Extending Process-Network Synthesis Algorithms with Time Bounds for Supply Network Design

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It is highly desirable to develop a software for estimating the quality of supply networks and synthesizing potential alternative realizations to improve the quality of service or to satisfy service level agreements. In the present paper, methodology, algorithm, and software are proposed to improve supply networks where the quality is measured by cost and response time. The methodology is based on the combinatorial foundations of algorithmic process synthesis. Structure generation and evaluation of a supply scenario are the two major steps of the algorithm elaborated for synthesizing the optimal network structure. The algorithm, besides generating alternative structures, takes into account alternative orders of the activities as well. The structural alternatives are evaluated on the basis of the cost and duration of each individual operation. The cost and response time for a single operation are expressed as concave functions of its volume.

1. Introduction

It has been shown that the P-graph approach to process-network synthesis (PNS) originally conceived for conceptual design of chemical processes (Friedler et al. 1992, 1993, 1995, 1996, 1998) provides appropriate tools for generating and analyzing structural alternatives for supply scenarios (Barany et al. 2010; Klemeš et al. 2010; Lam et al. 2010). However, extension of the original framework to handle constraints specific to supply scenarios may improve the practical applicability of the proposed methodology. To satisfy the deadline is a crucial aspect in supply chain design. In the present paper, time constraints on the availability of the resources, duration of the activities, and deadlines for the final targets are incorporated into the mathematical model as well as into the solution algorithms of PNS.

2. P-graph Framework for Algorithmic Process-Network Synthesis and Optimization

The approach based on the P-graph framework appears to be the only one being capable of executing process-network optimization giving rise to an algorithmically and mathematically proven solution for all steps involved, comprising superstructure generation, construction of the mathematical model, optimization, and the solution interpretation. In the P-graph framework, algorithm MSG produces the maximal structure, i.e. the superstructure, for the PNS problem (Friedler et al. 1992). This maximal structure serves as the input to the generation and solution of the mathematical model by algorithm ABB (Friedler et al. 1996).
3. Parametric PNS Problem with Fix Charged Linear Cost Functions

The combinatorial components of a PNS problem are given by a triplet \((P,R,O)\) where there exists a set \(M\) for which \(P \subseteq M\) is the set of final targets to be achieved, \(R \subseteq M\) is the set of resources, and \(O = \varphi(M)\times \varphi(M)\) is the set of candidate activities to form a network and reach each of the final targets by deploying any of the available resources. Each activity is defined by its preconditions and outcomes. A precondition can be the availability of a resource or an outcome of another activity. It is assumed that \(P \cap R = \emptyset\). A parametric problem definition with a fixed charged linear cost function for the volume of the activities is detailed below (Barany et al. 2011).

The lower bound \(L_p\) on the gross result is greater than zero for each final target \(m\), and it is equal to zero for any other entity: \(L_p = \begin{cases} > 0, & \forall m \in P \\ 0, & \text{otherwise} \end{cases}\). The upper bound \(U_p\) on the gross result for each resource \(m\) is equal to zero, and greater than or equal to \(L_p\) for any other entity: \(U_p = \begin{cases} 0, & \forall m \in R \\ \geq L_p, & \text{otherwise} \end{cases}\). The upper bound \(U_c\) on gross utilization of resource \(m\) is greater than zero, and equal to zero for any other entity: \(U_c = \begin{cases} > 0, & \forall m \in R \\ 0, & \text{otherwise} \end{cases}\).

The upper bound \(u_i\) for the volume of each activity \(a\) and the price \(c_m\) for each resource or target are also given. The cost of an activity is estimated by a linear function of its volume with a fixed charge. For the cost function of each activity the proportionality constant \(c_p\) and the fixed charge \(c_f\) are defined.

The lower bound \(L_i\) and the upper bound \(U_i = U_p\) denote the balance on result vs. utilization of the resources and outcomes of the activities. Furthermore, in the optimal structure let \(m^o \subseteq M\) denote the set of entities and \(o^o \subseteq O\) the set of activities. The relations between the entities and activities are given by parameters \(a_i\) denoting the difference of the volumes of entity \(m\) resulted and utilized by activity \(o\). \(x^o\) is the vector of the optimal volumes of activities for the problem, and \(z^o\) is its objective value. The aim is to determine the network \((m^o,x^o,z^o)\), which satisfies the following conditions (Eq. 1 – Eq. 6) where \(z^o\) is minimal as indicated in Eq. 7.

\[
m^o = \varphi(o^o), \text{ i.e., let } m^o \text{ be the set of entities resulting from or utilised by at least one activity in } o^o. \tag{1}
\]

\[
x^o = [x^o_1, x^o_2, \ldots, x^o_t]\tag{2}
\]

\[
0 < x^o_i \leq u_i \Leftrightarrow o^o_i \in o^o\tag{3}
\]

\[
\forall m_i \in m^o \cap R: -U_{c_i} \leq \sum_{i \in m^o} a_i x^o_i \leq 0 \tag{4}
\]

\[
\forall m_i \in m^o \cap P: L_{p_i} \leq \sum_{i \in m^o} a_i x^o_i \leq U_{p_i} \tag{5}
\]

\[
\forall m_i \in m^o \setminus R \setminus P: 0 \leq \sum_{i \in m^o} a_i x^o_i \leq U_{p_i} \tag{6}
\]
\[ z^* = \sum_{(a, \beta_i) \in \alpha} \left( ct + x_i^* \cdot \left( cp - \sum_{m \in \alpha, \beta_i} \right) \right) \]

4. Time Constrained PNS (TCPNS)

In the time constrained extension of PNS (TCPNS), each target must be achieved in time while the availability of the resources is constrained and the duration of each candidate activity is given as a fixed charged linear function of its volume. To the parametric PNS problem with fixed charged linear cost functions four additional parameters are given to define time constraints. \( tf \) is the fixed part; \( tp \) is the proportionality constant of the function, which estimates the duration of an activity based on its volume, respectively; \( Ut_j \) is the deadline for each result: \[ Ut_j = \begin{cases} \geq 0, \forall m_j \in P \\ \max \{ Ut_j \} \text{otherwise} \end{cases} \] and \( Lt_j \) is the time of the earliest availability of a resource: \[ Lt_j = \begin{cases} \geq 0, \forall m_j \in R \\ 0, \text{otherwise} \end{cases} \] which satisfies the following conditions (Eq. 1 – Eq. 6 and Eq. 8 – Eq. 10) where \( z^* \) is minimal as given in Eq. 7.

\[ \forall m_j \in M: Lt_j \leq t_{m_j} \leq Ut_j \] (8)

\[ \sigma_o = (\alpha, \beta_i) \in \alpha, \forall m_j \in \alpha_i: t_{m_j} \geq t_{o_i}, \text{i.e., the starting time } t_{o_i} \text{ of an activity } \sigma_o \text{ cannot precede the time of availability } t_{m_j} \text{ of any of its precondition } m_j; \] (9)

\[ \sigma_o = (\alpha, \beta_i) \in \alpha, \forall m_j \in \beta_i: t_{m_j} \geq t_{o_i} + tf_i + tp_i \cdot x_i, \text{i.e., the time of availability } t_{m_j} \text{ of any outcome of an activity } \sigma_o \text{ cannot precede the sum of the starting time } t_{o_i} \text{ and the duration } (tf_i + tp_i \cdot x_i) \text{ of the activity } \sigma_o. \] (10)

5. Structural Examination

In order to introduce time constraints in process synthesis, the structural model needs to be extended to express whether the outcome from an activity and precondition to another activity precede each other or not. As a result, artificial activities are included to the maximal structure; see, e.g., activities Ot1 and Ot2 in Figure 1. Note that including to or excluding from the solution structure these artificial activities clearly represent the decisions on the precedence; see Figure 1.

For the example in Figure 1, the deadline \( tb \) can only be satisfied if the generation of targets \( m_5 \) and \( m_6 \) are independent. It can be achieved by a solution structure where precondition \( m_4 \) for activity \( O4 \) is exclusively provided by activity \( O2 \), i.e., \( O4 \) does not wait for \( O1 \) to generate its results.
Figure 1: Extension of the Maximal Structure
6. Relaxation of Time Constraints

In the relaxed model of TCPNS, Eq. 10 is relaxed as Eqs. 11 and 12, where $tb$ denotes the latest deadline among those upper bounds defined for the final targets; thus,

$$\text{max} \{ \text{Ut}_i \} _{m, f} = \sum_{i, \beta} a_{i, \beta} \text{max} \{ \text{Ut}_i \} _{m, f} \quad \text{(11)}$$

$$y_i = 0 \Rightarrow t_{\alpha_i} \geq t_b - \delta$$

$$y_i = 1 \Rightarrow t_{\alpha_i} \geq t_b + x_i * t_p + t_f$$

Note that $y_i$ is a binary variable expressing the existence ($y_i = 1$) or the absence ($y_i = 0$) of activity $a_i$ in the structure. In the solution procedure prior to complete decisions on the existence or absence of the activities, the lower bound on the cost and the duration of alternative scenarios are estimated at each step. For the estimation, $y_i$ is relaxed as a continuous value in range $[0, 1]$, and its relation to the volume of the activity is expressed as:

$$x_i \geq u_i * y_i$$

The inequality in Eq. 12 is equivalent to Eq. 13 or Eq. 14 depending on whether $y_i$ is equal to 0 or 1. Note that constraint in Eq. 12 comes into play only if the value of $y_i$ is close to 1 in the relaxed model.

7. Conclusion

The current work presents a methodology to model formally supply chain networks and to algorithmically synthesize optimal supply scenarios by the P-graph framework. Recent extension introduces time constraint in the mathematical model and solution method for PNS. Need for extending the superstructure and the relaxed mathematical model for solving PNS problems with fixed charged linear cost function has been satisfied. A method for representing decisions on the precedence of activities by structural decisions has been introduced.

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References


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