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Comparing Attic Method with the Existing Techniques for Linear Programming

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The aim of the work is to compare the performances of the novel Attic method for linear programming (Buzzi-Ferraris, 2011) with the existing algorithms of the simplex and interior point families. Potentialities of the new method are demonstrated and quantified on the linear programming problem of thermal cracking refinery.

1. Introduction

Linear programming has been acknowledged for many years as the essential basis to face a wide number of optimization problems across a spectrum of different scientific and industrial areas, ranging from oil blending problems (Schrage, 1997; Mendez et al., 2006) to production scheduling (Afentakis et al., 1984; Barany et al., 1984; Dzielinski and Gomory, 1965; Lasdon and Terjung, 1971) to name but a few. Numerical roots can be traced back to the pioneering work done by Dantzig for the Simplex method (Dantzig, 1963; Dantzig, 1982; Dantzig and Orchard-Hays, 1954; Dantzig et al., 1955; Dantzig and Wolfe, 1961) and by Karmarkar for the Interior Point algorithms (Martin, 1999; Frisch, 1955; Fiacco and McCormick, 1968; Vanderbei, 2007).
The idea, on which the Attic method is based and for which we remind the reader to the pioneer dedicated paper of Buzzi-Ferraris (2011) for every numerical detail, is that whatever feasible point (vertex or nonvertex) sees the vertex where the solution is (it can be joined to the solution by a line). Often, this line must deviate from one or more constraints on which the working point is lying and pass through the attic to achieve the solution. If rather than moving from one vertex to another, one moves from one feasible point to another feasible point, large regions can be skipped before reaching a new vertex (connection of three tiles). This avoids many calculations. This strategy may seem similar to the one used by another important family of methods, the Interior Point algorithms (Martin, 1999; Vanderbei, 2007) which were developed to overcome the shortcomings of Simplex methods. Nevertheless, as it will be shown later, the Attic method is based on a totally different technique since it does not consider constraints as untouchable barriers, rather a specific constraint is touched at each iteration. Actually, in the Attic method, a direction that moves inside the feasible region and along which the function improves is adopted, and the search on is stopped when another constraint (tile) is encountered. The direction of search is selected by looking for the maximum function improvement. The new point on the roof is generally not a vertex and the number of active constraints for each iteration is usually smaller than the number of constraints required for a vertex. By iterating the procedure, the number of active constraints only sometimes equals the dimension of the linear programming problem n_V. Therefore, the Attic method could seem a middle course between the Simplex method and the Interior Point method: actually, the working point must not lay on a vertex,

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which makes the method to behave more like the Interior Point method, but, at the same time, the active constraints are actually satisfied, which makes to look like the Simplex method.

2. Thermal cracking refinery problem

The structure of the thermal cracking refinery proposed and solved by Manne (1963) is a well-established linear programming problem without any big issue in its solution (no degeneracy, zigzagging, cycling…). It is therefore the ideal application to test the normal performances of the novel Attic method with respect to the existing Simplex and Interior Point classes of methods, currently adopted to handle linear programming problems. The thermal cracking refinery consists of a crude-distillation column and associated steam strippers, a thermal cracking and fractionating system, and facilities for the blending and preparations of market products (Figure 1). The feed consists of a 38° API gravity Mid-Continent crude. The properties and amounts of the various materials present in this crude are listed in Table 1-2. The cracking section consists of four coils and a secondary fractionating column. There are provisions for the recycle of any portion of the three cracked distillate cuts. From the top of the column gasoline, gas and steam used for stripping are obtained. Such stream is sent to a condenser where gasoline and stream are condensed. Looking more in detail the cracking section, the cracked fraction per pass is taken to be 0.3 for distillates and 0.5 for residue (without considering the recycle). The yields for this section are given in Table 3. In the blending section the intermediate products are blended to form the desired final product respecting the specification reported in Table 4. In order to create the mathematical model, it is necessary to make an arbitrary choice: define the portion of the output of the primary distillation column (SL, SM, SH, SR) using an assigned cracking feedstock (FL,FM,FH,FR). This leaves as straight-run materials available for fuel oil blending (1-FL)SL, (1-FM)SM, (1-FH)SH e (1-FR)SR. Denoting by Zi each secondary distillation column output and with R_i each recycle, the cracking coils will process the following: $R_L Z_L$, $R_M Z_M$ and $R_H Z_H$.

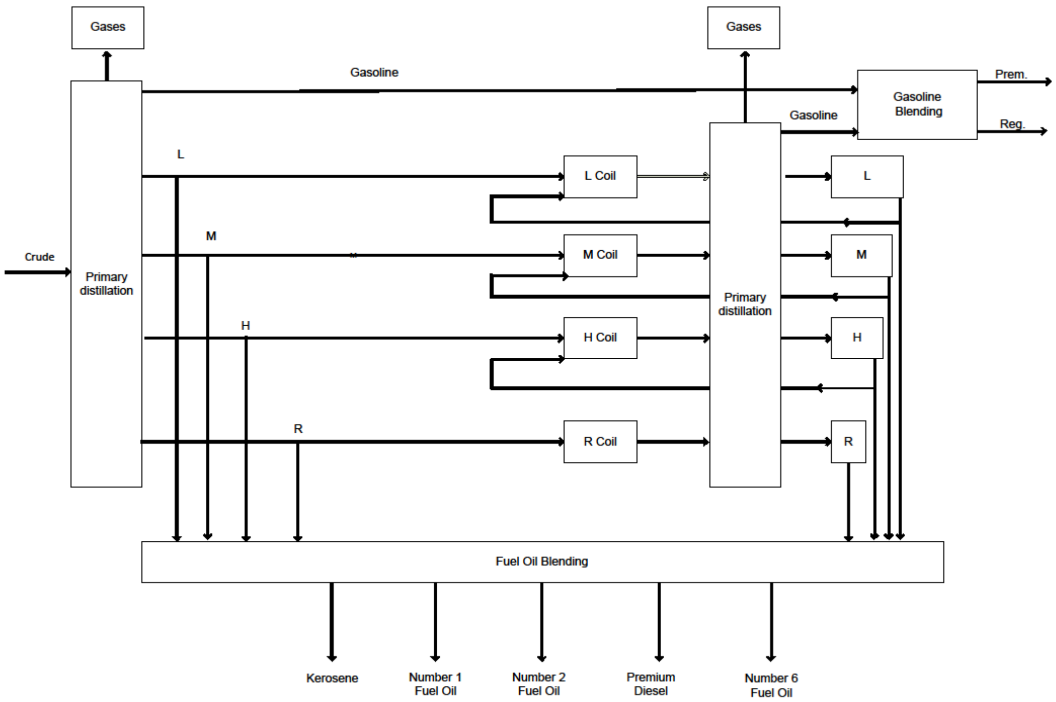

Figure 1: Manne's thermal cracking refinery

Table 1: Cuts from 38.0° API Gravity Crude Oil, Mid-continent (Mixed Base)

Cut	10%vol dist. temp. (°F)	End point (°F)	%w of crude	%w cumulative
C2 and lighter	-	-	0.02	0.02
C3 gases	-	-	0.22	0.24
C4 gases	-	-	1.46	1.70
Gasoline	150	400	24.8	26.5
Light distillate	400	520	14.6	41.1
Medium distillate	510	630	9.5	50.6
Heavy distillate	620	720	12.3	62.9
Residue	-	-	37.1	100.0

Table 2: Product streams, before blending

Material		10%vol dist. temp (°F)	End point (°F)	Specific gravity	Octane/Cetane number	Viscosity			
						At 100°F		At 122°F	
						cSt	μi	cSt	μi
Gasoline									
	Straight run	150	400	0.735	ON 58	-	-	-	-
	Cracked	150	400	0.755	ON 75	-	-	-	-
Light distillate									
	Straight run	400	520	0.816	CN 53	1.73	-0.422	1.44	-0.5214
	Cracked	400	520	0.840	CN 35	1.75	-0.419	1.45	-0.4919
Medium distillate									
	Straight run	510	630	0.830	CN 58	3.43	-0.215	2.6	-0.2965
	Cracked	510	630	0.865	CN 35	3.37	-0.220	2.65	-0.2853
Heavy distillate									
	Straight run	620	720	0.855	CN 61	7.4	-0.044	5.1	-0.1215
	Cracked	620	720	0.916	CN 40	11.2	0.030	7.5	-0.0417
Residuum									
	Straight run	-	-	0.944	-	-	-	88	0.2895
	Cracked	-	-	1.022	-	-	-	3.3	0.5464

Table 3: Yields of conversion products, single-pass cracking (%w of oil converted)

Yields	Light	Medium	Heavy	Residue
Straight-run charge stocks				
Gas	15	15	15	5
Gasoline	76	67	67	34
Light cycle oil	-	1	9	19
Medium cycle oil	4	-	5	21
Heavy cycle oil	3	4	-	21
Residue	2	3	4	-
Total	100	100	100	100
Total converted, %w of charge	30	30	30	30
Cycle oil charge stocks				
Gas	18	16	16	
Gasoline	51	48	48	
Light cycle oil	-	8	8	
Medium cycle oil	13	-	12	
Heavy cycle oil	10	14	-	
Residue	8	14	16	
Total	100	100	100	
Total converted, %w of charge)	30	30	30	

Table 4: Product streams, after blending

End product	Permissible components	Specifications						
		Maximum specific gravity	Minimum ON/CN	Market price (cents/gallon)	Viscosity At 100°F cSt	Viscosity At 100°F μi	Viscosity At 122°F cSt	Viscosity At 122°F μi
Premium gasoline	SR/cracked gasoline, C4 gases	-	88 ON	11.56		-	-	
Regular Gasoline	SR/cracked gasoline, C4 gases	-	82 ON	10.44		-	-	
Propane-propylene gases	C3 cut	-	-	4.00	-	-	-	
Butane-butylene gases	C4 cut	-	-	9.00	-	-	-	
Kerosene	SR light distillate	-	-	8.88	-	-	-	
Fuel oil 1	SR/cracked light and medium distillates	0.850	-	8.44	1.9	-0.400	-	
Fuel oil 2	SR/cracked light, medium, and heavy distillates	0.882	40 CN	7.88	4.3	-0.160	-	
Premium diesel oil	SR and cracked light, medium, and heavy distillates	0.840	55 CN	8.63	2.6	-0.290	-	
Fuel oil	SR and cracked light, medium, and heavy distillates; SR and cracked residues	1.014	-	2.26	-	-	375	0.411
Dry fuel gases	C2 cut		-	0.2 cents/ pound	-	-	-	

In order to evaluate the yields the following procedure must be performed:

- Arbitrary choice of F_i.
- Arbitrary choice of R_i.
- Solve with respect to Z_i the following equations:

$$Z_L = 0.1022F_L + 0.00314F_M + 0.00322F_H + 0.0352F_R + 0.7R_L Z_L + 0.024R_M Z_M + 0.024R_H Z_H \quad (1)$$

$$Z_M = 0.00175F_L + 0.0665F_M + 0.00185F_H + 0.039F_R + 0.039R_L Z_L + 0.7R_M Z_M + 0.036R_H Z_H \quad (2)$$

$$Z_H = 0.00131F_L + 0.00114F_M + 0.0861F_H + 0.0390F_R + 0.03R_L Z_L + 0.042R_M Z_M + 0.7R_H Z_H \quad (3)$$

- Calculation of the net material balance:

$$C_L = Z_L\left(1 - R_L\right) \quad (4)$$

$$C_M = Z_M\left(1 - R_M\right) \quad (5)$$

$$C_H = Z_H\left(1 - R_H\right) \quad (6)$$

$$C_G = 0.00657F_L + 0.00428F_M + 0.00554F_H + 0.00928F_R + 0.054R_L Z_L + 0.048R_M Z_M + 0.048R_H Z_H \quad (7)$$

$$C_{G_{aso}} = 0.0333F_L + 0.0191F_M + 0.0247F_H + 0.0631F_R + 0.1503R_L Z_L + 0.144R_M Z_M + 0.144R_H Z_H \quad (8)$$

$$C_R = 0.0009F_L + 0.009F_M + 0.0015F_H + 0.185F_R + 0.024R_L Z_L + 0.042R_M Z_M + 0.048R_H Z_H \quad (9)$$

The streams must be blended to meet the specifications on each product as follows:

- Gasolines: there are two market grades of gasoline derived from blends of straight-run gasoline, cracked gasoline, butane and tetraethyl lead fluid. The specifications to meet are: maximum Reid vapor pressure for both the types, 10 pounds per square inch, and minimum octane numbers of 82 and 88, respectively. Since butane is available at lower cost, it is estimated that 10 %vol of butane allows to meet the vapor tension specification on the octane number exactly.

- Kerosene: kerosene is made by straight-run light distillate without any more treatments.
- Distillate oil fuel 1: is made from a blend of straight-run and cracked light and medium distillates. The specifications to meet are a maximum viscosity of 1.9 centistokes at 100 °F and a maximum specific gravity of 0.850.
- Distillate oil fuel 2: is made from a blend of straight-run and cracked light, medium, and heavy distillates. The specifications are the following: viscosity less than 4.3 centistokes at 100 °F, a maximum specific gravity of 0.882; a minimum cetane number of 40; a flash point greater than 100 °F; an ASTM distillation end point at a maximum of 675 °F.
- Premium diesel fuel: This product is to be made from a blend of any of the six distillate materials, and must satisfy the following conditions: viscosity less than 2.6 centistokes at 100°F; specific gravity less than 0.840; cetane number greater than 55; 90 percent over in ASTM distillation at a maximum of 585 °F; and end point at a maximum of 646 °F.
- Number 6 bunker fuel oil: This product is composed primarily of straight-run and cracked residues. The specifications are as follows: viscosity less than 375 centistokes at 122 °F and specific gravity less than 1.014

3. Numerical comparison

Manne's problem has been solved using certain well-known algorithms. Hence specific programming languages have been used and code samples have been developed in this research activity so as to run them on the same machine. We selected some of the most common algorithms and languages to implement the Manne's refinery:

- Simpo (Vanderbei, 2007): based on the Simplex method
- CPLEX (IBM): which benefits from both the Simplex and Interior Point methods
- MINOS 5.5 (AMPL)
- Matlab (MathWorks): the linprog function is used and it is possible to select both the Simplex and the Interior Point method simply switching an option.
- The Attic method.

As reported in Figure 2, the number of iterations declared in the original article is similar to the one needed to Simpo e MINOS 5.5 to solve the problem. CPLEX solver, using non-linear techniques, is able to halve it. Matlab performances are even more interesting, with just 15 (Simplex) and 13 (interior Point) iterations. Attic results are even better: only 4 iterations are required to solve this problem.
Given the small size of the problem, no consideration could be made about the computing times of the different solver, a parameter particularly important for the industrial application, so it is necessary to perform further tests on the Attic method, either by solving a larger scale problem either solving a MILP problem where a large number of LPs has to be solved.

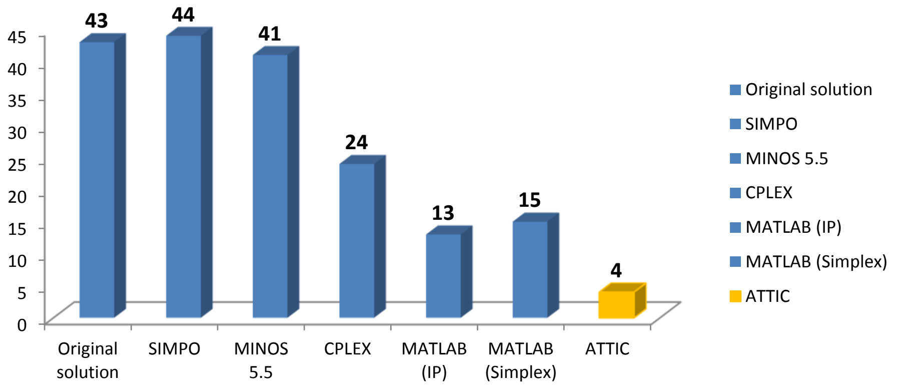

Figure 2: Numerical comparison. Attic method requires much less iterations than the existing methods

4. Conclusions

The Attic method results to be the most efficient method in terms of number of iterations with respect to well-known algorithms and packages. The gap with the existing algorithm in the number of iterations seems to be relevant, hence, further investigations are needed to compare the different algorithms in terms of calculation time and on different problems. For instance, an ongoing research activity with Linde Gas is focused on the industrial application of the Attic method and to the quantification of numerical performances with respect to the traditional algorithms.

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