extension of the works of Lins et al. (2011, 2012b). The proposed bootstrapped PSO + SVR is used for the prediction of TBFs of onshore oil wells located in the Northeast of Brazil. The onshore activities in this region date back to the beginning of oil exploration in Brazil (Zamith and Santos, 2007). In spite of being related to mature wells of low productivity, these activities were responsible for about 83 % of the regional production in the period 2000-2012 (ANP, 2013).

This paper unfolds as follows. In Section 2, an introduction to SVR, a description of the used PSO and the general ideas of bootstrap methods based on pairs and on residuals used in regression analysis are presented. In Section 3, the proposed combination of SVR with PSO and bootstrap is detailed. The application of the bootstrapped PSO + SVR framework in the prediction of TBFs of onshore oil wells from Northeast of Brazil is in Section 4. Finally, some concluding remarks are given in Section 5.

2. Theoretical Background

2.1 Support Vector Regression

Non-parametric regression can be formalized considering the response variable Y as generated by the model $Y = \mu_Y(\mathbf{x}) + u(\mathbf{x})$, where $\mu_Y(\mathbf{x})$ is the deterministic but unknown expected value of Y and $u(\mathbf{x})$ is a random error term with zero mean and variance $\sigma_u^2(\mathbf{x}) > 0$. SVR aims at estimating $\mu_Y(\mathbf{x})$ using an observed data set $D = \{(\mathbf{x}_1, y_1), ..., (\mathbf{x}_\ell, y_\ell)\}$, called the training set. More specifically, the weight vector \mathbf{w} and the linear coefficient *b* of a regression expression of the form $f(\mathbf{x}) = \mathbf{w}^T \phi(\mathbf{x}) + b$ are adjusted on the data of *D*. The operator ϕ maps \mathbf{x} into a higher dimensional space *F* to account for possible nonlinearities between the input vector and the response variable. The values for \mathbf{w} and *b* are obtained from the resolution of a convex and quadratic optimization problem, whose dual formulation is as follows (Schölkopf and Smola, 2002; Kecman, 2005):

$$\max_{\boldsymbol{\alpha},\,\boldsymbol{\alpha}^{\star}} \qquad -1/2\,\Sigma_{l}\,\Sigma_{o}\,(\alpha_{l}-\alpha_{l}^{\star})(\alpha_{o}-\alpha_{o}^{\star})\,\boldsymbol{\phi}(\mathbf{x}_{l})^{T}\boldsymbol{\phi}(\mathbf{x}_{o})-\Sigma_{l}\,[\varepsilon(\alpha_{l}+\alpha_{l}^{\star})+y_{l}(\alpha_{l}-\alpha_{l}^{\star})] \qquad (1)$$

$$\Sigma_{l} (\alpha_{l} - \alpha_{l}) = 0, \tag{2}$$
$$0 \le \alpha_{l}, \alpha_{l}^{*} \le C, \forall_{l}, \tag{3}$$

s.t.

where *I*, *o* = 1, ..., ℓ ; α and α^* are ℓ -dimensional vectors of Lagrange multipliers. The resolution of the dual training problem provides the adjusted regression function:

$$f_0(\mathbf{x}) = \mathbf{w}_0^T \boldsymbol{\phi}(\mathbf{x}) + b_0 = \sum_l \left(\alpha_{l0} - \alpha_{l0}^* \right) \boldsymbol{\phi}(\mathbf{x}_l)^T \boldsymbol{\phi}(\mathbf{x}) + b_0, \tag{4}$$

in which the index 0 represents optimal values.

In practice, an appropriate mapping ϕ is often difficult to be determined and the calculation of the dot products that appear in (1) and (4) may be computationally expensive. Fortunately, SVR allows the use of kernel functions $K(\mathbf{x}_i, \mathbf{x}_o) = \phi(\mathbf{x}_i)^T \phi(\mathbf{x}_o)$, which are defined in the original space and can be used to implicitly compute the dot products (Schölkopf and Smola, 2002). The Gaussian radial basis function (RBF), $K(\mathbf{x}_i, \mathbf{x}_o) = \exp(-\gamma ||\mathbf{x}_i - \mathbf{x}_o||^2)$ is the most widely used kernel function (Hsu et al., 2003; Lin and Lin, 2003). The substitution of the dot products in (1) by the RBF does not affect the way of solving the dual problem. Thus, the dot products in the estimated regression expression (4) may be replaced by the RBF kernel as well. Along with C and ε , the parameter γ in the RBF kernel has to be set a priori.

2.2 Particle Swarm Optimization for Variable Selection and SVR Hyperparameter Tuning

For PSO, the basic element is a particle *i*, $i = 1, ..., n_{part}$ that is characterized by its current position in the search space (**s**_{*i*}), the best position it has visited (**p**_{*i*}) and its velocity (**v**_{*i*}). Also, a fitness function is used to evaluate the particle performance. The adopted fitness function is the mean validation Mean Squared Error (MSE) given by (MSE₁ + ... + MSE_{*k*}) / *k*, which is obtained via a *k*-fold cross-validation. The n_{part} particles comprising the swarm fly throughout the search space towards an optimum until one of the stop criteria is met: maximum number of iterations (n_{iter}); 10 % of n_{iter} with the same best particle; the difference between two consecutive best fitness values is smaller than a tolerance Δ . This process is governed by the following update equations:

$$v_{ij}(t+1) = \chi\{v_{ij}(t) + c_1 \cdot u_1 \cdot [p_{ij}(t) - s_{ij}(t)] + c_2 \cdot u_2 \cdot [p_{gj}(t) - s_{ij}(t)]\},$$

$$s_{ij}(t+1) = s_{ij}(t) + v_{ij}(t+1),$$
(5)

in which *j* regards the *j*th dimension of the *d*-dimensional search space, *t* indicates the time step (i.e. PSO iteration), χ is the constriction factor used to avoid huge velocity values, c_1 and c_2 are constants, u_1 and u_2 are uniform random numbers in [0, 1] generated whenever the update takes place and for each *j*, $\mathbf{p}_g = (p_{g1}, ..., p_{gd})$ is the position associated with the best neighbour of particle *i* (Bratton and Kennedy, 2007). Basically, for the quest for SVR hyperparameters and variable selection, the PSO search space is formed by d = 3 + n dimensions, where the first three regard *C*, ε , γ , in this order, and the remaining *n* are the variables r_h associated with regressors x_h , h = 1, ..., n. The latter variables are defined in the range [0, 1] and if $r_h \ge 0.5$, the *h*th regressor is included in the model, otherwise it is not considered. PSO + SVR is summarized in steps 1-7 of Figure 1. For further details, the reader is referred to Lins et al. (2011).

2.3 Bootstrap for the Construction of Confidence and Prediction Intervals

Bootstrap is a computer intensive method, whose main idea is to resample from the original data, either directly or via a fitted model, in order to create replicate data sets. These replicates enable the variability assessment of the quantities of interest (Davison and Hinkley, 1997). In the case of regression models, the bootstrap samples can be obtained based on pairs or on residuals.

In the first case, the original data pairs (\mathbf{x}, y) are sampled with replacement and each of them have the same constant probability $1/\ell$ of being selected. A typical bootstrap sample based on pairs is of the form $D^b = \{(\mathbf{x}_{i_1^b}, y_{i_\ell^b}), ..., (\mathbf{x}_{i_\ell^b}, y_{i_\ell^b})\}$, in which the indices $i_1^b, ..., i_\ell^b$ are uniformly generated from 1, ..., ℓ and b = 1, ..., *B*. The residuals sampling, in turn, requires a fitted regression model over the original data set, henceforth called f_0^0 , and the computation of residuals. Thus, f_0^0 and the residuals are used to construct the bootstrap samples $D^b = \{(\mathbf{x}_1, \hat{y}_1^o + \breve{u}_{i_\ell}), ..., (\mathbf{x}_\ell, \hat{y}_\ell^o + \breve{u}_{i_\ell})\}$. In order to account for the more general case

of heteroskedastic errors, the various \vec{u}_{i} are sampled from the set of modified residuals $\vec{u}_{l} = (\hat{u}_{l} - \Sigma_{l} \hat{u}_{l} / \ell)^{-1}$

 η_l , in which $\hat{u}_l = y_l - \hat{y}_l^0$ is the ordinary residual and η_l is the Rademacher variable defined as -1 with probability 0.5 and as 1 also with probability 0.5 (Liu, 1988; Davidson et al., 2007).

For a given observation of the vector of input variables \mathbf{x}_+ , both bootstrap schemes allow for the construction of percentile confidence intervals for $\mu_Y(\mathbf{x}_+)$. However, only the residuals approach enables a straightforward manner to obtain prediction intervals concerning Y_+ (Davison and Hinkley, 1997; Lins et al., 2012b). In order to simulate the variation of Y_+ about $\mu_Y(\mathbf{x}_+)$, additional *M* samplings are required to

estimate the distribution of the prediction error $\delta_{+} = Y_{+} - f_{0}(\mathbf{x}_{+})$ via $\hat{\delta}_{+}^{bm} = (\hat{y}_{+}^{0} + \bar{u}_{+}^{m}) - \hat{y}_{+}^{b}$, m = 1, ..., M. Then, specific percentiles of the estimated prediction errors are added to the considered point estimate so as to give the related prediction interval. The bootstrapped SVR presented in Lins et al. (2012b) is summarized in steps 7-9 of Figure 1.

3. Bootstrapped PSO + SVR

In this section, the combination of SVR with PSO and bootstrap is presented (Figure 1). The first step to construct an SVR model to predict Y based on a set of observed pairs (**x**, *y*) is to divide the available data set into training + validation and test sets (step 1). The observations of the test set are treated as unknown data, thus they are neither used in the PSO nor in the construction of the bootstrap samples. The training + validation set feeds the PSO portion (steps 2-6) of the proposed methodology and, as an intermediate outcome, the selected hyperparameter values and chosen influencing variables are used for retraining SVR over the entire training + validation set (step 7); the obtained model is f_0^{0} . The construction of bootstrap samples D^{b} (either by pairs or residuals) based on resamplings of the training + validation set and the SVR trainings performed over each of them enable the estimation of the SVR models f_0^{b} (steps 8.1 and 8.2). For a new observation of the input variables **x**₊ (e.g. from the test set), f_0^{0} and the various f_0^{b} are used to provide estimates of the response variable, which are used in the calculation of the bagging estimate $\hat{y}_{+}^{bag} = (\hat{y}_{+}^{0} + \sum_{b} \hat{y}_{+}^{b}) / (B + 1)$ (Breiman, 1996) that is assumed as point estimate (step 9.1). Also, the values $\hat{y}_{+}^{0}, \hat{y}_{+}, \dots, \hat{y}_{+}^{B}$ are used in the construction of the percentile confidence interval with $(1 - \alpha) 100$ % confidence level for the mean response (step 9.2). By the residuals scheme, one may easily obtain

% confidence level for the mean response (step 9.2). By the residuals scheme, one may easily obtain prediction intervals for Y_{+} by the estimation of prediction errors (steps 9.3.1-9.3.2).



4. Application Example

In this Section, the bootstrapped PSO + SVR is used in the prediction of TBFs of oil wells located in the Northeast of Brazil. The data considered in this example was analyzed by Barros Jr. (2006) and is similar to the one used in Lins et al. (2011). The database contains observations from 1983 to 2006 of TBFs (in days) and of various aspects of different onshore wells. In addition, wells' failures are deemed to occur upon the failures of their installed rods. The regressors that are believed to influence the wells' performance are related to operational and environmental characteristics ($x_1 - x_{11}$) and to previous failure and maintenance of the rods (x_{12} - x_{18}). They are: x_1 – well depth (m); x_2 – well production (m³); x_3 – percentage of water and solids; x_4 - x_7 – presence of rods of 1", 7/8", 3/4" and 5/8", respectively; x_8 – level of

H₂S; x_9 – level of paraffin; x_{10} – type of artificial lifting; x_{11} – filter type; x_{12} – location of previous failure; x_{13} – mode of previous failure; x_{14} – substitution of rods in previous maintenance; x_{15} - x_{18} – state of the 1", 7/8", 3/4" and 5/8" rods in previous maintenance, respectively. The categorical variables x_4 - x_{18} were handled via 0-1 dummies (Montgomery et al., 2006).

The available data set comprised 242 observations related to 26 wells located in the same geographical area. From these, 192 were allocated to the training + validation phase and the remaining 50 formed the test data. For the PSO portion of the algorithm, a 5-fold cross validation technique was considered along with $n_{\text{part}} = 20$, $n_{\text{neigh}} = 8$, $n_{\text{iter}} = 5000$, $\Delta = 10^{-12}$, $\chi = 0.7298$, $c_1 = c_2 = 2.05$.

In order to avoid negative TBFs, the natural logarithm of the response variable was taken into account. Also, all variables were individually scaled in [0.1, 0.9] so as to minimize numerical problems during SVR trainings. In this way, the obtained values for the SVR hyperparameters are related to the transformed variables. The PSO provided *C* = 971.1587, ε = 0.0617, γ = 1.6886 and the selected input variables for the regression model were x_8 , x_9 , x_{11} , x_{14} , x_{15} and x_{18} .

The obtained results over the test set are shown in Figures 2 and 3. For this example, the point estimates provided by the residuals approach had overall better performance over the test set than the ones given by the pairs scheme ($MSE_p = 1.1586 \cdot 10^5$ vs. $MSE_r = 1.1557 \cdot 10^5$). As an illustration, for the first test point with input vector \mathbf{x}_+ : $\hat{y}_{+,p}^{bag} = 171.73$ and $\hat{y}_{+,r}^{bag} = 203.74$; [135.19, 368.61] and [71.92, 519.70] are the confidence intervals for $\alpha = 0.10$ given by the pairs and residuals sampling, respectively; the associated prediction interval for the same α is [25.83, 942.95], which was obtained by bootstrapping residuals. In this way, a conservative (resp., risky) attitude can be adopted if the maintenance action is performed near the lower (resp., upper) bounds of the given intervals. This would not be possible if only the estimated mean TBF had been provided. Surely, the definition of when maintenance should be carried out must consider the criticality of failure and the associated costs.



Figure 2. Test results by bootstrapping pairs

Figure 3. Test results by bootstrapping residuals

5. Conclusion

In order to construct accurate and informative failure prediction models, a combination of SVR with PSO and bootstrap methods was proposed. The selection of proper SVR hyperparameter values and the choice of the most relevant influencing variables were simultaneously performed by PSO. In this way, the problem of having inappropriate hyperparameters' values for the group of chosen regressors is avoided. Also, the reduction in the set of influencing variables is an indication on how future efforts should be allocated in the observation of the input variables. The bootstrap methods coupled with the PSO + SVR provided not only point estimates but also associated intervals of probable values, which is valuable information in practical maintenance-related decision-making.

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