

Online Model-Based Redesign of Experiments for Parameter Estimation Applied to Closed-loop Controller Tuning

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We present an approach to closed-loop online model-based redesign of experiments for system identification. Special attention is given to the compliance with safety restrictions and operating requirements during online experiments. For doing so, we propose the integration of a controller into the system identification algorithm. To avoid numerical problems regarding ill-conditioned matrices an algorithm for local parameter identifiability analysis is used. In order to demonstrate the benefits of our approach, the proposed procedure is validated in a real case study. Additionally, a PI-controller is tuned for the identified system. Moreover, the accuracy of the system parameters estimated by the proposed strategy was compared with results for a conventional open-loop step response technique.

1. Introduction

The idea of an optimum experimental design (OED) has been discussed since the 1960s (Mehra, 1974). In this work, we assume that the model structure of the process behavior is defined, whereas the values of the model parameters are unknown and need to be estimated. Thus, the discrimination between alternative models is not considered. According to the conventional concept of OED for parameter determination (Bauer et al. 2000, Franceschini and Macchietto, 2008), the parameter estimation and redesign task are usually computed for open-loop experiments, also referred to as sequential experiment design. Accordingly, the planning, execution and analysis of experiments is realized consecutively, and new information is used only after termination of an experiment. However, since model parameters are unknown, it is difficult or even impossible to execute experiments in compliance with safety restrictions and operating requirements.

In the related field of process control, there are two types of methods for system identification: the open-loop and closed-loop identification methods. The former sets the input variables, whereas the latter manipulates the set-points. The open-loop identification is simple, though experiments are sensitive to disturbances and are not applicable for unstable processes (Rajapandiyam and Chidambaram, 2012). Furthermore, a direct manipulation of design variables (for instance the stepwise change in the reactor temperature) is often impossible or prohibitive on a real plant. In contrast, closed-loop experiments are more robust against disturbances (Rajapandiyam and Chidambaram, 2012). To summarize, the conventional open-loop strategy may lead to undesirable state variable changes and as new information is processed with a possibly large time delay, it may also lead to longer and more expensive experiments.

We propose a closed-loop online model based redesign of experiments (CL-OMBRE) which overcomes the disadvantages of the open-loop method by introducing a controller to the system identification to comply with safety conditions. The controller gain and set-point represent the experiment design variables and are influenced directly.

2. Problem statement

We represent a dynamic model of a given process described by a set of differential algebraic equations (DAE's)

$$\dot{y}(t) = f(t, y(t), p, u(t)), \quad y(t_0) = y_0(p) \quad (1)$$

where $t \in [t_0; t_{Exp, max}]$ represents the time, $y(t) \in \mathbb{R}^{N_y}$ denotes state variables, $p \in \mathbb{R}^{N_p}$ is a set of parameters to be estimated and $u(t) \in \mathbb{R}^{N_u}$ represents the set of time-varying control variables or so-called design variables.

Parameter estimation (PE) is done using standard method based on maximum likelihood criterion:

$$\begin{aligned} \hat{p} &= \arg \min_p \varphi^{PE}(U^-, p) \\ \text{s.t. } \varphi^{PE}(U^-, p) &= (Y^-(U^-, p) - Y^m)^T \cdot \Sigma_y^{-1} \cdot (Y^-(U^-, p) - Y^m) \\ \text{DAE's (Eq. (1))} \end{aligned} \quad (2)$$

where $Y^-(U^-, p) \in \mathbb{R}^{N_y \cdot N_m}$ is the vector of the responses predicted by the model for all discrete time instances $i \in 1, \dots, N_m$, $Y^m \in \mathbb{R}^{N_y \cdot N_m}$ is the vector of the obtained measurement data, $U^- \in \mathbb{R}^{N_u \cdot N_m}$ denotes the piece-wise constant control actions. Finally, $\Sigma_y \in \mathbb{R}^{N_y \cdot N_m \times N_y \cdot N_m}$ represents the measurement-covariance matrix. The result of the parameter estimation is denoted by \hat{p} . Measured data is considered to be a random variable because of random measurement errors. As a result, the solution of the parameter estimation problem is also random. The model-based experimental design strategy (ED) aims to design experimental settings U^+ such that they maximize the accuracy of the parameter estimation. We describe the accuracy of the parameter estimation by the variance-covariance matrix $C \in \mathbb{R}^{N_p \times N_p}$. Here, the optimization problem is formulated as follows:

$$\begin{aligned} U^{+*} &= \min_{U^+} \varphi^{ED}(C(U^-, U^+, \hat{p})) \\ \text{s.t. DAE's (Eq. (1))} \end{aligned} \quad (3)$$

$$Y_{min} \leq Y(U^+, \hat{p}) \leq Y_{max}$$

where $U^+ \in \mathbb{R}^{N_u \cdot h}$ is the piece-wise constant trajectory of future control actions, h represents the length of a receding (future) horizon (see section 3). The functional φ^{ED} characterizes the chosen optimization criterion. Common design criteria are so-called A-, D- and E-optimal criteria whose definitions can be found in (Franceschini and Macchietto, 2008). In the following we use the A-optimal criterion that represents the trace of the variance-covariance matrix and minimizes the mean parameter standard deviations.

$$\varphi^{ED} = \frac{\text{trace}(C)}{\text{dim}(p)} \quad (4)$$

The variance-covariance matrix C is obtained from the inverse of the Fischer information matrix $F \in \mathbb{R}^{N_p \times N_p}$ (Galvanin et al., 2007).

$$C = F^{-1} = F^-(U^-, \hat{p}) + F^+(U^+, \hat{p}) = F_c + F^+(U^+, \hat{p}) \quad (5)$$

In Eq. (5) F_c denotes a constant part of the information matrix which depends on past control actions U^- . Accordingly, in Eq. (5) only vector U^+ is optimized. The calculation of the Fisher matrix is based on sensitivity coefficients $S \in \mathbb{R}^{N_y \cdot N_m \times N_p}$ for each estimated model output

$$F(U, \hat{p}) = S^T(U, \hat{p}) \cdot \Sigma_y^{-1} \cdot S(U, \hat{p}) \quad (6)$$

We apply parameter scaled sensitivities as $S_{ij} = \frac{\partial y_i}{\partial \hat{p}_j} \hat{p}_j$; $\forall i \in 1, \dots, N_y \cdot N_m, j \in 1, \dots, N_p$ (Franceschini and Macchietto, 2008).

Note that operation requirements or safety restrictions have to be considered by the formulation of the ED optimization problem. However, by the formulation of the PE optimization problem you do not need to consider these constraints because measured data already includes these process limitations.

3. Closed-loop online optimal model-based redesign of experiments (CL-OMBRE)

In the proposed strategy (see Figure 1), the system identification is done with the closed-loop control in order to satisfy safety requirements. Note that all experiments are executed only with a P-controller. We

select the controller gain k_c and the set-point y_c^{sp} as experiment design variables $U = [k_c, y_c^{sp}]^T$. The output of the controller (control actions) u_c and the controlled variable y_c are measured. The redesign technique implemented by the CL-OMBRE strategy adopts well known concepts from model predictive control (MPC) (see Camacho and Bordons, 2004). Accordingly, the experiment time $[t_0, t_{Exp,max}]$ is divided into equidistant time intervals with piece-wise constant experiment design variables u_k , with length $\Delta t = t_k - t_{k-1}$ and $k = 1, \dots, N_{Exp}$ being the number of intervals. In each interval k all prior measurements are considered in the parameter estimation problem in Eq. (2). Thus, the number of elements in the vector Y^m increases with the ongoing experimental time. In contrast, the future trajectory of control actions U^+ is computed for a receding horizon of a fixed length h (see section 2) (solution of Eq. (3)).

A major challenge for the experimental application of the CL-OMBRE algorithm is to determine the number of parameters that can be reliably estimated from available measurement data. Generally, those parameters whose sensitivities are low or non-existent are not identifiable. In this case, the sensitivity matrix is singular from a numerical point of view and leads to ill-conditioned PE and ED optimization problems. In order to avoid this problem we apply the Subset Selection technique (SsS) presented by Barz et al., 2012a. The SsS may reduce the parameter set N_p to a subset with dimension r . The set dimension r represents the rank of the sensitivity matrix with linear independent columns of S . In turn, the reduced sensitivity matrix $S^r \in \mathbb{R}^{N_y \cdot N_m \times r}$, with $S^r \subseteq S$, only represents sensitivities of remaining or active parameters \hat{p}^r . In the CL-OMBRE algorithm, first, the initial guess of the model parameters p_0 and an initial experiment design U_0^+ as well as the length of the receding time horizon are defined. At the end of each time interval k we gather measurement data Y_k^{m-} and set design variables U_{k+1}^+ . We also update the current parameter estimate \hat{p}_k (solution of the PE problem) based on available measurements Y_k^{m-} and U_k^- . After that, based on the last results of the parameter estimation the vector of simulated states variables $Y_k^-(U_k^-, \hat{p}_k)$ and the sensitivity matrix $S_k^-|_{p_k=\hat{p}}$ are generated. Next, we determine the reduced sensitivity matrix S_k^r with respect to the active parameters \hat{p}^r computed by SsS. Based on these results, we calculate an optimal action for the next sub-experiment U_{k+1}^{+*} (solution of the ED problem). The algorithm terminates if $t_{Exp,max}$ is reached, the accuracy of the validated parameters is sufficient (e.g. $\varphi_k^{ED} \leq \varphi_{min}^{ED}$) or the realized improvements in the parameter accuracy are smaller than a given threshold ($\varphi_k^{ED} - \varphi_{k-1}^{ED} \leq TOL$).

Note that all computations need to be performed in real-time, within Δt . Hence, we have set an upper limit to the number of iterations performed by the optimizer (see Eq. (2) and Eq. (3)).

4. Case study

The efficiency of the proposed strategy is evaluated in a real case study. We have applied our technique to the identification of a temperature controlled tank. All experiments are performed on the PCS Compact Work Station from Festo Didactic GmbH & Co. KG, Denkendorf, Germany. The process monitoring and control is realized using the ABB Freelance Controller AC 700F with Analog Input/Output Module AX 772F. All numerical computations are implemented in the programming environment Matlab R2010a. The communication is realized using the ABB OPC-Server and the OPC Toolbox from Matlab.

We conducted our experiments by changing the temperature controller settings (controller gain k_c and temperature set-point y_c^{sp}) which in turn impact the temperature in the tank by changing the heater power

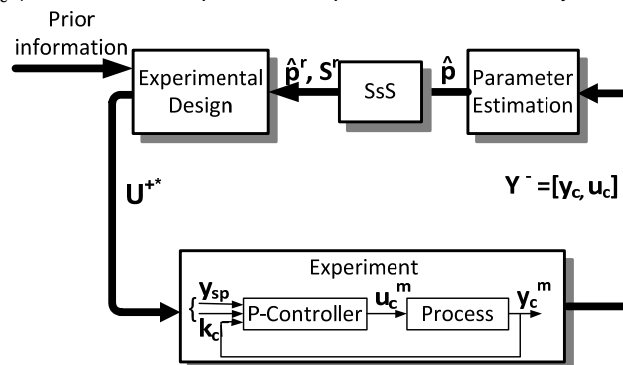


Figure 1: CL-OMBRE strategy

(manipulated control variable u_c). In these experiments we have two measured responses of the process: heater power and the temperature of the tank (controlled variable y_c). The systems dynamic is of first

order with time delay. In order to avoid discontinuities, the dynamic closed loop system is approximated by a DAE of fourth order (see Eq. (7)). Disturbances are neglected.

$$p_5 \cdot \frac{d^4 y_c(t)}{dt^4} + p_4 \cdot \frac{d^3 y_c(t)}{dt^3} + p_3 \cdot \frac{d^2 y_c(t)}{dt^2} + p_2 \cdot \frac{d y_c(t)}{dt} + y_c(t) = p_1 \cdot u_c(t)$$

$$y_c(t) = k_c(t) (y_c^{sp}(t) - y_c(t)) \quad (7)$$

$$0\% \leq u_c(t) \leq 90\%$$

All measurements are taken with a sampling time of 10 s. We define $\Delta t = 100$ s, $t_{exp,max} = 3700$ s. The receding horizon covers the length of three intervals. The initial parameter guess for the model parameters as well as the initial values of the design variables for the receding horizon are set to $p_0 = [2.0 \ 700.0 \ 3.0E4 \ 7.0E5 \ 6.0E6]^T$ and $U_0^+ = [k_{c,0}, y_{c,0}^{sp}]^T$, with $k_{c,0} = [3.0 \ 5.0 \ 15.0]^T$ and $y_{c,0}^{sp} = [24.0 \ 29.0 \ 25.0]^T$ °C, respectively. All computations were performed on a 32 bit Linux platform with an Intel® Core™ i7, 2.20 GHz and 2.6 GB RAM. The PE and ED optimization problem were solved with Matlab using its Optimization Toolbox solver *lsqnonlin* (trust-region-reflective) and *fmincon* (sqp), respectively. For the DAE system presented in Eq. (1) we used the sDACI solver which is a sparse DAE solver based on the orthogonal collocation on finite elements method. For more details see Barz et al, 2011; Barz et al, 2012b.

4.1 System identification with CL-OMBRE

The A-optimal criterion is used in the experimental design problem formulation. Though, in contrast to the algorithm described in section 3, the only termination criterion which was used is the total experimental duration.

First, an offline open-loop system identification based on step responses was conducted. Here, six experiments were performed with a total of six random step-wise changes in the heater power (controller output). Then, the model parameters (see Eq. (7)) were estimated based on the gathered measurements. The results for offline open-loop experiments are given in Table 1. Second, a so-called conventional ED based on step-wise changes of U was realized (the problem in Eq. (3) was not solved). The design variables U_k were randomly generated with an uniform distribution. The results are shown in Table 1. It was not possible to identify all model parameters using this conventional ED. The number of identifiable parameters selected by the SsS algorithm is four out of five. Third, a closed-loop optimum experiment was conducted. Here, both, PE and ED optimization problems as well as the SsS problem were solved online.

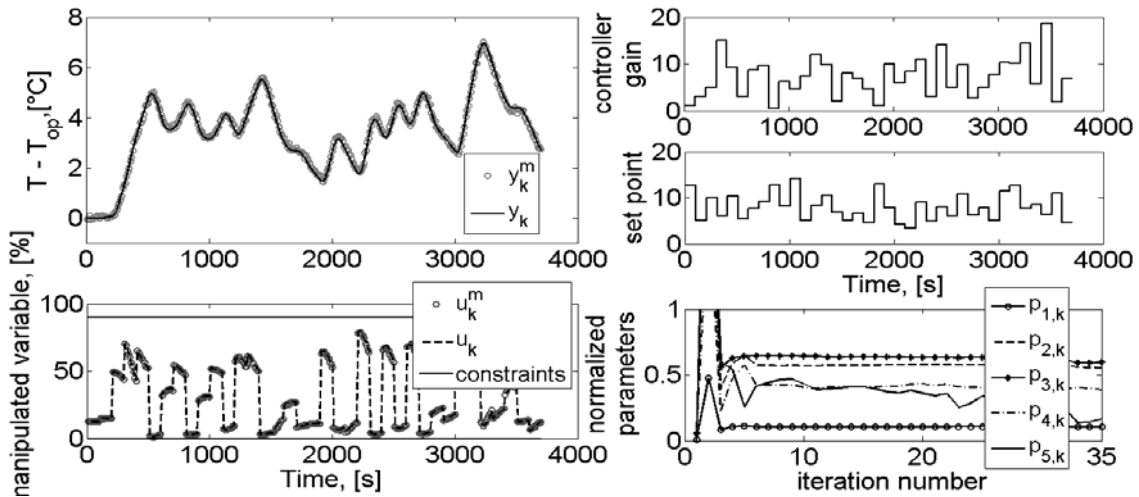


Figure 2: Results of the experiment with A-optimal design. Here T_{op} represents the operating point

The results of this experiment are presented in Figure 2 and Table 1. This experiment took the same time as the second one (same mode of operation, in both cases we used a controller). In contrast to the offline open-loop experiments, there is no need to wait for a steady state. Therefore, the closed-loop experiments are more time-efficient especially in online mode. Finally, in order to verify the accuracy of the estimated model parameters obtained from the different identification methods discussed above, a reference closed-loop experiment was executed, with random step-wise changes of U (same conditions as in the online closed-loop (conventional) ED). Here, both, PE and ED optimization problems were not solved and only

measured data was collected. The duration of this experiment was 1.03 h. We were then able to verify the accuracy of the previous three models based on how good they were able to predict the outcome of the fourth experiment. The results are shown in Figure 3, where y_{op} denotes the operating point of the process. The mean residual calculated as a weighted L_1 norm $1/N_m \cdot \sum_{i=1}^{N_m} \|y_i^m - y_i\|_1$ are presented in Table 1. All parameter values in Table 1 are normalized by their respective initial guess taken from section 4, with $\hat{p} = \hat{p}/p_0$. Symbol () denotes non-active (non-identifiable) parameters. It can be seen, that for the open-loop as well as the unplanned closed-loop experiment only a subset of the parameter space is identifiable (parameter p_5 was not identified), whereas, using optimum closed-loop ED, all parameters could be identified. Moreover, compared to the conventional design the accuracy of the model parameters was improved and the necessary experiment time was also reduced by a factor of three.

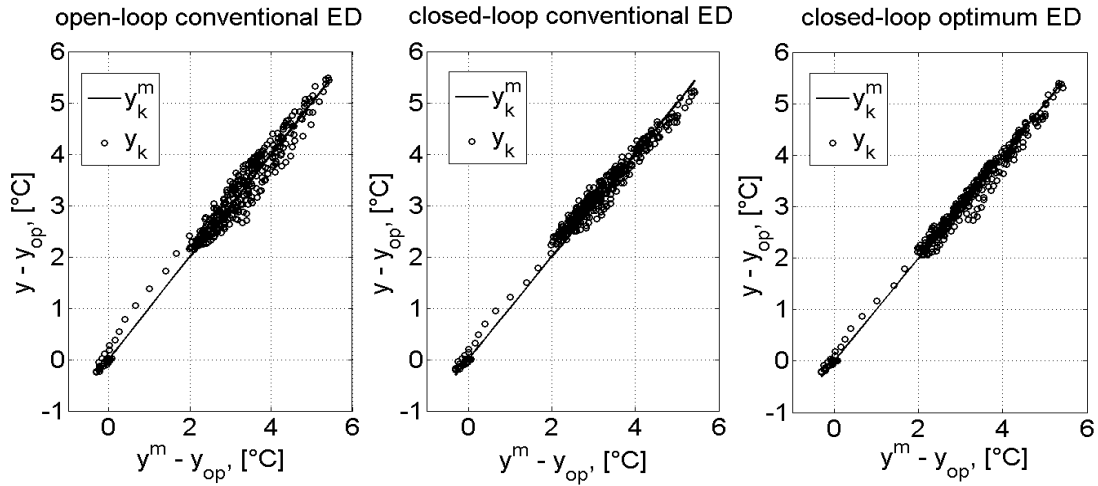


Figure 3: Model prediction versus measurement for the estimated parameter sets

Table 1: Normalized parameters and $t_{Exp,max}$ obtained from closed-loop and open-loop experiments and mean residual of the fourth experiment

Variables	\hat{p}_1	\hat{p}_2	\hat{p}_3	\hat{p}_4	\hat{p}_5	$t_{Exp,max}, [h]$	Mean residual
Offline open-loop (conventional ED)	0.10	0.58	0.53	0.59	(0.0034)	3.1	1.7541
Online closed-loop (conventional ED)	0.12	0.71	0.60	0.28	(0.00)	1.03	1.5589
Online closed-loop (optimum ED)	0.10	0.55	0.60	0.39	0.17	1.03	1.0109

4.2 Application of the CL-OMBRE strategy to tune a PI-controller

In this section we compare the CL-OMBRE technique with the conventional method by using the identified model to tune a PI-controller (controller parameter k_c, T_i). For this purpose we calculated the controller parameters by minimizing the integrated absolute error (IAE) constrained by a maximum overshoot of 2 %.

$$\min_{k_c, T_i} \int_0^{\infty} |E(t)| dt \quad \text{with } E(t) = y_c(t) - y_c^{sp}(t) \quad (8)$$

$$\frac{y_c(t) - y_c^{sp}(t)}{y_c^{sp}(t)} \leq 0.02$$

Results of the controller tuning based on model obtained from open-loop and closed-loop experiments are presented in Figure 4. The experimental data of the controller performance confirms that the model obtained from the CL-OMBRE technique describes the process behavior sufficiently and we were able to tune a controller which is able to comply with operating requirements (overshooting below 2 %). In contrast, the model obtained from open-loop experiments shows poor conformity with process behavior. Moreover, the overshooting exceeded operating requirements with a rate of 5 %.

5. Discussion

The CL-OMBRE technique has been presented, which allows online system identification through closed-loop experiments. The technique has been validated experimentally for a temperature control system and

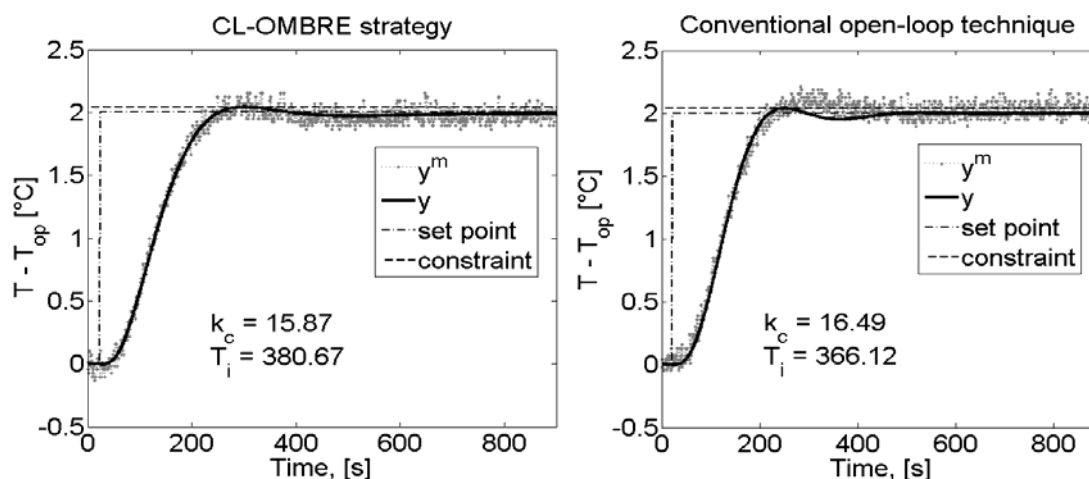


Figure 4: Step responses using PI-settings obtained through the CL-OMBRE and open-loop strategy.

has proved to be more efficient than a conventional open-loop method. The accuracy of the model parameters is also improved. In the CL-OMBRE method, design variables are represented by a set-point and a controller gain, all design variables are directly adjustable. The P-controller used in the CL-OMBRE strategy acts as a watchdog to ensure that the defined restrictions are kept during the whole experiment. Finally, the proposed procedure allows us to significantly reduce the experimental effort in comparison to the conventional method (for the presented application the proposed technique is three times faster in contrast to the open-loop method) and in turn decrease experimental costs. For the presented case study, it could be shown that the closed loop system was stable and safety restrictions were kept during online identification. However, the stability of the closed-loop system was not enforced directly. Generally, the controller gain k_c is responsible for keeping the stability of the closed-loop system and the changes in the set-point y_c^{sp} for compliance with safety restrictions. In both cases, stability requirements and safety restrictions need to be considered as constraints in the formulation of the ED problem. Thus the CL-OMBRE method needs an extension to consider stability conditions in the ED problem formulation, e.g. all the roots of the characteristic equation for the closed loop system dynamics must have negative real parts.

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