A Qualitative and Quantitative Functional Modelling Method for PHM System Design

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In the early stage of PHM system design, it is very important to analyse the failure modes and their effects, i.e. FMEA. A great number of model-based methods have been proposed recently; however, the analysis process might concern only qualitative or quantitative demands for the complex system. The method is usually difficult to adapt to the progressively definite research for modular modelling of the functional structure. It is necessary to provide an adaptive approach to represent the relationship between the functions and the faults, covering the period from initial planning to final performance assessment. This paper explores the challenges and needs of efforts to implement FMEA, laying an emphasis on functional modelling. In particular, this paper puts forward a qualitative and quantitative hybrid modelling method based on the bond graph theory framework, together with embedded block diagram modules and programming conditional statements. Afterwards, the presented method is exemplified with an EHA (electro-hydrostatic actuator) system, and the simulation results demonstrate the efficiency.

1. Introduction

FMEA is the important content in PHM research field. Although there are technical standards, for example MIL-STD-1629A, FMEA analysis appears to make difference with respect to the experts’ knowledge (Hunt, Price et al. 1993). Many model-based techniques have been utilized to figure out how faults occur and propagate (Gargama, Chaturvedi 2011). However, analysis activities might partially describe the complex system in a qualitative or quantitative way. Notice that the model is to be progressively definite with future study of system structure; an effective technique should be extensible and suitable for multi-level modular modelling, which has a basic and support role for PHM system design, especially in the early stage.

Given the fact that the power flow process is uniformly represented in the bond graph theory (GAUTHROP and BEVAN 2007), it offers a kind of foundation for interfaces of component modules. The bond graph model is intuitively corresponding to the system functional structure (Song and Hu et al. 2007). That seems suitable to characterize relationship between component modules at a certain modelling level, and someone module can be seen as a whole with known boundary features, but unknown inners at this level. Programming statements are occasionally required to fulfil the conditional association, concerning logical or numerical forms (Henrion and Werner 2012). Besides, block diagram is more convenient when expressing activities of signal flow instead of power flow. The hybrid modelling framework of power flow, signal flow and conditional connection is extensible, thus suitable for different phases of the design work. In this paper, the problem is studied, and an effective and quantitative modelling method is presented. Application shows the effectiveness.

2. Basic theory

2.1 Bond graph

The bond graph theory provides a uniform way of developing mathematical models for dynamic systems with different nature (mechanical, hydraulic, electrical, and thermal, etc.). This means that the same model
can be extended to different subsystems of a global system. In the bond graph theory, one needs to recognize only four groups of basic symbols, i.e., three basic one-port passive elements, two basic active elements, two basic two port elements and two basic junctions. The basic variables are effort, flow, time integral of effort and the time integral of flow.

Basic 1-Port elements: A 1-port element is addressed through a single power port, and at the port a single pair of effort and flow variables exists. Ports are classified as passive ports and active ports. The passive ports are idealized elements with no sources of power. The inertia or inductor, compliance or capacitor, and resistor or dashpot are classified as passive elements. The active ports are those, which give reaction to the source, i.e. the effort source and the flow source. Besides, there are two kinds of two port elements, namely "Transformer" and "Gyrator". As the name suggests, two bonds are attached to these elements. There are two kinds of junctions, the 1 and the 0 junction.

Each type of port should be assigned a specific causal property, with a half arrow indicating direction of power. However, due to the limited space, the different forms of constitutive relations and causal port properties are not discussed in this paper.

2.2 Block diagram
The block diagram provides a usual tool for algebraic description of the system based on signal flow. Block diagram elements are commonly corresponding to mathematic equations, abstract if compared with bond graph relations in some sense. There lies no definite physical meaning here, like power transformation and energy conversation. This implies that the block diagram model cannot intuitively represent the functional structure of the complex system.

A great variety of operations with block diagrams have already been developed, for instance the mature software environment of MATLAB/Simulink. Some commonly used blocks include Gain, Integrator, Switch, Logical operator, and etc. Besides, other blocks of Sources, Sinks, Math operations, Signal routing and etc. are also abundantly offered. Due to the limited space, they are not discussed here.

In addition, it should be pointed out that bond graph expression has bidirectional signals, i.e. directions of power and feedback, while block diagram indicates a single one. Block diagram is more convenient when expressing activities of a single signal flow of instead of power conserving.

2.3 Conditional statement
If a module can be identified with definite inner components and structure, a bond graph expression shall prove effectiveness with boundary interface of power, similarly block diagram with boundary connection of a certain signal. However, what if we get nothing about someone module with its inners, or not necessarily knowing about the details? Then it has to be seen as a whole and represented by the conditional boundary features, concerning logical or numerical forms. It means that a kind of knowledge-descriptive qualitative technique should be adopted rather than quantitatively characterizing the actual functional structure or dynamic behaviour of the module.

Here presents a feasible solution for conditional description by way of programming functional statement. Take the piecewise relation as an example, expressed as follows:

```plaintext
halfzone = 0.5 * zone;
output = begin
    if input > halfzone then input - halfzone
    else if input < - halfzone then input + halfzone
    else 0.0
end;
```

3. Qualitative and quantitative model
A general algorithm is illustrated as follows, based on the above principles of bond graph, block diagram and conditional statement, which constitute a hybrid framework of power flow, signal flow and conditional connection. The modelling process with that is hierarchic, as shown in Figure 1.

1) Make a survey of system hierarchy, i.e. the components and interaction relationships between them. This depends on the extent to which the functional structure should be analysed. If the current level could meet the demands for FMEA analysis, it should be seen as an indivisible whole unit. However, if a certain module’s inner failure mode is interesting, it should be divided into units at the next subsystem level. The boundary features of the subsystem are the same with the module’ interfaces with others. If a certain part of the subsystem is concerned, the functional structure of that should be analysed in a likely way. Similarly, the system is disassembled to multi-level units as required by failure analysis.

2) Considering a certain level, the composition of units should be identified. The information includes the
number and their interface features and association, i.e. power flow, signal flow and conditional connection. The boundary properties of this whole level keep concordant.

3) At the specific level, the power flow process among units is studied; thereby a bond graph model is established, intuitively representing the functional structure.

4) At the specific level, the signal flow process between units is studied except for power conserving. A block diagram model is correspondingly established.

5) At the specific level, the knowledge-descriptive requirements are studied, and programming functional statements are prepared to fulfill the conditional association.

6) At the specific level, bond graph model, block diagram model and conditional connection are linked as one hybrid model, called “Fault description model”.

7) Repeat step 2) to step 6) within the top-down hierarchy, according to practical requirement.

8) Considering a certain level with an interesting failure mode, inject faults from three aspects, namely the conditional input/output or model elements. Plotting curves are observed by simulating software, and signal features are extracted later.

9) Repeat experimental simulation. Respectively identify the quantitative influences of each injected fault over the relevant test points, mainly the range of deviation and failure distribution. That has a basic and support role for FMEA analysis.

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**System hierarchy**

A certain level

interface features and association with components

Bond graph model for power flow  Conditional connection  Block diagram model for signal flow

Fault description model

Fault injection and simulation

FMEA analysis

*Figure 1: Illustration of qualitative and quantitative modeling*

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**4. Application case: EHA system**

In this section, the above functional modelling method is simulated and applied to an EHA system. The EHA system mainly contains controller, chopper, electrical motor, hydraulic pump, hydraulic cylinder and aircraft rudder. The following is the illustration of application.

**4.1 System modelling**

At the system level, there lies a current feedback from electrical motor. Control signal flows from controller to chopper. Chopper passes power to electrical motor, and the causality is effort (voltage). Electrical motor also passes power to hydraulic pump, and the causality effort (torque). Hydraulic pump passes power to hydraulic cylinder with the causality of flow (rate), and there exists a conditional dependence of oil height in reservoir. Hydraulic cylinder passes power to aircraft rudder in an effort (force) way. Aircraft rudder has an output of angular velocity signal.

Suppose controller is the one we concerned. Controller is presented with a design of PI (Proportional-integration) algorithm, thus a simple block diagram model can be obtained, as shown in Figure 2.
As to the conditional dependence of oil height in reservoir, we know that it doesn’t matter even if there is little leakage; however hydraulic pump’s work capability will decrease if oil height is lower than needed. Such a causal relationship can be modelled by programming functional statements. It is pointed out that the conditional output is attached to the power flow from hydraulic pump, influencing the rate (flow) output to hydraulic cylinder with a certain manner, for instance, direct multiplication. Followed that, hydraulic cylinder is a relatively more complex subsystem in our study case. If we make deep investigation, the subsystem is composed of delivery line as a storage element of capacitor, DCV A-B as a resistor, black boxes of A-B cylinder and reservoir. The connections between them are power flows, and the outputs are consistent with the above. The model of hydraulic cylinder is outlined as Figure 3. And if we continue carrying out research, A-B cylinder can be further divided to hydraulic line A, hydraulic line B, actuator, and etc. It is not difficult to determine their constitutive relations, thus a detailed model is shown in Figure 4. According to practical requirement, it is seen as the bottom level. Similarly, the other modules of EHA system can be hierarchically modelled in the qualitative & quantitative way of functional description. Due to the limited space, they are not discussed here.

![Figure 2: Functional modeling at the subsystem level - Controller](image)

![Figure 3: Functional modeling at the subsystem level - Hydraulic cylinder](image)

![Figure 4: Functional modeling at the bottom level - A/B cylinder](image)
4.2 Simulation and analysis
Simulation is carried out based on the EHA system model. Four kinds of faults are separately simulated to validate our work, i.e. electrical motor sticking, hydraulic pump leakage, hydraulic pump blocking and hydraulic pump wear. Sticking fault is injected as decreased transformation efficiency of electrical motor, leakage fault is injected as added resistance of hydraulic pump, blocking is similarly as decreased transformation efficiency, and wear fault as increased dissipation.

In order to compare the deviation, each fault is independently injected at the unified simulating moment of 30s. That is to say, the former 30s is normal, and the latter is faulty, partly shown in Figure 5. The steady output values are chosen as extracted signal features, for the sake of simple. The quantitative influences of each injected fault over the test point (system output) are listed in Table 1.

From the curves and signal features, we can carry out further study on the range of deviation and failure distribution, which is supportive for FMEA analysis.

![Figure 5: Simulation of angular velocity output of Aircraft rudder](image)

- a) electrical motor sticking fault
- b) hydraulic pump leakage fault
- c) hydraulic pump blocking fault
- d) hydraulic pump wear fault

**Table 1: Measurement of faults over system output**

<table>
<thead>
<tr>
<th>Fault</th>
<th>Angular velocity of Aircraft rudder</th>
</tr>
</thead>
<tbody>
<tr>
<td>electrical motor sticking</td>
<td>0.31</td>
</tr>
<tr>
<td>hydraulic pump leakage</td>
<td>0.28</td>
</tr>
<tr>
<td>hydraulic pump blocking</td>
<td>0.18</td>
</tr>
<tr>
<td>hydraulic pump wear</td>
<td>0.31</td>
</tr>
</tbody>
</table>
5. Conclusion
In this paper, a qualitative and quantitative modelling method is presented, and its effect is simulated. Furthermore, the hybrid modelling framework is illustrated to figure out the relationship between functions and faults. This method overcomes the defect of former studies that meet partial demands for the system description. A hybrid method is more appropriate for the progressive design work. Finally, the method is applied to an EHA system and delightful results are obtained.

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References