CHEMICAL ENGINEERING TRANSACTIONS

VOL. 32, 2013

Chief Editors: Sauro Pierucci, Jiří J. Klemeš

ISBN 978-88-95608-23-5; **ISSN** 1974-9791

A publication of

AIDIC

The Italian Association of Chemical Engineering www.aidic.it/cet

DOI: 10.3303/CET1332095

Convection During the Fluid Heating by Laser Emission

Sergey P. Karlov, Dmitry A. Kazenin, Andrey V. Vyazmin*, Olga V. Pirogova, Kseniya A. Okhotnikova

Department of Chemical Engineering, Moscow State University of Mechanical Engineering, Staraya Basmannaya str. 21/4, Moscow, 107066, Russian Federation

*- av1958@list.ru

In this work hydrodynamics flows with surface tension forces appearing due to fluid heating by a laser emission are considered.

Several unsteady hydrodynamics model of such flows near the heating spot characterized by various evolution stages of the process are considered. For this model an axisymmetric flow approximation is used. The first class of models of unsteady flow with a spreading of the liquid layer describes the initial process phase. A structure of flows, where under surface tension forces the spreading of the layer on the substrate occurs with an unsteady velocity profile, is considered. The second class of models for long times takes into account the appearance of unsteady pressure gradients, which prevent the spreading. It is demonstrated that the pressure gradient rise and their growth lead to a generation of unsteady circulation flow near to the heating axis.

New experimental data showing the flow structure near the heating spot are reported. Such flow reveals a complex system of convection streams. A toroidal vortex structure appears near to the heating spot, where convective cells join to this toroidal vortex structure from outside.

1. Introduction

The interfacial instability generated near the gas-liquid or liquid-liquid interface results in the appearance of convective flows which increase significantly mass and heat transfer across the interface (Kaminsky et al., 1998). In addition, the convective flows may be considered as an example of dissipative structures in open systems.

Hydrodynamical flows under the influence of surface tension arising from superficial flow heating are considered experimentally and theoretically. The dependency of the surface tension coefficient on the temperature leads to the appearance of so-called capillary forces (Marangoni effect), which can completely determine flow characteristics in the case of low gravitation (see for example Polyanin et al., 2002).

Most theoretical approaches to this problem have considered either stationary solutions or weakly nonlinear disturbances to an unstable solution (Golovin et al., 1995). Most data on the structure of the convection flow is currently obtained from experimental investigations (see for example Karlov et al., 2009). The main achievements in the experimental study of interfacial convection are related to the application of optic methods of the study of liquid surfaces (see for example Kutepov et al., 2001). Using these methods, one is able not only to observe directly the sequence of formation of various instable regimes but also to estimate the characteristic scales of instability (Karlov et al., 2002).

Theoretical approaches for the description of unsteady convective flows are insufficient today. Exact solutions to the Stokes equations play an important role in forming the correct knowledge of the qualitative character of a flow and in certain limiting cases make it possible to calculate the generalized integral characteristics of a flow. In addition, these solutions serve as a starting point for constructing the models of transport processes in fluids based on an exact solution to convective mass and heat transfer equations (see Bird et al., 1960; Ganduli and Kenig, 2011).

As an example, we consider the pattern of a flow under the effect of surface forces that arise due to elliptic heating of a fluid by radiation. A similar problem for a round heating spot was considered by Aristov et al., (2009).

Please cite this article as: Karlov S., Kazenin D., Vyazmin A., Pirogova O., Okhotnikova K., 2013, Convection during the fluid heating by laser emission, Chemical Engineering Transactions, 32, 565-570 DOI: 10.3303/CET1332095

2. Heat problem solution

Let an optically partially transparent fluid layer with the initial thickness h_0 be on an absolutely transparent support that coincides with the plane *x*, *y* (the thickness of the layer is measured upwards along the coordinate *z*). The layer is illuminated from above by laser radiation with an ellipse-shaped spot with the *a* and *b* semiaxes along the *x* and *y* axes, respectively (the origin of coordinates coincides with the ellipse center). Radiation passes into a fluid and is uniformly absorbed; in this case, volumetric heat release can be assumed to be constant throughout (due to a small layer thickness) and equal to *q*, and there is no heating of the support due to its optical transparency. An arising flow is assumed to be slow; therefore, convective heat transfer inside the heating region can be neglected as a first approximation. Then, the temperature of the fluid *T* inside the heating zone is described by the Poisson equation

$$\Delta T = -q, \tag{1}$$

and, outside the zone, it is assumed be zero (the temperature is reckoned from ambient temperature).
The solution to heat problem (1) has the form

$$T = \frac{q}{2}\frac{a^2 b^2}{a^2+b^2}\left[1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2\right]. \tag{2}$$

This distribution of temperature occurs both in the bulk and on the surface of the fluid. We assume that, in the case under consideration, surface tension σ linearly depends on temperature on the surface of the fluid layer $\sigma = \sigma_0 - \alpha T$; then, the stress vector that acts on the surface of the fluid due to surface tension forces has the components

$$\nabla\sigma = \left(\frac{\partial\sigma}{\partial x}, \frac{\partial\sigma}{\partial y}, 0\right) = \left(\alpha q\frac{b^2}{a^2+b^2}x, \alpha q\frac{a^2}{a^2+b^2}y, 0\right). \tag{3}$$

Let us consider several examples of solving problems for these flows under the effect of capillary forces.

3. Axial flows

In the rectangular Cartesian coordinate system *x*, *y*, and *z*, a system of nonstationary Stokes equations is written as

$$\begin{aligned}
&\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right),\\
&\frac{\partial \mathrm{v}}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 \mathrm{v}}{\partial x^2}+\frac{\partial^2 \mathrm{v}}{\partial y^2}+\frac{\partial^2 \mathrm{v}}{\partial z^2}\right),\\
&\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right),\\
&\frac{\partial u}{\partial x}+\frac{\partial \mathrm{v}}{\partial y}+\frac{\partial w}{\partial z} = 0,
\end{aligned} \tag{4}$$

where *u*, v, and *w* are the fluid velocity vector components; *t* is time; *p* is pressure; ρ is the density of the fluid; and ν is the kinematic viscosity.
We consider the axial flow of a viscous incompressible fluid when the flow velocity vector on the *z* axis is directed along this axis. Near the *z* axis, the transverse components of the velocity vector are close to zero (tend to zero when the transverse coordinates tend to zero) and they can be expanded into a Taylor series in terms of the transverse coordinates *x* and *y*. If we restrict ourselves to the main terms of expansion in *x* and *y* in the velocity components, representation

$$u = f_1(z,t)x + f_2(z,t)y,$$
$$\mathrm{v} = g_1(z,t)x + g_2(z,t)y,$$
$$w = w(z,t), \tag{5}$$
$$p = p_0(z,t) + \frac{1}{2}p_{11}(t)x^2 + p_{12}(t)xy + \frac{1}{2}p_{22}(t)y^2,$$

can be derived.
Substituting Eqs. (5) into (4) yields polynomials of the first degree with respect to *x* and *y*. Equating the coefficients of these polynomials to zero, we obtain the following equations for the sought functions $f_n = f_n(z, t)$ and $g_n = g_n(z, t)$:

$$\frac{\partial f_1}{\partial t} = \nu\frac{\partial^2 f_1}{\partial z^2} - \frac{1}{\rho}p_{11}(t), \qquad \frac{\partial g_1}{\partial t} = \nu\frac{\partial^2 g_1}{\partial z^2} - \frac{1}{\rho}p_{12}(t),$$
$$\frac{\partial f_2}{\partial t} = \nu\frac{\partial^2 f_2}{\partial z^2} - \frac{1}{\rho}p_{12}(t), \qquad \frac{\partial g_2}{\partial t} = \nu\frac{\partial^2 g_2}{\partial z^2} - \frac{1}{\rho}p_{22}(t). \tag{6}$$

The remaining functions $w = w(z, t)$ and $p_0 = p_0(z, t)$ are determined from the formulas

$$w = -\int (f_1 + g_2)dz + w_0(t), \quad p_0 = \rho\int\left(\nu\frac{\partial^2 w}{\partial z^2} - \frac{\partial w}{\partial t}\right)dz, \tag{7}$$

where $w_0(t)$ is an arbitrary function of time.
Any flows that have two planes of symmetry are described by set of equations (6) in the neighborhood of the intersection line of these planes (that coincides with the *z* axis). Thus, solution (5) to set of equations (4) describes non-steady-state two-dimensional flows symmetric about a straight line; non-steady-state axisymmetric flows and their combinations with rotation about the *z* axis; non-steady-state flows in rectilinear impermeable and porous tubes with circular, square, elliptical, and rectangular cross sections; non-steady-state laminar fluid jets that issue from nozzles; etc.

4. Steady-state flow with fluid layer spread (problem 1)

We consider a flow when layer spreading over a support with a steady-state velocity profile occurs under the effect of surface stresses. It is assumed (Aristov et al., 2009) that, when this method of flow initiation is used, the radial pressure gradient that prevents spreading does not arise; therefore, the fluid velocity vector components in *x*, *y*, and *z* are specified by relations (5) at $f_2 = g_1 = p_{11} = p_{12} = p_{22} = 0$ and described by stationary Eqs (6) and (7) for $f_1(z)$, $g_2(z)$, $w(z)$, and p_0. The no-slip conditions are set on a solid support at the bottom of the layer (at $z = 0$) as follows: $f_1(0) = g_2(0) = w(0) = 0$. We impose the following stress balance conditions in the presence of surface forces on the free boundary (at $z = h(t)$):

$$-p_0 - 2\mu\left.\frac{dw}{dz}\right|_{z=h} = P_g, \quad \mu\left.\frac{df_1}{dz}\right|_{z=h} = \alpha q\frac{b^2}{a^2+b^2}, \quad \mu\left.\frac{dg_2}{dz}\right|_{z=h} = \alpha q\frac{a^2}{a^2+b^2}, \tag{8}$$

where μ is the dynamic viscosity of the fluid and P_g is the given external pressure, which is determined to within an arbitrary constant.
Based on this formulation of a hydrodynamic problem, the velocity vector components are specified by the relations

$$u = \frac{\alpha q}{\mu}\frac{b^2}{a^2+b^2}xz, \quad \mathrm{v} = \frac{\alpha q}{\mu}\frac{a^2}{a^2+b^2}yz, \quad w = -\frac{\alpha q}{2\mu}z^2. \tag{9}$$

The dependence of the variations in the thickness of the fluid layer on time is determined from the velocity of the upper bound of the layer as follows:

$$\frac{d\,h}{d\,t} = w\big|_{z=h(t)},\tag{10}$$

Based on (9) and (10), we derive the equation for determining the dependence $h(t)$ as follows:

$$\frac{d\,h}{d\,t} = -\frac{\alpha q}{2\mu} h^2.\tag{11}$$

Taking into account that the initial thickness of the fluid layer is h_0, based on Eq. (11), we have the following relationship:

$$h(t) = \frac{2\mu\, h_0}{2\mu + \alpha q h_0 t},\tag{12}$$

which is identical to the result of Aristov et al. (2009); i.e., the rate of layer spreading does not depend on the shape of a heating spot.

However, it should be noted that the condition for the balance of normal stresses on the free surface (the first relation in (8)) is only valid at a thickness of the fluid layer that is close to the initial thickness. With a decrease in the thickness of the layer, the condition is violated, since there is a constant value in the right-hand side of the first relation in (8) and the left-hand side decreases with time. Thus, relationships (9) and (12) are only valid at small spreading times.

The balance of normal stresses at the free boundary is maintained when pressures p_{11} and p_{22} (the last relation in (5)), which prevent fluid spreading, appear and increase with time. Probably, a steady-state flow with a constant fluid layer thickness and constant radial pressure gradients can originate at large times. In this case, the turn of a fluid that moves in the radial direction occurs at infinity and motion is no longer described by relations (5) in the entire flow region.

5. Steady-state flow in a layer with a constant thickness (problem 2)

We consider a steady-state axial flow in a fluid layer with a constant thickness that arises under the effect of surface tension gradient (3). Near the heating spot, the fluid velocity vector components in x, y, and z are set by relations (5) at $f_2 = g_1 = p_{12} = 0$ and described by stationary equations (6) and (7) for $f_1(z)$, $g_2(z)$, $w(z)$, and p_0 at constant values of p_{11} and p_{22}. On a solid support at the bottom of the layer (at $z = 0$), the no-slip conditions are set; at the free boundary (at $z = h$), we impose the stress balance conditions in the presence of surface forces (8).

Solving Eqs. (6) for $f_1(z)$ and $g_2(z)$ and taking into account the above boundary conditions, we obtain the following relations for the radial components of the velocity vector:

$$u = \frac{p_{11}}{2\mu} xz(z-2h) + \frac{\alpha q}{\mu}\frac{b^2}{a^2+b^2} xz, \quad \mathrm{v} = \frac{p_{22}}{2\mu} yz(z-2h) + \frac{\alpha q}{\mu}\frac{a^2}{a^2+b^2} yz.\tag{13}$$

It can be seen from (13) that, when there are no pressures p_{11} and p_{22} that prevent radial spreading, expressions for the radial components of the velocity vector are identical to similar expressions from (9). The steady-state values of these quantities (when p_{11} and p_{22} are constant) can be found from the conditions of the zero balance of the fluid flow rate in the layer through any plane (x = const and y = const), which have the form

$$\int_0^h u\,dz = \int_0^h f_1\,dz = 0, \qquad \int_0^h \mathrm{v}\,dz = \int_0^h g_2\,dz = 0.\tag{14}$$

It follows from Eqs. (13) and (14) that

$$p_{11} = \frac{3}{2}\frac{\alpha q}{h}\frac{b^2}{a^2+b^2}, \quad p_{22} = \frac{3}{2}\frac{\alpha q}{h}\frac{a^2}{a^2+b^2},\tag{15}$$

Substituting Eq. (15) into (13), using (7) to calculate w, and taking into account that the axial velocity component takes on the zero value on the support (at $z = 0$), we derive the following relationships:

$$u = \frac{\alpha q}{\mu} \frac{b^2}{a^2 + b^2} xz \left[1 + \frac{3}{4h}(z - 2h) \right], \quad \mathrm{v} = \frac{\alpha q}{\mu} \frac{a^2}{a^2 + b^2} yz \left[1 + \frac{3}{4h}(z - 2h) \right],$$
$$w = -\frac{\alpha q}{4\mu h} z^2 (z - h). \tag{16}$$

It follows from Eqs. (16) that the axial velocity component *w* also takes on the zero value on the layer surface *h*. This means that the fluid layer does not change its thickness (nonstationarity associated with a decrease in the thickness of the layer is absent). The thickness of the layer can be determined from the first condition in (8). Steady-state circulation motion arises in the layer, and the fluid flow in the radial direction changes its direction at $z = (2/3)h$.

6. Non-steady-state flows (problem 3)

We consider a problem similar to problem 1; however, we take into account the possibility that a non-steady-state velocity profile will arise at small times from the beginning of motion; i.e., until constant profile (9) is formed. We assume that temperature distribution corresponds to Eq. (2) and generates surface stresses (3). The slow convective flow that arises does not change the temperature profile, and the pressure that prevents the radial spreading of fluid is absent.
We assume that the components of the fluid velocity vector in *x*, *y*, and *z* are set by relations (5) at $f_2 = g_1 = p_{11} = p_{12} = p_{22} = 0$ and described by nonstationary equations (6) and (7) for $f_1(z, t)$, $g_2(z, t)$, $w(z, t)$ and p_0. The initial condition (at $t = 0$) is as follows: the fluid is at rest and $f_1 = g_2 = w = 0$. The stress balance conditions in the presence of surface forces (8) are satisfied at the free boundary (at $z = h(t)$). At small times $t << h^2/(4\nu)$, the real no-slip conditions on a support are replaced by the following model conditions: at $z \to -\infty$: $f_1 \to 0$, $g_2 \to 0$, and $w \to 0$.
The formulated problem is identical (to within notation) to the first modified Stokes problem (Polyanin et al., 2002). For the radial velocity components, we derive the relationships

$$u = \frac{2\alpha q}{\mu} \frac{b^2}{a^2 + b^2} x \sqrt{\frac{\nu t}{\pi}} \left[\sqrt{\pi}\xi \operatorname{erfc}\xi + \exp\left(-\xi^2\right) \right],$$
$$\mathrm{v} = \frac{2\alpha q}{\mu} \frac{a^2}{a^2 + b^2} y \sqrt{\frac{\nu t}{\pi}} \left[\sqrt{\pi}\xi \operatorname{erfc}\xi + \exp\left(-\xi^2\right) \right], \qquad \xi = \frac{h - z}{2\sqrt{\nu t}}. \tag{17}$$

The axial velocity component *w* and pressure p_0 are found using relations (7). Since the expression between square brackets in formulas (17) is equal to unity at $z = h$, the velocity of the fluid on the surface at the beginning of the process increases with time according to the square root law.

7. Experimental results

In order to validate mathematical models for the convection observed at the surface of a liquid heated by a laser an experimental visualization of the flow is performed. The experiment is carried out in a glass cell of dimension 20 x 18 mm, filled with distilled water. Radiation is produced using a standard industrial CO_2 laser with a wavelength of 10.6 nm and a power of 5.5 W. Focusing of the laser beam using a mirror produces alight spot of elliptical shape with axes 1.5 x 0.5 mm on the liquid surface. The wavelength of the laser is chosen under the constraint of complete absorption of the radiation energy by the boundary layers of the liquid.
Visualization of the convection flow is achieved using small particles of aluminum powder with an average particle size of 0.07 mm. The movement of the particles at the fluid surface completely corresponds to the directions of the surface flow, which is recorded by a camera.
The experimental video shows that at all stages of the development of unsteady convection, the spreading of the liquid over the surface and the reduction of the thickness of the liquid layer are not detected. A local change in the height of liquid near the spot is also not fixed. Hence we conclude that the quasi-steady regime of the liquid spreading on the surface does not exist under realistic conditions.

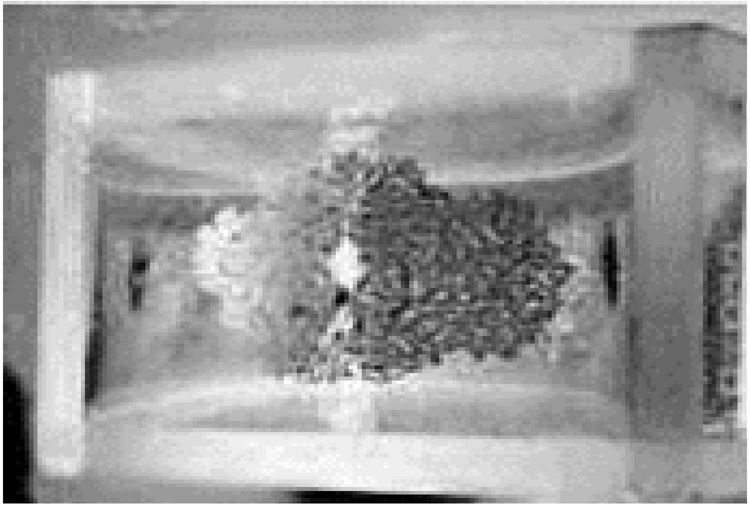

Figure 1: Visualization of the fluid flow near the heating laser spot (steady regime).

In the early stages of the heating by the laser, particles start to move from the center of the spot in the radial direction. As a result, close to the heating spot the flow (without particles) appears. The size of such «free» space increases with time and equilibrates at a steady value equal to 1.5 - 2.5 times the size of the spot. Outside of this «free» zone particles show complex circular motion on the fluid surface (Figure 1).
This result can be considered as a qualitative proof of the validity of the second and third theoretical models (see above). The presence of the zone «free» of particles close to the heating spot can be interpreted as a steady toroidal convective vortex structure near the flow axis, when the motion on the fluid surface occurs from the axis towards the periphery. Complex convection structures far from the heating spot show the limitation of the applicability of the model.

8. Conclusions

It is shown that exact solutions of the nonstationary problem of convection during the fluid heating by laser emission are derived with the linear dependence of the velocity components on two space variables of a nonstationary three-dimensional system of the Stokes equations that describes axial flows.
The exact solutions of three different initial-value problems are obtained. The unsteady regime of developing convection is found experimentally near the flow axis (heating spot). This regime evolves towards the steady toroidal convective vortex structure close to the flow axis. No quasi-steady regime with the surface flow spreading is observed.

References

Aristov S.N., Knyazev D.V., Polyanin, A.D., 2009, Exact solutions of the Navier–Stokes equations with the linear dependence of velocity components on two space variables, Theoretical Foundation Chemical Engineering, 43, 642-661.

Bird R.B., Stewart W.E., Lightfoot E.N., 1960, Transport Phenomena, Wiley, New York, USA.

Ganguli A., Kenig E., 2011, Prediction of interfacial mass transfer in liquid-liquid systems with moving interfaces, Chemical Engineering Transactions, 24, 1357-1362, DOI: 10.3303/CET1124227

Golovin A.A., Nepomnyashchy A.A., Pismen L.M., 1995, Pattern formation in large-scale Marangoni convection with deformable interface, Physica D, 81, 117-147.

Kaminsky V.A., Vyazmin A.V., Kulov N.N., Dilman V.V., 1998, Marangoni effect in the presence of bulk turbulence, Chemical Engineering Science, 53, 3347-3353.

Karlov S.P., Kazenin D.A., Vyazmin A.V., 2002, The time evolution of chemo-gravitational convection on a brim meniscus of wetting, Physics. A, 315, 236-242.

Karlov S.P., Kazenin D.A., Vyazmin A.V., 2009, Interfacial instability on bubble surface, Chemical Engineering Transactions, 17, 495-500.

Kutepov A.M., Pokusaev B.G., Kazenin D.A., Karlov S.P., Vyazmin A.V., 2001, Interfacial mass transfer in the liquid-gas system: An optical study, Theoretical Foundation Chemical Engineering, 35, 213-216.

Polyanin A.D., Kutepov A.M., Vyazmin A.V., Kazenin D.A., 2002, Hydrodynamics, Mass and Heat Transfer in Chemical Engineering. Taylor & Francis, London, UK.