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Gear System Time-varying Reliability Analysis Based on Elastomer Dynamics

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This paper establishes nonlinear dynamic equation based on the consideration of elastic deformation of shaft which happens in gear meshing process. In addition, the stiffness incentive, error incentive and external incentives caused by eccentric phenomenon during the process are analysed. The Newmark algorithm is also been used. Then the dynamic load coefficient of the gear could be gotten. Based on this, a time-varying reliability algorithm is used to calculate the degree of reliability. Monte Carlo algorithm is used to prove the accuracy of this method. Finally the numerical example is given.

1. Introduction

At present, the high speed machine and high power generating set mostly use flexible rotor. The gear dynamics model that does not consider axis transverse vibration is not applicable. (Shao et al, 2005) In addition, the gear system reliability analysis and strength calculation mostly use the method that inquires the mechanical design manual to get dynamic load coefficient. So the subject factor cannot be avoided. (Li ,et al, 2011). The torsion vibration model is used to establish the gear dynamic model. But this model does not considering the effect of the gear bending. (Baguet et al, 2010) the finite element dynamics method is used to establish gear dynamic model. Although the influence between the shaft and gear is considered, the method is too complicated to be used to solve the engineer problem. (Mejri et al,2010) PHI2 method is used. But it cannot provide the accurate time-varying reliability. Moreover, the failure rate which is important in reliability analysis is not gotten. (Huang et al, 2003) Stress maximum and strength minimum values are selected in the time process to establish limit state function. Then the moment method is used to get reliability calculation result. As a result, the change rule of reliability could not be gotten.

This paper deduces the dynamic equations of gear-shaft system which includes elastic vibration model and plane vibration model of spur gear. The time-variant excitation of gear is offered by FEM. Then the dynamic load coefficient of spur gear under this circumstance would be gotten. Finally, the time-varying failure rate and reliability level is obtained by a new reliability calculation method.

2. Dynamic Model

It is certain that there will be a radial force when gears mesh with others. This influence can be ignored if the gear shaft is short enough. Ignoring the influence of a long and flexible shaft deformation will result in a big error, as shown in figure 1. So the influence of shaft flexibility must be taken into consideration when dynamic equations are deduced.

Differential equations can be deduced for the dynamic analysis on elastic body in this model. But it's too complicated to be used to solve an engineering problem. To deal with this problem, Finite Element Method (FEM) would be used.

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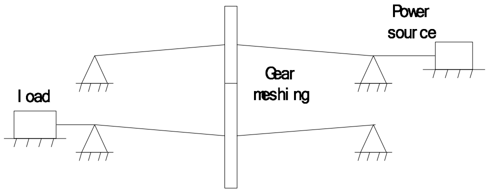

Figure 1：Gear rotating system diagram

2.1 Dynamic equations of shaft in FEM

Gear shaft will have a vibration in the rotational and horizontal direction under radial force and bending moment. Just as Timoshenko beam element, the shape function of horizontal and rotational displacement can be interpolated independently. Because deflection curve is cubic when beam is endured concentrated load, the shape function can be formulated in cubic equation:

$$u(x,t) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 \tag{1}$$

Substitution boundary condition：

$$u(x,t) = \sum_{i=1}^{4} \varphi_i(x) u_i(t) \tag{2}$$

Analogy in Timoshenko beam element Angle shape function can use linear interpolation ：

$$\theta(x,t) = a_0 + a_1 x \tag{3}$$

Substitution boundary condition：

$$\theta(x,t) = \sum_{i=1}^{2} \varphi_i \theta_i(t) \tag{4}$$

Getting beam element potential energy ：

$$v(t) = \frac{1}{2}\int_0^1 EI\left[\frac{\partial^2 u(x,t)}{\partial x^2}\right]dx + \frac{1}{2}\int_0^1 GI_P\left[\frac{\partial^2 \theta(x,t)}{\partial x^2}\right]dx \tag{5}$$

The kinetic energy T of beam element:

$$T(t) = \frac{1}{2}\int_0^1 \rho A\left[\frac{\partial u(x,t)}{\partial t}\right]dx + \frac{1}{2}\int_0^1 \frac{\pi}{2}R^4\rho\left[\frac{\partial \theta(x,t)}{\partial t}\right]dx \tag{6}$$

Substitution Lagrange equation can get beam element vibration equation:

$$\left[M_{shaft}\right]\left\{\ddot{y}\right\} + \left[K_{shaft}\right]\left\{\dot{y}\right\} = \left\{F_{shaft}\right\} \tag{7}$$

2.2 Gear dynamic equation

Straight gear contact can be described by two dimensional vibration model system as figure 2 showing , y_1 y_2 said end displacement of beam element that connect with gear.

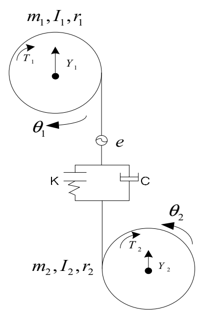

Figure 2：Straight gear plane drive model diagram

The gear contact dynamics equation:

$$\left[M_{gear}\right]\left\{\ddot{y}\right\}+\left[C_{gear}\right]\left\{\dot{y}\right\}+\left[K_{gear}\right]\{y\}=\left\{F_{gear}\right\} \tag{8}$$

2.3 Driving system dynamic equation

Stacking all the elements vibration equation and gear vibration equation, the force of various elements and between gear and element as internal force is offset, the system dynamic equation as follows:

$$[M]\left\{\ddot{y}\right\}+[C]\left\{\dot{y}\right\}+[K]\{y\}=\{F\} \tag{9}$$

3. Gear drive incentive analysis

3.1 Stiffness incentive

During the gear meshing, due to the meshing tooth logarithmic change and elastic deformation from top to root, mesh comprehensive stiffness would be periodic changed by time. It leads gear tooth meshing force periodic to change .It is not practical to use analytical method to calculate the time-varying stiffness of gear. Finite element simulation method to calculate elastic deformation of the gear tooth is easier. After using FEM to get displacement and force, the calculate gear meshing of time –varying stiffness can be calculated by equation 10 (S. Baguet et al, 2010).

$$K=\frac{\sum_{i=1}^{n}F_i}{\delta_{pi}+\delta_{ni}} \tag{10}$$

By using ANSYS APDL language programming to establish the model of gear engagement, as shown in figure 3.Then the gear meshing of time-varying stiffness can be obtained, as shown in figure 4.

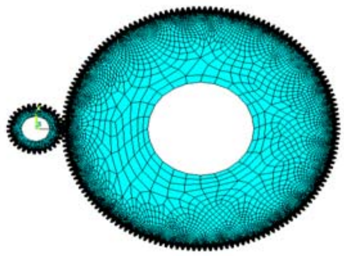

Figure 3 :Gear meshing finite element model diagram

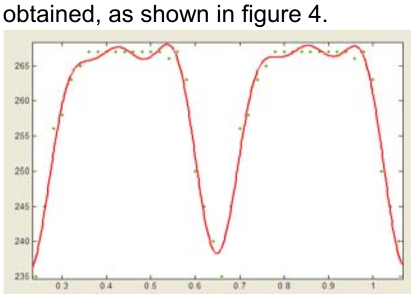

Figure 4:Gear meshing time-varying stiffness diagram

All of periodic functions can be regarded as superposition of sine function. Fourier series is suitable to fit simulation data. Then the time – varying stiffness expression of gear meshing can be expressed by equation 11.

$$K(t)=K_m+\sum_{i=1}^{n}k_i\sin\left(a_it+b_i\right) \tag{11}$$

3.2 Error incentive

Gear meshing error is deviation between tooth profile actual position and ideal position during the gear meshing caused by gear machining error and installation error. Similarly, Gear error incentive is also a kind of cyclical incentive, it is suitable for using sine function to describe (M. Mejri, et al ,2010).

$$e(t)=e_0+e\sin\left(\frac{2\pi t}{T}+\varphi\right) \tag{12}$$

3.3 External incentive

Gear transmission system will be inevitable into manufacturing error. It will lead to the rotor eccentricity. Eccentric rotor will produce the dynamic load incentive, the expression as following:

$$F_w = mr\omega^2 \tag{13}$$

4. Dynamic load coefficient calculation

We can get the dynamic equation of gear transmission system which is shown by equation 9 :

$$[M]\left\{\ddot{y}\right\}+[C]\left\{\dot{y}\right\}+\left[K(t)\right]\{y\}=\left[K(t)\right]\left[e(t)\right]+\left[F_w\right]+\left[T_{in}(t)\right]+\left[T_{out}(t)\right] \tag{14}$$

From above, the conclusion that the gear transmission system stiffness matrix, internal and external incentives are time-varying and nonlinear is gotten. The traditional transfer matrix method and vibration mode Superposition method are not suitable to this situation. The paper uses the Newmark algorithm shown by equation 15 :

$$M\ddot{Y}_{t+\Delta t}+C\dot{Y}_{t+\Delta t}+K(t+\Delta t)Y_{t+\Delta t}-K(t+\Delta t)e(t+\Delta t)-F_W-T_{in}(t+\Delta t)-T_{out}(t+\Delta t)=0 \tag{15}$$

$$\dot{Y}_{t+\Delta t}=\dot{Y}_t+\left[(1+\alpha)\ddot{Y}_t+\alpha\ddot{Y}_{t+\Delta t}\right]\Delta t \tag{16}$$

$$Y_{t+\Delta t}=Y_t+\dot{Y}_t\Delta t+\left[\left(\frac{1}{2}-\beta\right)\ddot{Y}_t+\beta\ddot{Y}_{t+\Delta t}\right]\Delta t^2 \tag{17}$$

Using above three equations can obtain $Y_{t+\Delta t}$, $\dot{Y}_{t+\Delta t}$, $\ddot{Y}_{t+\Delta t}$

As long as $Y_{t=0}$ $\dot{Y}_{t=0}$ $\ddot{Y}_{t=0}$ at initial time are given, by increasing Δt the displacement, velocity and acceleration can be gotten at each time. Thus, the gear meshing line displacement can be obtained. If $\alpha=1/2$, $\beta=1/4$, it can ensure to get unconditional stability solution. By gear dynamic load calculation formula 18 and 19 dynamic load coefficient can be gotten. As following:

$$F_d = K_t\,\dot{y}_t \tag{18}$$

$$K_v=\frac{F_d+F_t}{F_t} \tag{19}$$

5. Time Variant Reliability Analysis of Gear

Logarithmic normal distribution or Wei Bull distribution can be used to represent failure probability in traditional reliability analysis of gear system. But this method is not suitable to solve the problem failure mechanism. In addition, the load on the gear is time variant, as shown in fig 6, it is unscientific to analyze the reliability in dynamic load coefficient stable value. Some documents used out-crossing rate method , which can just estimate upper and lower boundary of breakdown rate. To the time variant and nonlinear character of dynamic loading coefficient, this paper uses FORM method to calculate reliability index $\beta(t)$ and normal vector $\alpha(t)$ .and uses reliability calculation method that based on failure rate and uses Monte Carlo method to check the accuracy of this method.

5.1 Reliability calculation based on gear bending stress

Limit state function :

$$G=\sigma_F-\sigma_{F\lim}=\frac{F_t}{bm}Y_SY_bK_v-\sigma_{F\lim} \tag{20}$$

5.2 Time – varying reliability calculation method

Order $X(\omega,t)$ as mechanical problem random variables, t represent time course , ω represent a sample point in sample space Ω .therefore, gear time-varying limit state function as following: G(t , X(ω,t))=0 then the reliability function can use $R(t)=prob\left(G\left(t,X(\omega,t)\right)\right)\geq 0$ to describe. At the same time failure rate function can be defined:

$$\lambda(t)=\lim_{\Delta t\to 0}\frac{prob\left(A|B\right)}{\Delta t}=\lim_{\Delta t\to 0}\frac{prob\left(A\cap B\right)}{\Delta t\cdot prob\left(B\right)} \tag{21}$$

Note : $A=\left\{G\left(t+\Delta t, X\left(\omega, t+\Delta t\right)\right) < 0\right\}$; $B=\left\{G\left(t+\Delta t, X\left(\omega, t+\Delta t\right)\right) \geq 0\right\}$; As we know $prob\left(B\right)$ can be expressed by (22)

$$prob\left(B\right)=\phi\left(\beta\right) \tag{23}$$

Introduce two dimensional normal distribution function ϕ_2 and correlation coefficient ρ_t , so

$$prob\left(A\cap B\right)=\phi_2\left(\beta\left(t\right),-\beta\left(t+\Delta t\right),\rho\left(t,t+\Delta t\right)\right) \tag{24}$$

Note: $\rho\left(t,t+\Delta t\right)$ can expressed by limit state surface normal vector α

$$\rho\left(t,t+\Delta t\right)=-\alpha\left(t\right).\alpha\left(t+\Delta t\right) \tag{25}$$

ϕ_2 calculation method as follows :

$$\begin{aligned}&\phi_2\left(\beta\left(t\right),-\beta\left(t+\Delta t\right),\rho\left(t,t+\Delta t\right)\right)=\\&\int_{-\infty}^{\beta(t)}\int_{-\infty}^{-\beta(t+\Delta t)}\varphi\left(x,y,\rho\right)dxdy=\phi\left(\beta\left(t\right)\right).\phi\left(-\beta\left(t+\Delta t\right)\right)\\&+\int_0^{\rho}\frac{1}{2\pi\sqrt{1-z^2}}\exp\left(-\frac{1}{2\left(1-z^2\right)}\left(\beta\left(t\right)^2+\beta\left(t+\Delta t\right)^2+2z\beta\left(t\right)\beta\left(t+\Delta t\right)\right)dz\right)\end{aligned} \tag{26}$$

Using reliability function $R\left(t\right)=e^{-\int_0^T\lambda(t)dt}$ can obtain reliability of the whole time history

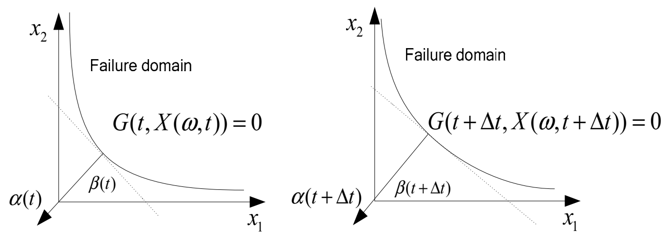

Figure 5 : Limit state surface at different time

6. Calculation example

Gear related parameters as is shown in Table 1:

Table 1 : Gear related parameters

	Number of teeth	Modulus	Tooth width (mm)	Mass (kg)	Pressure Angle	Center Distance (mm)
Small gear	26	4	50	2	20	366
Big gear	157	4	40	74	20	366

Shaft related parameters as shown in Table 2:

Table 2 : Shaft related parameters

	Length (mm)	Radius (mm)	Elastic modulus(Gpa)	Eccentricity Distance (mm)
Shaft	800	35	210	1

6.1 Gear dynamic load coefficient calculation results and Gear system reliability evaluation

Using above method to write corresponding calculation program can get out load coefficient calculation results. As shown in figure 6Time step is 0.02 seconds, due to the influence of damping gradually after 6 s it become stable. The mean and variance of various parameters in formula 20 are as follows :

$F_t \sim N\left(34644,519.66\right), Y_{s1} \sim N\left(2.6,0.012\right), Y_{b1} \sim N\left(1.595,0.011\right), Y_{s2} \sim N\left(2.14,0.015\right)$, $Y_{b2} \sim N\left(1.83,0.021\right), \sigma_{F\lim} \sim N\left(1300,156\right)$.

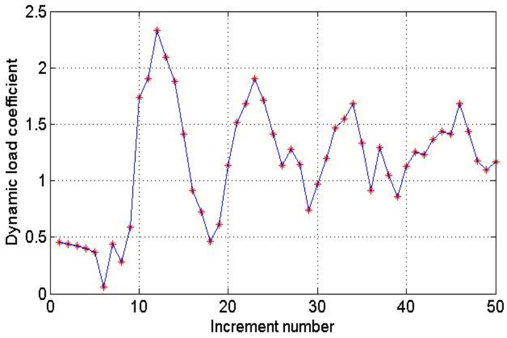

Figure 6： Dynamic load coefficient diagram

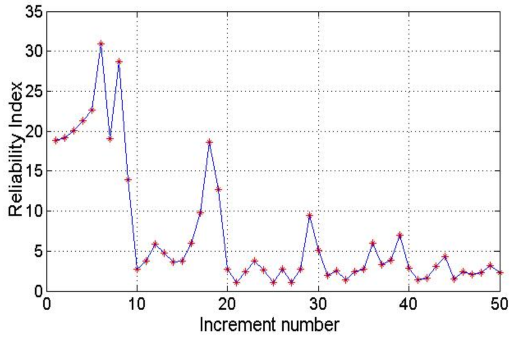

Figure 7：Reliability index at different time

Time discrete reliability index can't reflect the relationship between time points. Use formula 22 can get time-varying failure rate curve the method of numerical integration for time-varying reliability and Monte Carlo simulation comparison results was shown in Figures 8 and 9:

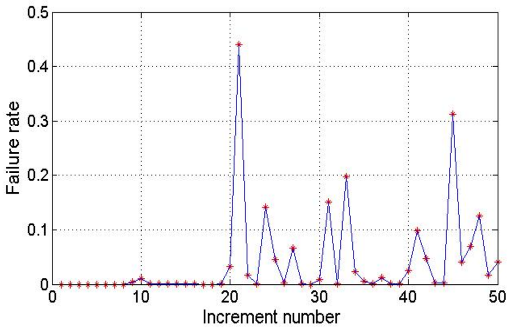

Figure 8：Failure rate

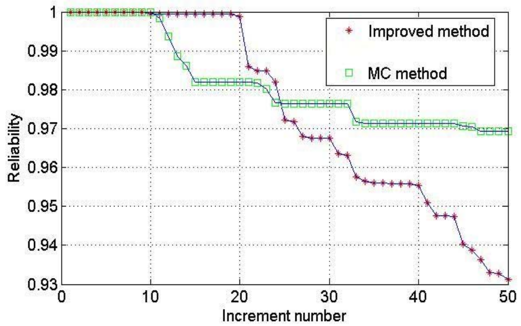

Figure 9：Comparison for different method

The maximum error is less than 0.01, but its calculation time significantly less than Monte Carlo algorithm.

7. Conclusion

This paper considers axis elastic deflection and gear time-varying stiffness to establish the nonlinear dynamic equation which is solved by Newmark method. then the dynamic load coefficient is gotten. Finally a improved time-varying reliability calculation method is used to calculate reliability of the gear system. In addition, Monte Carlo algorithm is used to verify the accuracy and the feasibility of this method.

The improved time-varying reliability calculation method has the advantage that can get time-varying failure rate. At the same time, it can also be applied at the situation that the limit state function is highly nonlinear. It is expected to have a certain guiding significance for failure and life prediction.

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