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A Bayesian Framework for Crack Detection in Structural Components Under Dynamic Excitation

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Structures under dynamic excitation can undergo phenomena of crack growth and fracture. For safety reasons, it is of key importance to be able to detect and classify these cracks before the structural failure. Additionally, the cracks will also change the dynamic behaviour of the structures, impacting their performance. Here, a Bayesian model updating procedure has been introduced for the detection of crack location and length on a numerical model. A high-fidelity finite element model has been used to simulate experimental data, by inserting cracks of different lengths in different locations and obtaining reference frequency response function. Then, a low fidelity model has been used in the Bayesian framework to infer the crack location and length by comparing the dynamic responses. It is shown that the proposed technology can be successfully adopted as a structural health monitoring tool.

1. Introduction

Fatigue is one of the most dangerous failure modes for mechanical components subject to alternating loads: one or several cracks can be initiated and propagated through the cross section of the structure. Once a critical crack length is exceeded, the structure will catastrophically fail even for stress level much lower than the design stress (Paris & Erdogan, 1963). In particular, interactions may occur between the structural responses and cracks in components subject to high frequency dynamic excitations, leading to vibration-induced fatigue. In this case, the stress field in the structure is mainly determined by the high frequency resonance modes, leading to very fast cycles of loading and accelerated fatigue crack growth. Hence the service life of the structure may be considerably reduced.
Several strategies are possible to avoid fracture; for instance, non-destructive inspections may be performed at predetermined time intervals in order to detect the cracks; however failure can happen between inspections (Beaurepaire, et al., 2012). Alternatively, a continuous monitoring of the dynamic response of the structure can allow for real-time crack detection and for a timely intervention with maintenance procedures (Chang, et al., 2003). Repair actions are taken in case the monitoring procedure successfully identifies a crack which jeopardizes the structure.
In both cases, the procedure may fail in identifying a crack, leading to fatigue failure. Thus, efficient crack detection before fracture occurs is required in order to avoid the loss of the structure, and more importantly to mitigate the consequences that such loss could cause, both from an economical and safety point of view. Once the crack is successfully detected, corrective maintenance or substitution of the damaged part can be performed.
Thanks to the advancements in the field of computational mechanics, new detection techniques can be developed to assist in the monitoring of the health of the structures. These numerical techniques allow for a synthetic analysis of experimental and sensor data. More specifically, these techniques modify some specific parameters in a numerical model to ensure a good agreement with the data, leading to a so-called *inverse problem*. A computational framework well fitted for the solution of such inverse problems is the model updating (Fritzen, et al., 1998).
In this paper, an efficient numerical framework, based on a Bayesian model updating procedure (Goller, et al., 2011), is implemented for the identification of cracks within structures under dynamic excitation. The

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influence of the cracks on the mechanical behaviour is quantified using the Frequency Response Functions (FRF) at a specific location. A suspension arm, as normally used by automotive industry, has been analysed. Two finite elements models have been used: one high-fidelity model to simulate experimental data of the arm under dynamic excitation (i.e., the reference model). Cracks of random location and dimension are introduced in the simulated experimental suspension arm.
The second, "low-fidelity" model is used in the Bayesian model updating procedure The updating procedure adjust the length of cracks inserted at candidate crack locations in the "low-fidelity" model in order to minimize the difference between the frequency response function of this model and the reference frequency response. This allows reconstructing efficiently the crack pattern of the reference model.
Particular attention is given to the efficiency of the numerical simulation. As a matter of fact, a high number of model evaluation is required, thus a strategy for the parallelization of the simulations as provided by the general purpose software COSSAN-X (Patelli, et al., 2012) is employed.
The paper is structured as follows: Section 2 deals with the modelling of fracture in a Finite Element framework. Section 3 outlines the main concept of Bayesian model updating and the efficient simulation algorithm employed in the particular case of structure with cracks under dynamic excitation. A numerical example is introduced in Section 4, and the crack detection routine is tested in different configurations. Finally, some conclusions and final remarks are pointed out in Section 5.

2. Fracture mechanics and modelling

The mechanical behaviour of structures may be modified by the presence of cracks. The cross section of the component is reduced, which causes a reduction of the stiffness. Moreover, the stress field is also modified in the vicinity of a crack.
Specific methods allow modelling efficiently the mechanical behaviour of structures containing cracks. The extended finite elements method (XFEM), first introduced by Moes et al. (1999), has received considerable attention over the past few years. It consists of enriching the elements affected by a crack by introducing additional shape functions, which increases the number of degrees of freedom associated with the nodes. The stress field in these elements is then expressed using a combination of the standard and of the enrichment shape functions
In case an element is crossed by a crack, a Heaviside function centred on the crack is introduced as an additional shape function. This step function accounts for the discontinuity of the displacements between the two lips of the crack. In case an element includes the crack tip, the corresponding nodes of the finite element model are enriched with specific shape functions. These functions correspond to the asymptotic displacement field at the vicinity of a crack tip, which can be determined analytically (see Moes, et al., 1999 for more details about the enrichment of the tip elements). This allows capturing efficiently the displacement and strain fields near the crack tip, without excessive refinement of the mesh.
Details on the implementation of the extended finite element method as implemented in the analysis here presented may be found in the work of Zi and Belytschko (2003) and Abdelaziz and Hamouine (2008). However, mesh refinement in the vicinity of the crack tip may be necessary when the extended finite elements method is used, in spite of the enrichment of the nodes at the crack tip (Geniaut, 2011). Nevertheless, the mesh does not have to be compatible with the crack, which considerably simplifies the re-meshing.
In case the behaviour of a cracked structure under dynamic excitation needs to be determined, the stiffness matrix may be computed using the XFEM, as stated above. The mass matrix is not modified by the presence of cracks, and no special action needs to be taken. The problem is subsequently solved using the standard procedure for linear dynamics: the modes and frequency of vibration are determined by solving the eigen-value problem associated with the mass and stiffness matrices; and the FRF associated with any node of the finite element model are determined.

3. Bayesian model updating for crack detection

3.1 Bayesian updating of structural models

A Bayesian model updating procedure is based on the very well known Bayes' theorem (Bayes, 1763). Its general formulation reads

$$P(\theta \mid D, I) = \frac{P(D \mid \theta, I) P(\theta \mid I)}{P(D \mid I)} \tag{1}$$

where θ represents any hypothesis to be tested, e.g., the value of the model parameters, *D* is the available data or observations, and *I* is the background information. Three main terms can be identified in the Bayes' theorem: $P(\theta \mid I)$ is the prior probability density function (PDF) of the parameters θ; $P(\theta \mid D, I)$ is the posterior PDF; and $P(D \mid \theta, I)$ is the likelihood function of the data *D*

Finally, the term $P(D \mid I)$ at the denominator is a normalization factor ensuring that the posterior PDF integrates to 1.The theorem introduces a way to *update* some a-priori knowledge on the parameters θ by using data/observations, conditional to the available information.

Bayes' law has been successfully applied in the updating of structural models (Beck & Katafygiotis, 1998, Katafygiotis & Beck, 1998); in particular the Bayesian structural model updating has been successfully used to update large finite element models using experimental modal data (Goller, et al., 2011).

In a structural model updating framework, the initial knowledge about the unknown adjustable parameters, e.g., from prior expertise, is expressed through the prior PDF. A proper prior distribution can be a uniform distribution in the case when only a lower and upper bound of the parameter is known, or a Gaussian distribution when the mean and a relative error of the parameter is known.

The likelihood function gives a measure of the agreement between the available experimental data and the corresponding numerical model output. Particular care has to be taken in the definition of the likelihood, and the choice of likelihood depends on the type of data available, e.g., whether the data is a scalar or a vector quantity.

Finally, the posterior distribution expresses the updated knowledge about the parameters, providing information on which parameter ranges are more probable based on the initial knowledge and the experimental data.

3.2 Transitional Markov-Chain Monte-Carlo

The Bayesian updating expressed in equation (1) needs the computation of a normalizing factor, that can be very complex to compute and computationally expensive. An efficient stochastic simulation algorithm, called Transitional Markov Chain Monte-Carlo (Ching & Chen, 2007), has been used in this analysis. This algorithm allows the generation of samples from the complex shaped unknown posterior distribution through an iterative approach.

In this algorithm, *m* intermediate distributions P_i are introduced,

$$P_i \propto P(D \mid \theta, I)^{\beta_i} \, P(\theta \mid I) \tag{2}$$

where the contribution of the likelihood is scaled down by an exponent β_i, with $0 = \beta_0 < ... < \beta_i < ... < \beta_m = 1$, thus the first distribution is the prior PDF, and the last is the posterior. The value of these exponents β_i is automatically selected to ensure that the dispersion of the samples at each step meet a prescribed target, see (Ching & Chen, 2007) for additional information on the exponents selection. These intermediate distributions show a more gradual change in the shape from one step to the next when compared with the shape variation from the prior to the posterior. In a first step, samples are generated from the prior PDF using direct Monte-Carlo. Then, sample from the P_{i+1} distribution are generated using Markov chains with the Metropolis-Hasting algorithm (Hastings, 1970), starting from selected samples taken from the P_i distribution, and β_i is updated. This step is repeated until the distribution characterized by $\beta_i = 1$ is reached. By using the Metropolis-Hasting algorithm, samples are generated from the posterior PDF without the necessity of ever computing the normalization constant.

3.3 Model updating for crack detection

The approach introduced so far has been successfully applied to update structural model, thus to reduce the uncertainties in the numerical models and improve the agreement between the numerical results and experimental data (Goller, et al., 2011). Within the model updating framework, the cracks present in the damaged structure are seen as uncertain model properties.

Knowing that cracks will develop most probably in locations characterized by high concentration of stresses, they are inserted in these position in the undamaged model, modelled using XFEM and assuming the crack length and shape as random parameters. In the updating procedure, the prior will use uniform distribution for the crack parameters, allowing the possibility of crack in any stress concentration point and with any possible physically acceptable length.

Experimental data from the reference structure are available in the form of Frequency Response Functions (FRF). These reference data are compared with the numerical FRF of the numerical model, by computing the root mean squared error (RMSE), in order to include the experimental data information in the likelihood. It is assumed that the prediction error is distributed according to a Gaussian PDF, thus the likelihood can be expressed as

$$P(D \mid \theta, I) \propto \exp\left(-\frac{1}{2\delta} \sum_{k=1}^{N_{freq}} \left(h_{\exp}(\omega_k) - h(\omega_k;\theta) \right)^2 \right) \tag{3}$$

where $h_{exp}(\omega_k)$ is the experimental FRF, $h(\omega_k;\theta)$ represents the numerical FRF, and δ is the variance of the RMSE. After the updating procedure, the posterior will provide a qualitative indication on both the crack length and position, i.e., the length of the crack will go towards zero if the crack is not present in the candidate location, and it will concentrate around the unknown length otherwise.

4. Numerical example

In this numerical example, the proposed framework is applied to detect cracks in a suspension arm similar to those used in the automotive industry (Mrzyglod & Zielinski, 2006). The structure, shown in Figure 1, can freely rotate along the axis indicated by the dashed line, and the suspension spring and the wheel structure is connected at the location indicated by “S”. The stress concentration points, and candidate crack locations, are indicated in the figure by the numbers 1 to 6.

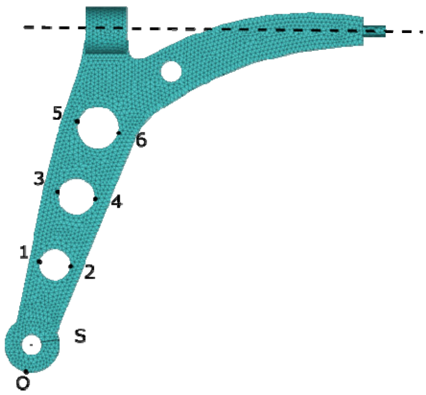

Figure 1: FE model of a suspension arm.

In this example, “simulated” experimental data are generated using a high-fidelity FE model, characterized by a very refined mesh. The software used to construct the model and in the analysis is Code_ASTER (Code_Aster, 2004). A crack with fixed length is inserted in one of the candidate position, and the reference FRF is computed at the position indicated by “O”. Both the FRF in direction X and Y are considered, while no FRF is obtained in the direction Z since the structure is not constrained in that direction. Figure 2 shows the FRF in the two directions X and Y when a crack with the length of 5mm is inserted in the possible positions. A clear difference is shown between the FRF, especially at the high frequencies.

The Bayesian model updating procedure is employed using the low-fidelity FE model, where a more coarse mesh is used. The crack lengths are considered as uncertain parameters, and are modelled using uniformly distributed random variables. Since the crack is physically constrained to not touch the flanges of the arm, a maximum crack length of 5mm is assigned to the cracks in position 1 and 2, while the length is limited to 10mm for the cracks in positions 3 to 6. The sampled values of the random variables are inserted into the FE model by using the ASCII file injection routine provided by COSSAN-X (Patelli, et al., 2012). Additionally, the simulations are run in parallel on a computer cluster allowing a reduction of the overall computational time.

The Bayesian updating procedure has been executed with one crack. The reference FRF has been obtained with a 6mm crack located in position 3. Figure 3 shows the variation of the FRFs of the low-fidelity model with the prior distributions. The posterior distribution shown in Figure 4 converged successfully in mean value towards the correct crack length for the position 3, while it converged towards a

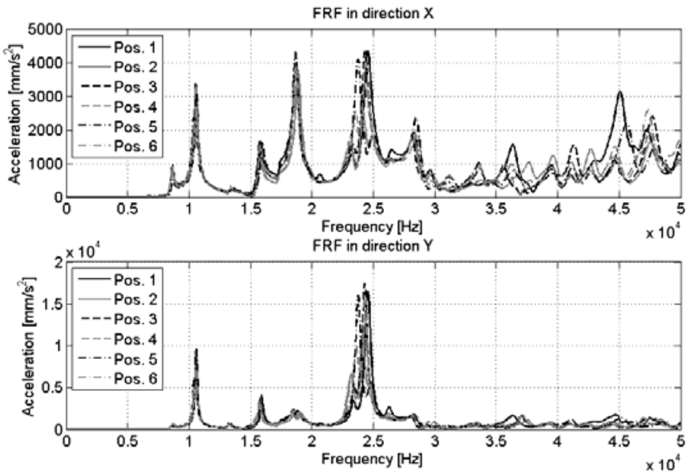

Figure 2: Frequency response functions of the high fidelity FE model. A single crack of 5mm length is inserted in each of the stress concentration points

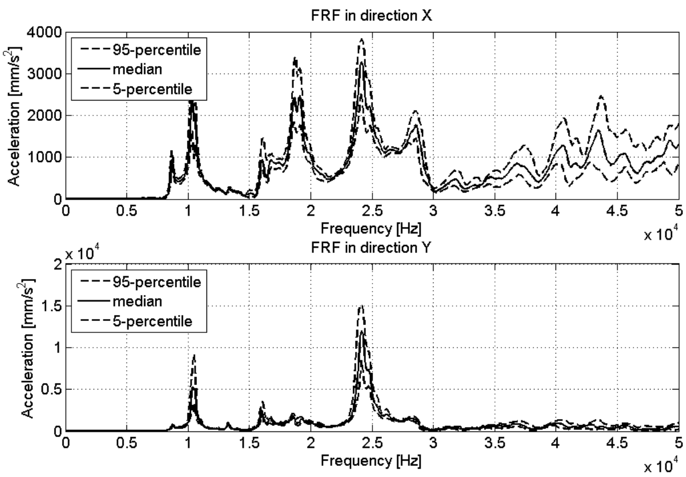

Figure 3: Variation of the FRFs with the crack length sampled from the prior distributions

length of zero for the positions 1, 5 and 6. On the other hand, the posterior samples are not converged around zero for the length parameters of the cracks in position 2 and 4. This remaining uncertainty can be explained by a lack of experimental data, and can be eliminated by measuring the FRFs at additional locations.

5. Concluding remarks

A Bayesian updating procedure has been successfully employed as a computational framework to detect crack location and length. Reference dynamic data from vibration analysis has been used as target for the updating procedure.

To further validate this procedure, the updating will be run with additional experimental cases, with varying position and length of the crack. The results will show the performance of the framework, in particular with different crack length and location. Future development and additional research will be taken by using real experimental data to further validate and expand the proposed approach.

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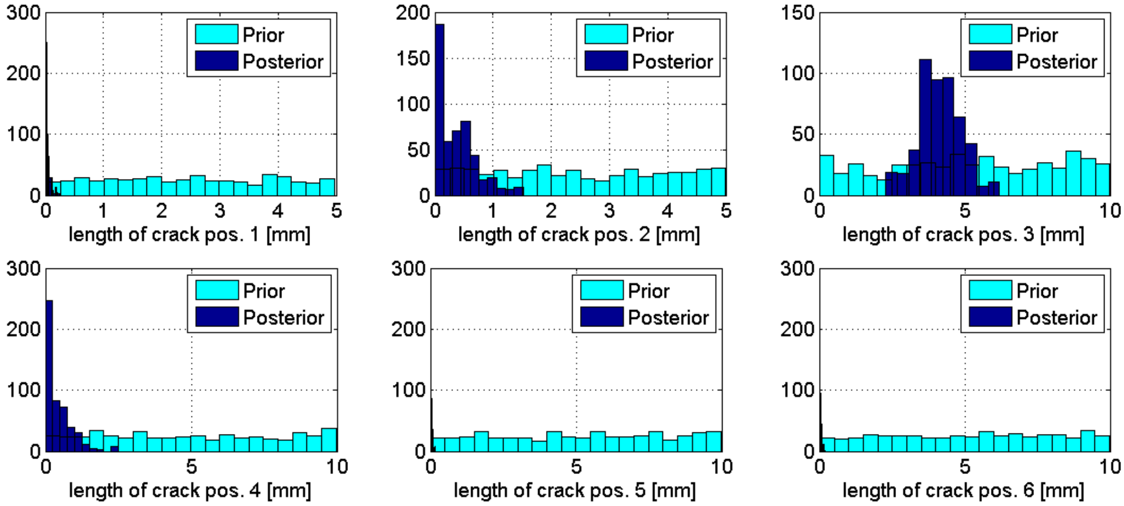

Figure 4: Histogram of the prior and posterior distribution of the crack length parameters

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