CHEMICAL ENGINEERING TRANSACTIONS

VOL. 31, 2013

Guest Editors: Eddy De Rademaeker, Bruno Fabiano, Simberto Senni Buratti

ISBN 978-88-95608-22-8; **ISSN** 1974-9791

A publication of AIDIC

The Italian Association of Chemical Engineering
Online at: www.aidic.it/cet

DOI: 10.3303/CET1331067

Optimal allocation of safety and security resources

Genserik L.L. Reniers*[a,b], Kenneth Sörensen[a]

[a]Universiteit Antwerpen, Antwerp Research Group on Safety and Security (ARGoSS), Prinsstraat 13, 2000 Antwerp, Belgium
[b]Hogeschool-Universiteit Brussel, KULeuven, Research Group CEDON, Stormstraat 2, 1000 Brussels, Belgium
genserik.reniers@ua.ac.be

A model was developed to execute a cost-efficiency analysis for taking prevention investment decisions. Using the knapsack problem solving technique well-known in the field of operational research, the approach suggests how to prioritize bundles of prevention measures based on their costs and benefits within a predefined prevention budget. This way, an efficient allocation of safety and security measures is achieved using the suggested methodology.

1. Introduction

It is essential for companies to be able to reduce and control the wide variety of existing occupational risks in a cost-efficient way. Such risks are reduced through safety management, risk reduction policies and prevention measures. To determine an optimal set of prevention measures, organizations need to take into account in a systematic way the measures' costs as well as their (hypothetical) benefits. Prevention benefits can be calculated by determining the difference between hypothetical accident costs before and after implementing precaution measures. In other words, the *avoided accident costs* are calculated, or the financial benefits of *accidents not happening*. The model builds on these insights.
A concept well-known in organizations throughout the world for ranking risks is the risk matrix, allowing to make a classification of risks in a systematic and transparent way (CCPS, 1992; Middleton, 2001; Cook, 2007; Wilkinson and David, 2008; Cox, 2008 and 2011; Smith *et al.*, 2009). This method can be used to measure and categorize risks on an informed judgment basis as to both likelihood and consequence and as to a relative level of importance. Different possible ways to use and refine the risk matrix are possible, as Garvey (2009) explains.
Despite the fact that the risk matrix merely allows to roughly assess risks, many decision takers and consultants are convinced that the method is very useful to make a qualitative distinction between different levels of risks. The qualitative difference between risks which is based on the risk matrix is preferable over ad random decision taking (Cox, 2008). A risk matrix which is divided into four consequence grades and five likelihood grades, can for example be employed. Consequence grades can be expressed in financial terms, while likelihood grades are expressed in the number of times per year that a risk leads to an accident in an organization.
Table 1 illustrates the risk matrix used in the remaining of this article. Every cell of the risk matrix corresponds to a risk class. Per cell (thus per risk class), the financial consequence value is multiplied by the likelihood value, and the total yearly costs per risk class are determined and shown.

Please cite this article as: Reniers G. and Sorensen K., 2013, Optimal allocation of safety and security resources, Chemical Engineering Transactions, 31, 397-402 DOI: 10.3303/CET1331067

Table 1: Risk matrix used in this article

Likelihood [$year^{-1}$]	Cell assignments (in €/year)			
> 1	7,500	75,000	750,000	2,500,000
> 10^{-1}	750	7,500	75,000	250,000
> 10^{-2}	75	750	7,500	25,000
> 10^{-3}	7.5	75	750	2,500
> 10^{-4}	0.75	7.5	75	250
Consequence classes / financial impact [€] →	< 7,500	75,000	750,000	< 2,500,000

Cox (2008) indicates that a certain risk in each of the cells of any risk matrix is not equally large (or small) due to the classification into risk classes. Hence, a risk cell may contain different varieties of risks. This does not pose a problem in our research, since to decide on bundles of prevention measures, we aim at comparing bundles of risks, not individual risks.

A discretization of the risk matrix into *n* cells is illustrated in Figure 1. Every risk cell is numbered from *1* to *n* (in our example*, n = 20*).

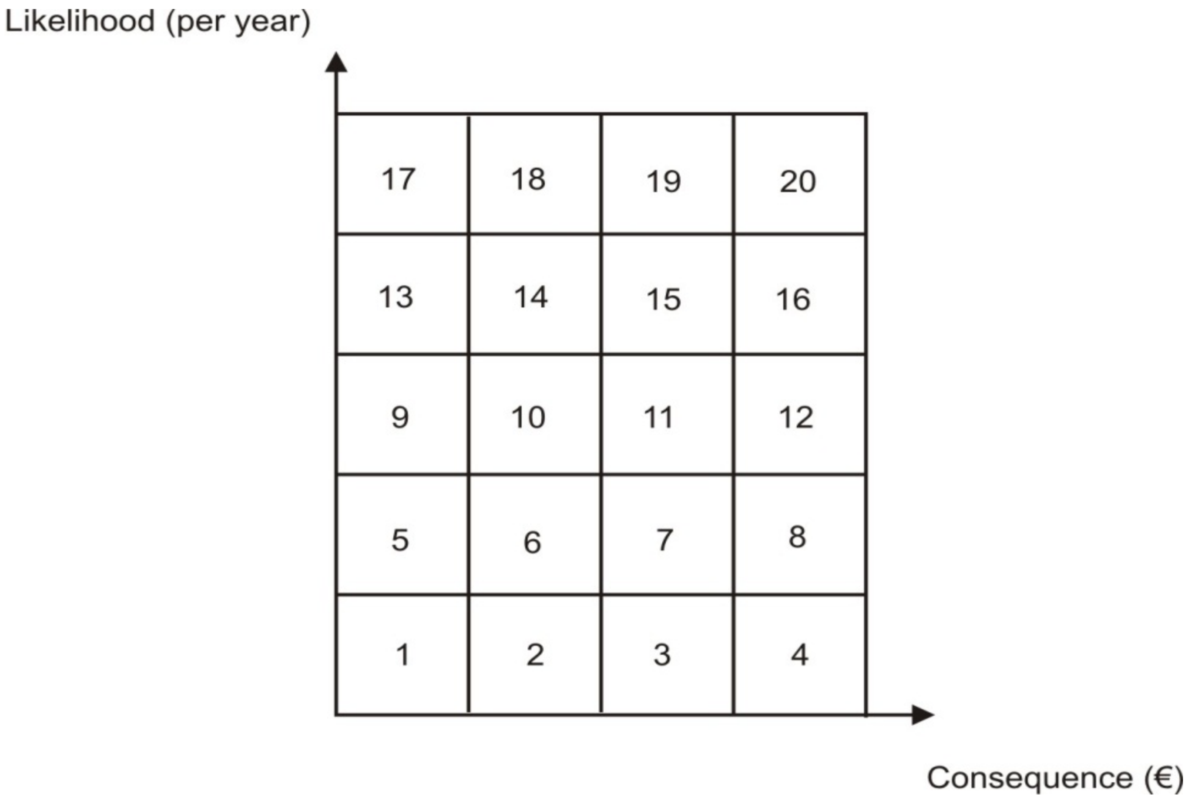

Figure 1: Discretization of the risk matrix

The risk matrix can be refined by relating the risk classes to a cost-benefits analysis. This way, a decision support instrument can be developed which can be used to determine, taking a certain safety budget into account, the risk reducing measures or precaution measures leading to the most optimal and cost-efficient result within an organization.

2. Basic model development

In industrial practice, companies are confronted with budget limitations. In this paper, we call the available yearly budget for safety prevention Bu_{tot}. Let us assume that; when possible prevention investments exceed this budget, they cannot be carried out. Therefore, only the preventative measures having a cost within Bu_{tot} will be considered in the model.
To employ a model for taking cost-efficient prevention decisions, certain actions should have been carried out by the user and certain input information is needed. All risks should have been classified into one of the risk cells of the risk matrix. Every cell *i* corresponds to a potential cell cost C_i, determined by:

$$C_i = l_i \times c_i \tag{1}$$

with:

C_i = *Costs resulting from an accident related to a risk from risk cell i*

l_i = *likelihood corresponding to risk cell i*

c_i = *financial impact (consequences) corresponding to risk cell i*

Table 1 illustrates the cost figures (expressed in € per year) for this article's risk matrix. Other risk matrix configurations are of course possible in real industrial practice. When precaution investments are made to decrease risks situated within cell *i* towards cell *j* (remark that *j* is characterized with lower consequences and/or likelihood), the potential cell costs become C_j. Hypothetical benefits in that case can be calculated as C_i - C_j.
The required information for application of the model is shown in Table 2.

Table 2: Required information for application of the model

n
Number of cells where risks do exist for the organization (= Nc; $Nc \subset n$ *)*
C_i
Bu_{tot}
Costs of Prevention for going from risk cell i to risk cell j (CoP_{ij}*),* $\forall i, j$ *whereby* $i \in Nc$

When all these data are known, it is possible to use the suggested approach to determine the most cost-efficient prevention measures. The first step of the model is the categorization of risks into the risk classes of the risk matrix. We assume that *Nc* risk cells (out of the *n* risk cells in total) contain one or more risks. Prevention costs to go from risk cell *i* to risk cell *j* (remark that $j < i$) are written as CoP_{ij}. If the prevention costs are higher than the yearly prevention budget Bu_{tot}, no investment will be made in these prevention measures, hence these prevention costs are excluded at the beginning of model execution. The hypothetical benefits corresponding to a decrease in risk cell from *i* to *j* are calculated by subtracting C_j from C_i.
Furthermore, a list of prevention measures will have been drawn up. Using this list, the optimal risk portfolio can be determined using optimization. In its simplest form, determining the optimal risk portfolio is equal to solving a knapsack problem. The knapsack problem derives its name from the fact that a person having to fill his fixed size knapsack with the most valuable items faces a similar problem. The knapsack problem is one of the most fundamental problems in combinatorial optimization and has many applications, e.g., in stock portfolio management, as well as many extensions.
In the basic version of this problem, a set of decision variables x_i is defined where variable x_i (corresponding to measure *i*) takes on value 1 if this measure *i* is chosen as part of the portfolio and 0 if it is not. A mathematical formulation of the knapsack problem is the following:

$$\max B_i x_i \tag{2}$$

$$s.t. C_i x_i \leq Bu_{tot} \tag{3}$$

$$x_i \in \{0,1\} \tag{4}$$

The first equation (2) expresses the total benefit from the selected portfolio, which should be maximized. The second equation (3) expresses the fact that the total cost of the selected measures should not exceed the budget. The third constraint (4) implies that a measure is either fully taken or not taken at all.

A number of assumptions are implicitly taken in this formulation:

- A measure is either taken or not (it cannot be partially taken);
- The total benefit of all measures taken is the sum of the individual benefits of the chosen measures;
- The total cost of all measures taken is the sum of the costs of the individual measures;
- Measures can be independently implemented, without consequences for the other measures.

Some of these assumptions are not entirely realistic. In the following section, we will suggest some possible model refinements.

Although the knapsack problem is NP hard[1], it can be solved efficiently even for very large instances (Martello *et al.*, 2000). The advantage of using the knapsack-based formulation is that it can be solved by standard off-the-shelf commercial software for mixed-integer programming, such as CPLEX (ES1) or Gurobi (ES2) or their open source counterparts such as GLPK (ES3) or lpsolve (ES4). Moreover, even spreadsheet software such as Excel or LibreOffice include a solver that can be used to model and optimize the safety measures portfolio using the method described in this paper.

In industrial practice, an optimal allocation of safety measures with maximum one prevention measure from each of the risk cells which can be assigned, needs to be determined by using the model. As explained before, to solve this problem, four conditions have to be met: (i) the total benefit of measures taken, needs to be maximized; (ii) the available budget constraint needs to be respected; (iii) maximum 1 decrease per risk cell is allowed; and (iv) a measure can be taken, or not. These conditions translate into the following mathematical expressions:

$$\max_{i,j} \sum B_{ij} x_{ij} \tag{5}$$

$$\sum_{i,j} CoP_{ij} \leq Bu_{tot} \tag{6}$$

$$\sum_{j} x_{ij} \leq 1 \tag{7}$$

$$x_{ij} \in \{0,1\} \tag{8}$$

Solving these equations for a certain concrete problem yields the optimal solution for the allocation of safety and security measures.

3. Model refinements

In general, the portfolio of safety measures chosen by a company is subject to a number of extra constraints, that express relationships between these measures. Fortunately, these relationships are generally easily added to the knapsack-based model, usually by introducing additional constraints.

For example binary relationships are possible: if risk cell *r* is decreased, risk cell *t* also has to be decreased and vice versa. This situation occurs when measures are mutually dependent on each other and taking one measure without the other makes no sense. An example is when the use of a new device that enhances safety requires training. It does not make sense to install the device without the training, and it does not make sense to give the training without installing the device.

---

1 An optimization problem is NP hard if the running time of the fastest known algorithm to solve it increases exponentially in the problem size.

This relationship between risk cell decreases from *r* to *s* and from *t* to *u* can be expressed in the model by the extra constraint

$$x_{r \to s} = x_{t \to u} \tag{9}$$

Another situation which might occur is the following: if risk cell *r* is decreased, risk cell *t* also has to be decreased, but the reverse is not true. As an example, to prevent fire from spreading between departments, a company is considering installing a fire-resisting door. The time the door resists fire can be increased by adding an extra layer of fireproof coating to it. Clearly, installing the coating and not the door makes no sense, but the reverse does.
The relationship between risk cell decrease *r→s* (installing the door) and risk cell decrease *t→u* (installing the fireproof coating) can be expressed as:

$$x_{r \to s} \leq x_{t \to u} \tag{10}$$

Yet another possible situation is that either risk cell *r* or risk cell *t* needs to be decreased, but not both risk cells at the same time. This situation can occur if two measures duplicate each others' effects and the company judges it superfluous to invest in both measures simultaneously. E.g., a machine can be protected by a concrete casing or a steel casing, but not by both.
This can be mathematically expressed as follows:

$$x_{r \to s} = 1 - x_{t \to u} \tag{11}$$

Yet other binary relationships are possible, and a mathematical answer for solving the knapsack problem can usually easily be found. Actually, in principle, all relationships between measures can be expressed as constraints in the knapsack problem.
However, for some situations, the benefits or costs of measures are not simply additive. Suppose e.g., that two fire doors can be installed in series to prevent fire from spreading to the next room. Clearly, the effect of installing one door instead of none will be larger than the effect of installing two doors instead of one. In other words, there will be a diminishing rate of return on the second door.
This can be easily handled by identifying such situations and creating "virtual" measures in the cost-benefit table to represent the action of taking both measures. To ensure that each measure is only taken once, some additional constraints are also necessary.

4. Conclusions and recommendations

Preventing occupational accidents in the industry, is an important expenditure on a yearly basis, for any organization. Optimizing prevention investments and making investment decisions in a cost-efficient way is therefore essential. To this end, we suggest a user-friendly knapsack-based model for taking cost-efficient prevention decisions. The model uses some data that can easily be determined by any organization and that can be displayed using a risk matrix. The most cost-efficient preventive measures are determined following the knapsack algorithm, given a certain prevention budget available.

References

CCPS (Center for Chemical Process Safety) (1992). Guidelines for Hazard Evaluation Procedures, 2nd Ed., New York: American Institute of Chemical Engineers.

Cook, R. (2007). Simplifying the creation and use of the risk matrix. In F. Redmill & T. Anderson (Eds.). Improvements in System Safety (p. 239-263). Bristol: Springer.

Cox, L. A., Jr. (2008). What's wrong with risk matrices. *Risk Analysis* 28(2).

Cox, L.A., Jr. (2011). Evaluating and improving risk formulas for allocating limited budgets to expensive risk-reduction opportunities. Accepted for publication (2011) in Risk Analysis. DOI: 10.1111/j.1539-6924.2011.01735.x

ES1 (2012). Accessible via: http://www.ibm.com/software/integration/optimization/cplex-optimizer/

ES2 (2012). Accessible via: http://www.gurobi.org

ES3 (2012). Accessible via: http://www.gnu.org/software/glpk/

ES4 (2012). Accessible via: http://lpsolve.sourceforge.net

Garvey, P.R. (2009). Analytical methods for risk management. A systems engineering perspective. Boca Raton, FL: CRC Press.

Martello, S., Pisinger, D., Toth, P. (2000). New trends in exact algorithms for the 0-1 knapsack problem. *European Journal of Operational Research* 123(2).

Middleton, M., Franks, A. (2001). Using risk matrices. *The Chemical Engineer* 34–37.

Smith, E., Siefert, W., Drain, D. (2009). Risk matrix input data biases. *Systems Engineering* 12(4).

Wilkinson, G., David, R. (2009). Back to the basics: Risk matrices and ALARP. In C. Dale & T. Anderson (Eds.). Safety-critical system: problems, process and practice (p.179-182). London: Springer.