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Physical Modeling and Control of a Distributed Parameter System

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The aim of this work was to test model based controllers with pilot-scale water heating equipment. The controlled variable is the outlet temperature, and the manipulated variable is the volumetric flow rate. The main disturbances are the heat duty and the inlet temperature. The power of the electric heater can be manipulated to imitate solar radiation, thus our system could be the physical model of a solar collector.
The first principle based dynamic modeling of the system yields partial differential equations (PDEs). One way to simplify the model is integrating the heat balance equation along the geometric space, and estimating the needed average variables using the measured signals. Another approach is to divide the geometric space into discrete units, thus transforming the PDE into a system of ordinary differential equations (ODEs).
Controller structures were synthesized using the constrained inversion technique, which yields nonlinear feed-forward controllers that accomplish a pre-defined control specification (relation between the setpoint and the controlled variable). Perfect models are not available, and IMC structure has been used for feedback compensation of the model error. Constrained PI controller has been used as reference.
The effect of different modeling techniques and the different control specifications were examined. All the model based controllers outperformed the fixed parameter PI controller.

1. Introduction

Controlling distributed parameter systems (DPS) has always been a challenge, as concentrated parameter models are most commonly used to describe systems, and many times linearity is also assumed. A great question of the early attempts to model and control DPS is to reduce the model consisting of PDEs to a set of ODEs, and if possible, linearization of these equations (Padhi and Ali, 2009).
In this paper the behavior of a pilot-scale water heating equipment is studied. The model, that is presented later, shows resemblance to that of solar collectors. In Camacho et al. (2007a and 2007b) a thorough review summarizes the achievements in the control of industrial solar collectors. In recent applications nonlinearity is neutralized by gain scheduling, feed-forward and predictive control, while model error is reduced by adaptation, and by using new model types, like fuzzy and neural network models.
It is a problem in the study of solar collector control, that measurements are not reproducible due to weather conditions. Another problem is that each measurement lasts

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long, probably for several days, and if reliable simulators are not present, it is very time consuming to test controller algorithms. Our physical system is a good compromise in transition from simulators to the physical controlled system, but with more independence from the environmental effects compared to solar collectors.

2. The controlled physical system

In the system of our case-study there is a tube, in which an electric heater is used for heating up water flowing through. The flow rate can be adjusted by a control valve. The flow rate is measured by an orifice plate and the temperatures of the inlet and outlet flow are also measured. The power of the electric heater can be adjusted with a PWM signal.

Most of the signals are transmitted via Fieldbus H1. Only the PWM signal of the heater is driven by the conventional 4-20 mA analog output. OPC servers were also used to transfer data, the main control strategy was implemented in MATLAB Simulink environment, and the interface to physical system was Experion PKS.

The aim of the control was to keep the outlet temperature as close as possible to the reference signal, by using the control valve as manipulator. Measured disturbances are the heating power and the temperature of the inlet flow. Ambient temperature and heat loss to the environment are examples for unmeasured disturbances, though none of these had severe effect on the system.

3. Modeling the system

First we started our work with building the first-principle model of the process. Only the heat transfer in the liquid phase was analyzed, as our goal is to control the outlet temperature of the liquid flow. Perfect plug flow of the liquid without axial mixing was assumed. It was also assumed that the fluid is perfectly mixed in radial direction, thus the geometric space could be reduced to one dimension, the length. Following these assumptions the resulting model for an inner point of the system is:

$$\frac{\partial T}{\partial t} = -\frac{F}{A}\frac{\partial T}{\partial x} + \frac{Q}{V\rho c_p} \tag{1}$$

Where T is the temperature of the liquid (°C), F is the volumetric flow rate (m^3/s), A is the area available for the flow (m^2), Q is the power of the heater (W), V is the volume of the equipment between the two thermometers (m^3), ρ is the density of the liquid (kg/m^3), c_p is the specific heat capacity of the liquid (J/(kg°C)), t is the time (s), x is the length coordinate (m).

The initial state and the boundary condition are (starting from steady state):

$$\mathrm{T}(0,\mathrm{x}) = \mathrm{T}_{\mathrm{in}}(0) + \frac{\mathrm{Q}}{\mathrm{V}\rho \mathrm{c}_{\mathrm{p}}}\frac{\mathrm{Ax}}{\mathrm{F}} \qquad T(t,0) = T_{in}(t) \tag{2}$$

Where $T_{in}(t)$ is the inlet temperature (°C), function of time.

For control purposes it is difficult to handle PDEs, thus we should approximate it with ODEs. The first approach is dividing the system into multiple perfectly mixed domains (cascades). In this case all these domains are identical in structure and size, and each one contains one state variable, the temperature. The model equation of one domain is the following:

$$\frac{dT_i}{dt} = -\frac{nF}{V}\left(T_{i-1} - T_i\right) + \frac{Q}{V\rho c_p} \qquad i = 1,2,\ldots,n \tag{3}$$

Where n is the total number of domains, and index i marks the number of the actual domain. T_0 means the inlet temperature, T_n is the outlet temperature.
This is a common approximation of PDE (Han-Xiong Li and Chenkun Qi, 2010), and as n rises the solution of the ODE set converges to that of the PDE. In Ding et al. (2009) this effect is described in relation with identification. In our case n=20 was found reasonable according to simulations.
Another approach of model reduction is to eliminate the length coordinate by introducing average variables. In our case the average temperature and its approximation is:

$$\theta = \frac{1}{L}\int_0^L T dx \approx \frac{T_{in} + T_{out}}{2} \tag{4}$$

Where L is the total length of the tube. If we integrate the original balance equation (1) by the length coordinate the resulting equation is as follows:

$$\frac{d\theta}{dt} = \frac{F}{V}\left(T_{in} - T_{out}\right) + \frac{Q}{V\rho c_p} \tag{5}$$

By substituting and rearranging the (4) equation we get:

$$\frac{dT_{out}}{dt} = \frac{2F}{V}\left(T_{in} - T_{out}\right) + \frac{2Q}{V\rho c_p} - \frac{dT_{in}}{dt} \tag{6}$$

Although we constructed first principle models of the process, we have to understand that the measurement devices and manipulators also affect the observed dynamics of the process. In our case these time constants are of the same magnitude as those of the process itself. This is the reason why the first attempts, neglecting dynamics other than the proper process, described poorly the observed dynamic behavior.
No further a priori knowledge is available about the time delays coming from the measurements, process control system and the manipulators, thus we have to identify filter parameters using measured data sets. The measurement signals can be seen on Fig.1.
The electric heater has linear characteristics, and fast dynamics, but the communication system caused a pure time delay on the signal. This time delay was greater than the time delay of the control valve. The valve also had fast dynamics except for the pure delay. The thermometers have latency because heat reaches the case earlier than the thermally

sensitive part. This effect can be modeled with a first order filter. In the case of the inlet temperature, which is an input of the model, the filter has to be inverted, and a lead-lag was built in for this purpose.
The calculated and measured outlet temperatures were compared (Fig. 1), and a numeric optimizer was used for minimizing the sum of square errors, varying the parameters of the above mentioned elements. The identification did not affect the steady-state characteristics, and it is a great achievement, that the model constructed solely from a priori information described the steady states satisfactory.

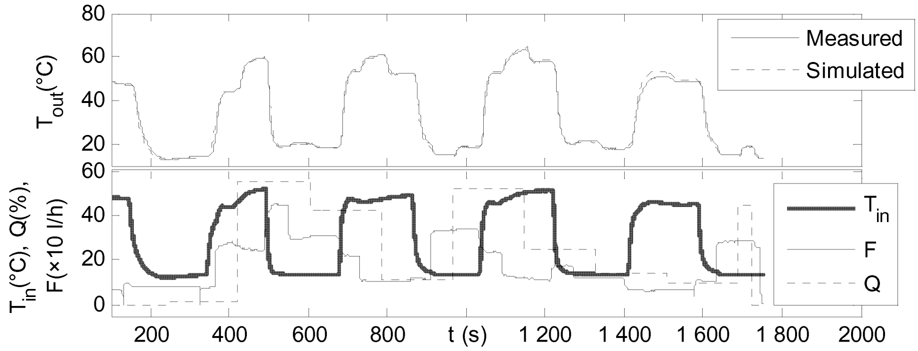

Figure 1: Measurement data set and simulation for identification

4. Controller synthesis

The controller algorithms to be designed are compared to a constrained PI (Abonyi et al., 2005) controller, with parameters optimized for the overall operating range. Two controllers were tested, each based on model equation (3) and (6). In both cases the same method was applied. Based on the model the constrained inverse (Szeifert et al., 2007) was set up (feed-forward), and a standard IMC structure was used for model error correction (feed-back). When inverting the models, a specification of the same order as the relative order of the system needs to be set up. The relative order in both models is one, as the manipulated variable (F) appears explicitly in the first derivative of the controlled variable (T_{out}). The inversion rule is:

$$\tau_c \frac{dT_{out}}{dt} + T_{out} = T_{ref} \tag{7}$$

Where T_{ref} is the reference signal, τ_c is time constant tuning the controller. By substituting the derivative from model equations (3) and (6) and rearrange the equation, we get an algebraic equation for the manipulated variable:

$$F = -\frac{V}{n(T_n - T_{n-1})}\left(\frac{T_{ref} - T_n}{\tau_c} - \frac{Q}{\mathrm{V}\rho c_{\mathrm{p}}}\right) \qquad \text{from cascade model} \tag{8}$$

$$F = -\frac{V}{T_{out} - T_{in}}\left(\frac{T_{ref} - T_n}{2\tau_c} - \frac{Q}{\mathrm{V}\rho c_\mathrm{p}} + \frac{1}{2}\frac{dT_{in}}{dt}\right) \qquad \text{from integrated model} \qquad (9)$$

It should be noted that the two control equations are very similar, but eq. (9) counts with the effect of the measured disturbance T_{in}, adding extra information about the system. In the denominator a temperature difference appears. To ensure robustness only positive values are accepted for this term, otherwise they are substituted with a small positive value that is 0.01 °C. The manipulated variable is constrained in the 50-300 L/h interval.

Results and discussion

A measurement was carried out to test the controllers both in servo and regulatory mode. Response for the changes in the reference temperature was studied along with response for changes in the heating power, to ensure steady state feasibility (Fig. 2). There were both step and continuous changes in the signals. The effect of inlet temperature was studied separately (Fig. 3). The disturbance signal slightly differs in these measurements. The applied disturbances were quite large.

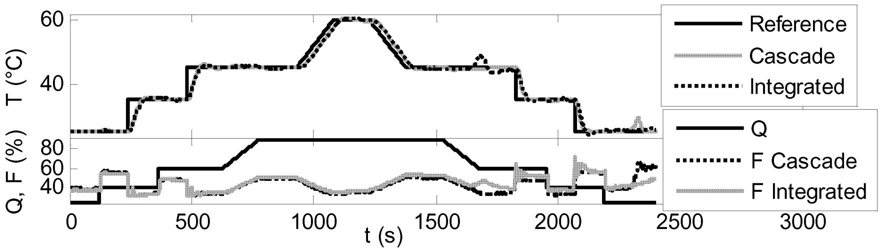

Figure 2: Measurement data set for servo and heat disturbance rejection

Although the parameters of the constrained PI controller were optimized, the results are poor for servo (not represented on Fig. 2) and for both disturbance compensations. It might be unfair to apply linear controller on a nonlinear system, but it is a common practice in the industry. Maybe it would perform better in a narrow interval around the operating point.

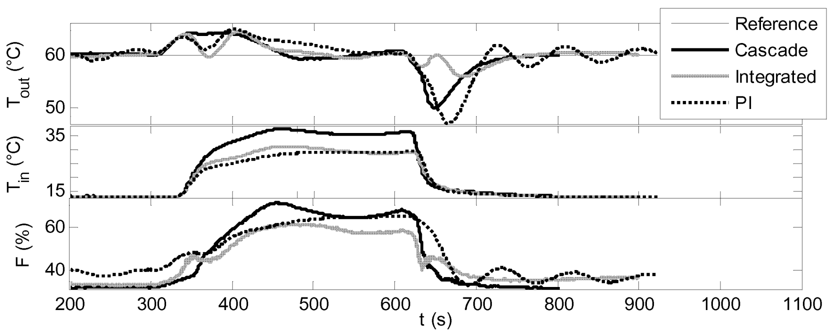

Figure 3: Measurement data set for inlet temperature disturbance rejection

The IMC controllers performed much better, and the outlet temperature signal is hard to be distinguished in the two cases for the servo and heat disturbance compensation. For inlet temperature compensation the integrated model based controller is faster in settling, and the manipulator is more dynamic, just as we expect from the inverse functions. The better performance of the integrated model based controller can be explained by the introduction of extra information about inlet temperature.

5. Summary

Pilot-scale water heating equipment was successfully modeled by first-principle models. Two ways of model reduction was introduced, and inverse controller was synthesized using both. The controllers were tested both for servo and for regulatory mode. Both controller structures outperformed fixed parameter PI controller, and some differences were observed when the inlet temperature was the load.
The physical system has proven to be useful in the study of model based controllers, and can be a potential step towards testing solar collector controller algorithms in an economically friendly way.

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